ISSN 2410-2547
DOI: 10.32347/2410-2547.2020.104

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

КИЇВСЬКИЙ НАЦІОНАЛЬНИЙ УНІВЕРСИТЕТ
БУДІВНИЦТВА І АРХІТЕКТУРИ

# ОПІР МАТЕРІАЛІВ І ТЕОРІЯ СПОРУД

# STRENGTH OF MATERIALS AND THEORY OF STRUCTURES

Науково-технічний збірник
Scientific-and-technical collected articles

Випуск
Issue **104**

Заснований у 1965 р.

КИЇВ 2020

УДК 539.3/6
ББК 30.121+38.112
О-61



Рекомендовано до випуску Вченою радою Київського національного університету будівництва і архітектури 10 вересня 2020 р. (протокол № 33).

**Опір матеріалів і теорія споруд:** Науково-технічний збірник. – Вип. 104 / Голов. ред. В.А. Баженов. –К.: КНУБА, 2020. – 330 с. – Укр. та англ. мовами.

У збірнику наведено статті з результатами досліджень у галузі опору матеріалів, будівельної механіки, теорії пружності і пластичності. Особливу увагу приділено розробці й розвитку методів розрахунку міцності, стійкості, динаміки просторових конструкцій з урахуванням геометричної нелінійності, пластичних властивостей руйнування матеріалів; питанням чисельної реалізації рішень; дослідженню напружено-деформованого стану тіл складної структури при сталих і змінних у часі навантаженнях, включаючи випадкові впливи.

Призначений для наукових працівників, викладачів, виробничників, докторантів, аспірантів та студентів.

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УДК 539.3/6
БББК 30.121+38.112
О-61

UDC 624.04

# KYIV SCHOOL OF THE THEORY OF STRUCTURES[1]

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DOI: 10.32347/2410-2547.2020.104.3-88

The paper presents a review of more than a century-long history of Kyiv school of the theory of structure, the foundation of which was laid by world-famous scientists V.L. Kirpichov and S.P. Tymoshenko. The birth of the Kyiv scientific school of the Theory of structures is associated in this paper with the establishment at the Kyiv Polytechnic Institute the Strength of Materials Department. It is noted that further formation and development of the theory of structures was facilitated by the creation in 1918 of the Ukrainian Academy of Sciences, the Institute of Mechanics of the NAS of Ukraine, expansion of relevant research in higher education institutions, creation of new academic and sectoral research institutions, most of which is located in Kyiv. The contribution of Kiev scientists to the development of methods for analyzing spatial structures of bar and shell type, their inelastic behavior, as well as dynamics and stability is reflected.

Particular attention is paid to the fundamentally new opportunities for the development of the theory of structures in the era of numerical analysis. The successes of Kiev mechanics in the field of development and improvement of structure analysis numerical methods, such as the finite difference method and various modifications of finite element methods, are emphasized. Kiev engineers and scientists are also known for their developments in the field of design and calculation of modern cable-stayed structures, as well as optimal design. The activities of the scientific school of structural mechanics of the Kyiv National University of Construction and Architecture are also covered in the review.

In the final part of the paper the new issues connected with the justification of calculation models and the analysis of reliability of constructions are considered. Some of this problems are dictated by the demands of practice, in particular those that arosed in the process of Chernobyl New Safe Confinement designing.

The publication contains a wide bibliography.

**Keywords:** bar systems, stability, shells, structural mechanics, finite difference method, finite element method, calculation model.

**Foreword.** Theory of structures is complex discipline which largely lost its original essence of the “teachings about the life of structures”[2]. Now it is

[1] Dedicated to the 90th Anniversary of the Kyiv National University of Construction and Architecture

[2] Such a definition is given in the book by P.A. Velikhov "Theory of engineering structures" (Moscow: Gosstroyizdat, 1924).

identified with a number of other disciplines such as strength of materials, structural mechanics, theory of elasticity and theory of plasticity (strength disciplines) and general sections of the engineering analysis of the bearing structures. Many of these disciplines are actively engaged in their own lives, naturally, serving the theory of structures, but often focused on some of the internal problems of general scientific orientation. So, to the theory of structures that is adjacent (but, in our opinion, are not included as part of) to the analysis of mechanical properties of structural materials, as well as such problems of mechanics of solids as the mathematical theory of elasticity, theories of plasticity, thermoelasticity, thermoplastic elasticity, etc. There is no clear boundary, but in this review we will not enter any seriously on “neighbouring territory”.

The emergence of the Kyiv scientific school of the Theory of structures is associated with the establishment in 1899 at the newly organized Kyiv Polytechnic Institute[3] (KPI) the Strength of Materials Department.

Further, the formation and development of the theory of structures was facilitated by the creation in 1918 of the Ukrainian Academy of Sciences, the Institute of Mechanics of the NAS of Ukraine, expansion of relevant research in higher education institutions, creation of new academic and sectoral research institutions. This review presents the substantive side of these studies. The bibliography is quite extensive.

When selecting quoted sources, monographic publications are indicated first. References to articles in periodicals, as a rule, represent specific examples and are not exhaustive lists of sources. The accents in the thematic selection, of course, are determined by the interests of the authors, are open to expansion and do not pretend to be ranked according to the importance of certain problems.

The authors hope that the book will be useful and will evoke pleasant memories for many students of the Kyiv School, who now work fruitfully in many countries of the world.

Studying the history of knowledge helps to familiarize the reader with historically objective evaluations of certain research results and priorities, to realize the sometimes long and extraordinarily thorny path of formation even small and obvious scientific achievements and truths. Equally important, it helps to reflect that atmosphere of deep mutual respect and goodwill which, despite the lengthy and intense discussions, has prevailed and should prevaile in the scientific community of modern society.

The authors of the work are graduates of Kyiv Civil Engineering Institute (now Kyiv National University of Construction and Architecture), which celebrates its 90th anniversary in 2020. The authors dedicate this publication to this significant event, as well as the blessed memory of their dear teachers and colleagues.

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[3] Over the 120 years of its existence, the Institute has repeatedly changed its name, in (1934-1944) it was the Kyiv Industrial Institute, today it is the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

## 1. FIRST STEPS

An outstanding scientist and organizer of engineering education, the first Rector of the KPI Professor Emeritus V.L. Kirpichov clearly understood the need for radical reform of higher engineering education in accordance with urgent demands of industry, as well as the development of mechanics in general and structural theory in particular (and he was personally interested in this section of mechanics) in physics and technology direction.

In 1899, a year after the opening of the Institute, V.L. Kirpichov began to lecture at the Strength of materials Department on a wide range of strength disciplines [253, 256, 255] (strength of materials, graphical statics, etc.). His classic work [254] presented the theory of statically indeterminate structures in a very compact and transparent way and completed the period of the theory of structures formation in Russia.

***Kirpichov Viktor Lvovych*** *(1845-1913) was a Privy Councillor and Professor Emeritus.*

*In 1868 he graduated from Mikhailovskaya artillery Academy. From 1869 he started lecturing on strength of materials. In 1873 he trained abroad. He listened the course of experimental and theoretical physics by G.R. Kirchhoff in Heidelberg. Then he got acquainted with the machine-building plants and hydraulic facilities in Germany, Belgium and Switzerland.*

*Later he worked under the direction of Thomson and J.C. Maxwell. In 1876 V.L. Kirpichov became a Professor of the Petersburg technological Institute.*

*In 1885 V.L. Kirpichov was instructed to organize practical technological Institute in Kharkiv. Viktor Lvovych brilliantly fulfilled the order. Under his leadership, the Kharkiv technological Institute quickly gained a high reputation. In early 1898 V.L. Kirpichov organized Kyiv Polytechnic Institute and headed it until 1902.*

*In 1902 he was appointed as a member of the Council of Minister of Finance, and in the spring of 1903 he became the Chairman of the Construction Commission of St. Petersburg Polytechnic Institute. Until the end of his days he lectured on applied mechanics and engineering*

On the initiative of V.L. Kirpichov many talented scientists from different universities of the former Russian Empire was invited to KPI. Among them were the bridge building specialist Professor Yevgeni Oscarovych Paton and specialist on the mechanics of materials and structures Professor Stepan Prokopovych Timoshenko. It was they, together with V.L. Kirpichov, who laid the Foundation of Kyiv scientific School of mechanics.

By the time of moving to Kyiv, Ye.O. Paton published the first volume of his four-volume course “Iron Bridges”, as well as the work that played an important role in the theory of structures [318]. In this work, a study was carried out, revealing the conditions under which it was possible to use a hinged design scheme of a truss, nodes of which were not perfect hinges. A notable influence on the development of the theory of structures was made by the studies of Ye.O. Paton, dedicated to the "laws of weight" of bridge structures [320].

***Paton Yevgenii Oscarovych*** *(1870-1953) was a famous scientis and engineer who worked in the field of welding, bridge engineering and structural mechanics. Hero of Socialist Labor (1943), laureate of Stalin Prize. He graduated from the Dresden Polytechnical Institute (1894), and the St.-Petersburg communications engineers institute (1896). He taught at the Moscow engineering school of communications (1899-1904), at the Kyiv Polytechnic Institute (1904-1938) and at the Kyiv construction College of railway transport.*

*In 1909-1911 the Mukhrani bridge across the Kura River in Tiflis was built according to his design.*

*In 1925 the chain bridge named after Eugenia Bosch was built in Kyiv according to the project of Paton. He was the author and project leader for more than 100 of the welded bridges. In 1929 he founded Welding lab and Welding Committee in Kyiv. E.O. Paton was head of Welding lab and Welding Committee (1929-1933), on the basis of which the Electric Welding Institute was created in 1934. Now the Electric Welding Institute bears his name.*

Further, Yevgenii Oskarovych led the department of bridge engineering in KPI for 25 years and many of his students have become famous scientists. Among them are the author of the Dneproges project, Academician I.G. Aleksandrov, Vice-President of the Academy of Sciences of the Ukrainian SSR K.K. Syminsky, Academicians of the Academy of Sciences of the Ukrainian SSR F.P. Belyankin, M.V. Kornoukhov and S.V. Serensen, corresponding member of the Ukrainian SSR Academy of Sciences B.M. Gorbunov, Doctor of Technical Sciences O.A. Umansky and others.

Combining teaching with practical engineering, E.O. Paton had already done a lot in the 1920s: according to his projects, bridges were built in Tiflis (Tbilisi), two bridges across the Ros River, a pedestrian bridge across the Petrivska alley in Kyiv. The third and fourth volumes of his fundamental work “Iron Bridges” were published, as well as a separate no less serious work “Wooden Bridges”. In 1917, as many as two textbooks, one atlas of drawings, and nine scientific articles appeared from his pen. The Kyiv bridge named after Eugenia Bosch (Fig. 1) was restored under his leadership.

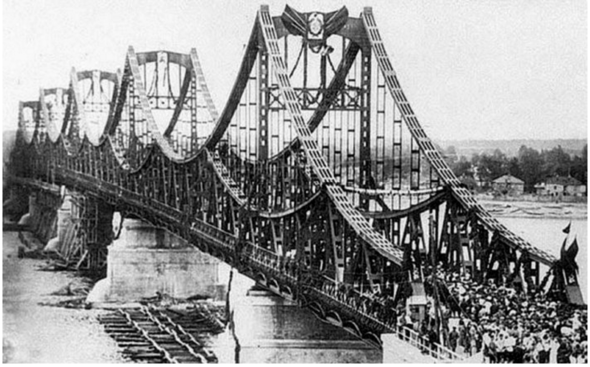


Fig. 1. Eugenia Bosch bridge

Yevgenii Oskarovych ran the Kyiv test station of the Central institute of structures of the People's Commissariat for Railways, and did a great job of summarizing the test results on operated and restored railway bridges.

In particular, an assessment of the errors that occur due to neglecting the rigidity of the nodes, when calculating bridge trusses, by comparing the data on the measured stresses with their calculated values was made [319]. Interesting data were also obtained while testing the bridge, specially designed and built by the Kyiv test station for experiments [324].

Together with his student B.M. Gorbunov, he continuously improved his multi-volume course of steel bridges, which was published in several editions and was a true encyclopedia of bridge building, in which the issues of structural resistance were presented in an exhaustively detailed form.

***Timoshenko Stepan Prokopovych*** *(1878-1972) - Professor, Academician of the Ukrainian Academy of Sciences (1918), Foreign Member of the Academy of Sciences of the USSR (1964, Corresponding Member, 1928). Honorary member of the Academies of Sciences, Scientific Societies, Honorary Doctor of the most famous universities in many countries of the world. In 1901 he graduated from the St. Petersburg communication engineers institute.*

*In 1906 he defended his dissertation. In 1906-1911, 1918-1920 - Professor of the Strength of Materials Department in the Kyiv Polytechnic Institute, 1912-1917 - Professor in the Polytechnic, Electrotechnical Institutes and Communication engineers institute in St. Petersburg.*

*In 1919 -1920 - the first director of the Institute of Technical Mechanics (now the S.P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine). In 1920-1921- Professor of the Zagreb Polytechnic Institute. From 1923 to 1927 - scientific consultant of the company "Westinghouse". He organized the mechanics section at the American Society of Mechanical Engineers (1927). In 1927-1936 - Professor of the University of Michigan; 1936 -1943 - Head of the Department of Mechanics, 1943 -1960 - Professor of the Department of Mechanics in Stanford University (California). From 1960 to 1972 he lived in Wuppertal, Germany.*

The years of S.P. Timoshenko work in Kyiv became a bright stage in the development of the theory of structures in the KPI. He worked in the KPI from 1906 to 1920 with a break from 1911-1917. V.L. Kirpichov knew him very well from their joint work in Petersburg and offered Stepan Prokopovych to take part in the competition for the head of the strength of materials department.

Combining teaching with active research, S.P. Timoshenko received a number of outstanding results on various issues of strength analysis [451, 450, 452, 448], including those in a series of studies on the theory of the elastic systems equilibrium stability [453, 454, 455, 456]. The work [456] was awarded the D.I. Zhuravsky prize and medal. In 1908 S.P. Timoshenko published a textbook on the strength of materials [449, 458], which became a classic and was later reprinted many times in many languages of the world.

The energy approach was developed in the works of S.P. Timoshenko on the problem of equilibrium stability. Since his first work in 1907 [455], where the

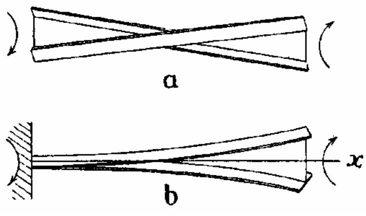


Fig. 2. Free and restrained torsion of a I-beam (picture is taken from [457])

energy method was used, he has widely applied it to the most diverse problems of elastic systems stability [456], including the energy derivation of the Euler formula [454].

The problem of lateral buckling of beams with narrow rectangular cross-section was first considered by Prandtl. Further development of this problem belongs to Timoshenko, who in 1905-1906 obtained the basic differential equation for the torsion of symmetrical I-beams and on this basis investigated the lateral buckling of transversely loaded high I-beams [457]. This study by S.P. Timoshenko was later represented at the Kyiv Polytechnic Institute for the adjunct in applied mechanics degree defense (the opponents were V.L. Kirpichov, A.A. Radtsig and N.B. Delone).

But the main thing is that S.P. Timoshenko turned the Prandtl academic problem into a problem of great importance for the practice of bridge engineering. In connection with the problem of spatial buckling, the issue of torsional loss of stability, which was also considered by S.P. Timoshenko [457], became very important.

It should be noted that in solving this problem S.P. Timoshenko found that the Saint-Venant principle is not applicable for torsion of an I-beam. The twist angle depends not only on the magnitude of the torque and the torsional rigidity of the beam, but also on the way of fixing its ends (Fig. 2).

In [455] S.P. Timoshenko considered the problem of buckling of rectangular plates, under various conditions of supporting the edges parallel to the acting compressive forces. He also considered the buckling of rectangular plate whose unloaded side is unrestrained.

## 2. ORGANIZATIONAL ACTIVITY. FOUNDATION OF THE UKRAINIAN ACADEMY OF SCIENCES. S.P. TIMOSHENKO INSTITUTE OF MECHANICS OF THE NAS OF UKRAINE

S.P. Timoshenko did a great deal of organizational work. From 1909 to 1911 he was dean of the mechanical and engineering departments. From the post of dean, he, along with two other deans of the KPI, was dismissed by order of the Minister of Education Kasso for refusing to dismiss Jewish students who had been accepted in excess of the so-called “percentage rate”. Returning to Kyiv in 1918, S.P. Timoshenko took an active part in the work of the V.I. Vernadsky commission drafting a law on the establishment of the Ukrainian Academy of Sciences.

S.P. Timoshenko’s participation in the creation of the Ukrainian Academy of Sciences should be especially noted.

After the collapse of the Russian Empire, the Ukrainian Scientific Society (USS), created in 1907 in Kyiv and headed by M.S. Grushevsky, at a joint meeting on July 8, 1917 formed the Commission for the establishment of the

Ukrainian Academy of Sciences. And on April 3, 1918, the USS addressed to the Ministry of Education of the Ukrainian People's Republic (UPR) with a proposal to consider the possibility of financing the work of reorganization the USS into the Ukrainian Academy of Sciences. National Academy of Sciences on the concept of M.S. Grushevskogo was to become a non-governmental institution without its own scientific institutions.

Almost at the same time, another concept of creating a national academic center emerged. In September-October 1917 in Petrograd, N.P. Vasilenko, who was a friend of the Minister of Public Education of Russia, together with V.I. Vernadsky, who was also a fellow of the Minister of Education of Russia, and others advocated the creation of state-owned research organizations in Ukraine, Georgia, and Siberia.

For the organization of the Ukrainian Academy of Sciences (UAS) M.P. Vasylenko invited V.I. Vernadsky, who was in Poltava during this period. The famous organizer of science V.I. Vernadsky was a supporter of the creation of a state network of research institutes. He, as a man of advanced views, openly believed that "the task is not a state organization of science, but state assistance to the scientific creativity of the nation." On June 7, V.I. Vernadsky discussed the creation of the UAS with M.S. Grushevsky, who stood on the principles of building the UAS as a free union of high scientific authorities. His position was against the concept of Vernadsky-Vasylenko. The same thing happened with respect to the staff and direction of the UAS. V.I. Vernadsky insisted on creating the "Academy of Ukrainian Studies", at least at the initial stage. M.S. Grushevsky himself categorically refused to participate in any activities proposed by the government of P.P. Skoropadsky.

The first meeting of the Commission for the development of the bill on the establishment of the Ukrainian Academy of Sciences was held in the office of Minister M.P. Vasilenko on July 9, 1918. It was attended by V.I. Vernadsky, N.F. Kashchenko, D.I. Bagaley, S.P. Timoshenko, P.A. Tutkovsky and others. Later A.E. Krymsky, M.I. Tugan-Baranovsky and others became members of the Commission.

Mykola Prokopovych Vasylenko (1866-1935)

Volodymyr Ivanovych Vernadsky (1863 – 1945)

Pavlo Petrovych Skoropadsky (1873 – 1945)

The Commission identified the fundamental problems associated with the development of the structure of the Academy and the composition of its departments, a list of departments, scientific institutions, and the procedure for their formation. At this meeting, they came to the conclusion that the appointment of the first Academy staff by the highest authority is logical, given that the UAS was created by the state.

The initiative group, headed by V.I. Vernadsky instructed S.P. Timoshenko to make a report on the organization of the unit of applied sciences in the physical and mathematical department of the Ukrainian Academy of Sciences. The idea of bringing science closer to the demands of life has always been attractive to S.P. Timoshenko, and he joyfully and with great interest began to compose a note.

S.P. Timoshenko wrote in the introductory part, "A characteristic feature of the modern development of industry and technology is the widespread use of the scientific method and the facts gathered by science. The times when science and technology have taken different paths is over, and now they often use the powerful tool that mathematics and mechanics give us to solve purely technical problems. They use the methods of experimental sciences and widely adapt them to solve technical problems in the laboratory. … The Academy of Sciences should take the initiative in combining science and technology. Due to its central position and scientific authority, it will be able to gather around itself the few scientific forces that currently exist in Ukraine and combine them in a common work where cooperation between people of technology and science will be possible." S.P. Timoshenko further noted that "representatives of technical science will be able to use scientific methods and knowledge accumulated by pure science to a greater extent than now. On the other hand, representatives of pure science in the field of applied natural sciences will encounter a number of new, unexplored issues, the solution of which will not only enrich science, but will also contribute to the development of industry and the technical life of the region. In the field of experimental activity, people of science will be able to use those powerful tools that modern technology gives into the hands of the experimenter."

S.P. Timoshenko believed that the newly formed Ukrainian Academy of Sciences should pay more attention to combining pure science with the solution of technical problems, in accordance with the needs of technology.

The commission completed its work on the draft law on the creation of the Ukrainian Academy of Sciences on September 17, and on October 12, the Minister of Education and Arts N.P. Vasilenko submitted a package of documents to the Council of Ministers. On November 14, 1918 Hetman of Ukraine P.P. Skoropadsky approved the "Law of the Ukrainian State on the Formation of the Ukrainian Academy of Sciences in Kyiv" adopted by the Council of Ministers, as well as the Charter and staff of the Academy and its institutions attached to it.

In the period from November 1918 to January 1919, the UAS carried out active scientific and organizational work. The Department of Physics and Mathematics of the UAS consisted of 14 departments of the main class and 16 departments of the class of applied natural sciences.

At this time, academic departments of the physics and mathematics subdivision were founded, including the Department of Applied Mechanics, which was headed by S.P. Timoshenko. At the same time, elections and approval in the posts of directors were held. On November 27, 1918, the first General Meeting of the UAS was held, at which V.I. Vernadsky was elected Chairman-President of the Academy. And at the second General Meeting of the UAS, which held on November 30, the Institute of Technical Mechanics of the UAS was formed and S.P. Timoshenko was approved as its director. During the first decade of its existence, the institute occupyed a leading place in the physical and mathematical subdivision.

In 1929 the Institute of Technical Mechanics was divided into the Institute of Structural Mechanics and the Cabinet of Transport Mechanics. In 1959, by the Decree of the Council of Ministers of the Ukrainian SSR, the Institute of Structural Mechanics was renamed the Institute of Mechanics of the Academy of Sciences of the Ukrainian SSR, and in 1993 by the Decree of the Presidium of the National Academy of Science of Ukraine (NASU), the Institute of Mechanics was named after S.P. Timoshenko.

Actually the entire history of the development of the institute, on the basis of which a number of other independent scientific institutions of the Academy of Sciences of the Ukrainian SSR was created, is based to some extent on the principles laid down by S.P. Timoshenko at the dawn of the emergence of Ukrainian academic science. It is worth recognizing that the ideas expressed by him almost a hundred years ago during the organization of the Ukrainian Academy of Sciences turned out to be quite progressive and embodied in practice. The experience of the first Ukrainian Academy of Sciences, where technical sciences were included in the number of academic sciences for the first time in world practice, was later spread in the practice of the USSR Academy of Sciences and in all other republican academies. Ideas of S.P. Timoshenko, laid down by him during the creation of the Ukrainian Academy of Sciences, over time having received further development, today contributes to the widespread implementation of the results of scientific achievements in practice.

The Institute was headed by well-known academicians of the NAS of Ukraine: S.P. Timoshenko (1918 1920), D.A. Grave (1921), K.K. Syminsky (1921 1932), S.V. Serensen (1932-1940), M.V. Kornoukhov (1940-1944), F.P. Belyankin (1944-1958), G.M. Savin (1958-1959), A.D. Kovalenko (1959-1965), V.O. Kononenko (1965-1975). Since 1976, the Institute has been headed by O.M. Guz.

Today, Ukraine has a high world level of development of mechanics and related sciences.

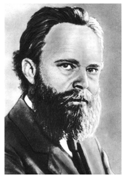
The level of science is determined, for example, by the existence of the Department of Mechanics and a number of scientific institutions at the National Academy of Sciences of Ukraine. These are such well-known in world science institutes as S.P. Timoshenko Institute of Mechanics, G.S. Pisarenko Institute for Problems of Strength, Institute of Hydromechanics, Ya.S. Pidstrihach Institute for Applied Problems of Mechanics and Mathematics, Karpenko Physico-Mechanical Institute, Institute of Applied Mathematics and Mechanics, M.S. Polyakov Institute of Geotechnical Mechanics, Institute of Technical Mechanics and a number of institutions close to mechanics: V. Bakul Institute

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104

for Superhard Materials, E.O. Paton Electric Welding Institute, A. Pidgorny Institute of Mechanical Engineering Problems and others. It should also be noted that in Kyiv, Lviv, Odessa, Dnieper, Kharkiv there are world-famous scientific schools on mechanics and 4 scientific journals on mechanics are published, which are translated into English by the world's largest scientific publishing house Springer Group. Significant research has been carried out in departments and research institutes of leading universities, research institutes and other institutions.


Stepan Prokopovych Timoshenko (1878-1972)


Dmytro Olexandrovych Grave (1863-1939)


Kostiantyn Kostiantynovych Syminsky (1879-1932)


Serhii Volodymyro-vych Serensen (1905-1977)


Mykola Vasyliovych Kornoukhov (1903-1958)


Fedir Pavlovych Beliankin (1892-1972)

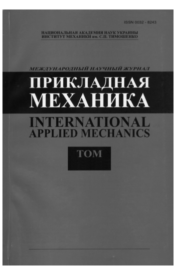

Gurii Mykolaiovych Savin (1907-1975)

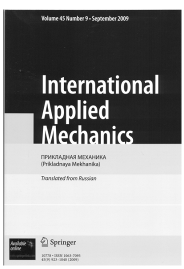

Anatolii Dmytrovych Kovalenko (1905-1973)

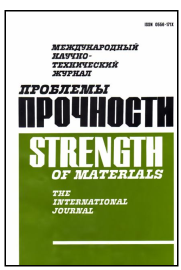

Viktor Olimpanovych Kononenko (1918-1975)

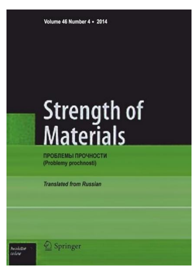

Olexandr Mykolaiovych Guz

The National Committee of Ukraine for Theoretical and Applied Mechanics (NCU) was formed in accordance with the Decree of the Presidium of the Academy of Sciences of Ukraine No. 191 dated July 3, 1992. The same Decree entrusted the Institute of Mechanics of the Academy of Sciences of Ukraine (IM) with the functions of the base institution of the NCU.




The constituent meeting of the committee, which was attended by 202 leading scientists of Ukraine, working in the field of mechanics and related sciences and representing various scientific centers of Ukraine, was held on June 7, 1993.

The main tasks of the NCU are: preparation and holding of scientific forums on theoretical and applied mechanics and related sciences; facilitating the coordination of scientific research on selected issues of mechanics conducted by scientists in various institutions, departments and industries; strengthening the relations of Ukrainian mechanics with foreign scientists, as well as organizations and international societies with the aim of developing mechanics; spread of scientific and technical information on mechanics; representation of Ukrainian mechanics at the International Union of Theoretical and Applied Mechanics (IUTAM) and other international organizations in mechanics and related sciences.

IUTAM was organized on September 22, 1946 at the constituent assembly of the world's leading mechanics at the Sorbonne University in Paris and is currently the most prestigious scientific union in the field of mechanics. It is appropriate to indicate that world physics is united into the International Union of Pure and Applied Physics - IUPAP.

Second most important after IUTAM is the European Mechanics Society, this society assumes only the personal membership of scientists. The third is the Society for Applied Mathematics and Mechanics (GAMM - Gesellschaft fur angewandte Mathematik und Mechanik), where Ukrainian scientists have their representatives along with other scientists from Eastern Europe.

The last at the time of writing this work, the Presidium of the NCU was elected at the General Reporting and Election Meeting on February 10, 2014. It consists of 5 people: the chairman - Guz O.M., the deputy chairman - Bazhenov V.A., Matveiev V.V., Martynyuk A.A., Shevchenko V.P., Scientific Secretary - Ruschitsky Ya.Ya. Chairman of the Committee O.M. Guz is an academician of the National Academy of Sciences of Ukraine, member of European Academy of Sciences (Brussels), Member of Academia Europaea (London), member of the World Innovation Foundation (London).

NCU is an affiliate of IUTAM (in total, 55 countries in which mechanics have reached a certain level of development are affiliated members). In 2000 at the session of the IUTAM General Assembly in Chicago (USA), following a vote, Ukraine was admitted to IUTAM. O.M.Guz was elected representative of Ukraine in the General Assembly.

NCU promotes the development of all important and relevant areas of basic research in the field of mechanics and related sciences, using various means (support for publications in the international scientific journal International Applied Mechanics, discussion at scientific seminars and scientific conferences, etc.).

In total, the NCU currently consists of 282 members - doctors of sciences in mechanics and related branches of science, who represent approximately equally academic and university science.

NCU is the only all-Ukrainian public organization that brings together scientists from all areas of Ukraine working in the field of mechanics and related sciences, and which creates a platform for discussing all the important problems of the development of mechanics and related sciences. The influence of NCU members on scientific research in the field of mechanics is dominant and determining.

The activity of the NCU is aimed both at maintaining the world level of development of mechanics achieved by previous and modern generations of Ukrainian scientists, and at developing new areas of mechanics (for example, nanomechanics, biomechanics, tribomechanics, etc.)

## 3. FURTHER DEVELOPMENT AND FORMATION OF THE THEORY OF STRUCTURES

The further formation and development of the Kyiv School of Theory of Structures is associated with graduates of Kyiv University and the Kyiv Polytechnic Institute.

So, the practical classes in mechanics were supervised by Olexandr Mykolaiovych Dynnyk, who graduated from Kyiv University in 1899 and became a laboratory assistant in physics at the newly organized Kyiv Polytechnic Institute. In 1909, he wrote a paper [87], in which he first gave a solution to the shock problem for the case of linear contact, and also determined the maximum tangential stresses and the points where they are observed.

KPI graduate of 1907 K.K. Syminsky began his pedagogical activity, which lasted 25 years at the same institute.

Thesis for the title of Adjunct of Structural Mechanics K.K. Syminsky defended in 1914, after which he was elected as a professor of the strength of materials department and head of the mechanical laboratory of the KPI. While working in the KPI, he lectured on the strength of materials course, as well as on graphic statics, and led the practice of students. K.K. Syminsky was the first to study the problems of fatigue and strength of steels, spatial trusses for bridges [439], created a number of instruments for testing bridges and lattice structures, and developed the theory of granite strength.

***Syminsky Kostiantyn Kostiantynovych*** *(1879-1932) - Professor (1914), full member of the Academy of Sciences of the Ukrainian SSR (1925).*

*He graduated from the Kyiv Polytechnic Institute in 1907, and in 1907-1932 he worked as a teacher at the Kyiv Polytechnic Institute. In 1920-1921 - Dean of the Faculty of Civil Engineering, 1924-1926 - Vice-Rector for Academic Affairs.*

*At the same time, he worked in 1921-1932 as director of the Institute of Technical Mechanics of the Academy of Sciences of the Ukrainian SSR (now S.P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine), in 1929-1932 - Director of the Kyiv department of the Scientific and Research Institute of Structures.*

He led the strength of materials department and mechanical laboratory from 1914 to 1932. Beginning in 1921, in connection with the appointment of K.K. Syminsky to the post of Director of the Institute of Technical Mechanics of the Academy of Sciences of the Ukrainian SSR, the scientific work at the department was closely associated with the scientific activities of this institute. Most of the department staff worked at the same time at the institute. These were such apprentices of K.K. Syminsky as famous scientists in the field of mechanics of materials and engineering structures M.M. Davydenkov and F.P. Belyankin.

Own work of K.K. Syminsky on structural mechanics [442, 443, 444, 445], especially on the analysis of spatial systems [438, 440], contained a number of new interesting results. In particular, in the textbook [443], a method for the gradual introduction of increasingly complex basic determinate structure has been developed. It should also be noted that in his article [441] the problem of prestressing was apparently considered for the first time as a design technique.

The works of K.K. Syminsky had a great influence on the direction of researchthat was carried out at the KPI and at the Institute of Technical Mechanics. On his initiative, the focus of the Institute's work was clarified, and the Institute itself was reorganized in 1929 into the Institute of Structural Mechanics of the Academy of Sciences of Ukraine.

The graduate of the department was S.V.Serensen, the world-famous founder of the scientific direction in the cyclic and thermocyclic strength of modern mechanical engineering, in particular aircraft engine building, the fracture and durability of engineering structures. He was a graduate of 1926 and passed the way from a laboratory assistant to a professor, head of the department (1931). From 1932 to 1940 S.V. Serensen worked as director of the Institute of Structural Mechanics of the Academy of Sciences of the Ukrainian SSR and head of the department of aircraft engineering at the Kyiv Aviation Institute (1933-1941). In 1939 he was elected an academician of the Academy of Sciences of the Ukrainian SSR.

In the prewar years, KPI conducted research on strength in power engineering, the results of which were defended in the form of a number of candidate dissertations. Among them were dissertations of graduate students A.D. Kovalenko "Study of stresses in the wheels of turbomachines" (1938) and G.S. Pisarenko "Determination of deflections and stresses in detachable elements of steam turbines" (February 1941). Both of them later became academicians of the Academy of Sciences of the Ukrainian SSR.

Due to the occupation of Kyiv, many teachers of the department were evacuated together with the Academy of Sciences of Ukraine to Ufa. On returning after evacuation, the strength of materials department of KPI was headed by a corresponding member of the Ukrainian Academy of Sciences F.P. Belyankin (full member since 1948). Simultaneously with the leadership of the department (1944-1952), he was the director of the Institute of Mechanics (1944-1958), and gave lectures on "Strength of Materials" and "Theory of Elasticity".

In 1952-1959 and 1961-1984 the department was headed by Professor G.S. Pisarenko. Out of 32 years of heading Department, 26 he worked part-time, having the main job in the Academy of Sciences of the Ukrainian SSR - as director of the Institute for Problems of Strength (1966-1988), Chief Scientific

Secretary of the Presidium (1962-1966), First Vice-President of the Academy of Sciences of the Ukrainian SSR (1970-1978). From 1959 to 1961 the strength of materials department was headed by prof. V.V. Khilchevsky.

Mykola Mykolaiovych Davydenkov (1879 - 1962)

Fedir Pavlovych Beliankin (1892 -1972)

Anatolii Dmytrovych Kovalenko (1905-1973)

Serhii Volodymyrovych Serensen (1905 -1977)

Managing the department, prof. G.S. Pisarenko did a lot for the development of creative cooperation of the department with scientific institutes of the Academy of Sciences of the Ukrainian SSR, attracting leading scientists from the Academy to work simultaneously in the department, conducting scientific seminars with the participation of scientists from other universities. For the preparation of scientific personnel not only the experimental base of the department was exploited, but also the one of Academy of Sciences.

G.S. Pisarenko considered the training of highly qualified scientific and pedagogical personnel as one of his main targets. He took care of improving the teaching of the course as well as the preparation of textbooks and tutorials on the strength of materials. He considered laboratory studies as a necessary condition for increasing the level of assimilation of educational material by students and made significant efforts to attract talented youth to work at the department.

In the sixties, actual researches on the strength and durability of heat-resistant materials received further development at the department. The results of these researches were introduced into the design practice of a number of enterprises and organizations of the aerospace complex. These studies were led by a student of academician G.S. Pisarenko prof. M.S. Mozharovsky, who headed the department in 1984.

The staff of the department G.S. Pisarenko, V.A. Agarev, O.L. Kvitka, V.G. Popkov and E.S. Umansky wrote several textbooks, including a full course for mechanical specialties, first published in 1963 under the title “Strength of Materials”. This textbook was reprinted in 1967, 1973, 1979, 1986, 1993 and 2004.

An important stage in the development of the school of mechanics of the KPI was the opening of specialty "Dynamics and strength of machines" at the strength of materials department in 1970.  The necessity to introduce such a specialty was caused by the needs of both the institutes of the Academy of Sciences of the Ukrainian SSR (such as Institute for Problems of Strength, Institute of Mechanics, Institute for Superhard Materials, Institute for Problems in Materials Science, Electric Welding Institute), and large machinebuilding, aircraftbuilding and shipbuilding enterprises of Ukraine.

The most famous scientists in this field are academicians: O.M. Dynnyk, S.P. Timoshenko, F.P. Belyankin, M.M. Davidenkov, V.V. Kharchenko, A.D. Kovalenko, M.V. Kornoukhov, A.O. Lebedev, V.V. Matveyev, M.V. Novikov, Ye.O. Paton, S.V. Serensen, K.K. Syminsky, G.I. Sukhomel, V.T. Troshchenko, corresponding members: B.M. Gorbunov, I.Ya. Shtaierman, V.A. Strizhalo, A.Ya. Krasovsky, M.I. Bobyr.

In 1989 Doctor of Technical Sciences, professor Ye.O. Antypov was elected the head of the department of dynamics and strength of machines and strength of materials.

Today, the graduating department of dynamics and strength of machines and strength of materials has become the leading department in terms of strength and reliability of machines and structures. It provides at the same time teaching of the general engineering course of strength of materials to students of many faculties and institutes of KPI. Nowadays it is headed by Professor S.O. Pyskunov.


Yevgenii Oscarovych Paton (1870 -1953)


Oleksandr Mykolaiovych Dynnyk (1876 -1950)


Georgii Yosypovych Sukhomel (1888 - 1966)


Illia Yakovych Shtaierman (1891-1962)


Borys Mykolaiovych Gorbunov (1901–1944)


Mykola Vasyliovych Kornoukhov (1903 -1958)


Georgii Stepanovych Pisarenko (1910 -2001)


Anatolii Oleksiiovych Lebediev (1931—2012)

A significant role in the development of structural mechanics and theory of structures was played by G.S. Pisarenko, who headed the Strength of Materials Department of KPI for many years. He was the founder and first director of the Institute for Problems of Strength of the Academy of Sciences of Ukraine (now G.S. Pisarenko Institute for Problems of Strength of the National Academy of Sciences of Ukraine). From 1989 to 2011, the Institute was headed by Academician of NAS of Ukraine V.T. Troshchenko, and from 2011 to the present - Academician of NAS of Ukraine V.V. Kharchenko

In October 2016, the institute celebrated its 50th anniversary and during that time it became a recognizable scientific center for fundamental and applied research on the strength of materials and structural elements under extreme thermomechanical loading. The main activities of the institute are: ultimate state and strength criteria of materials and structures; computational and experimental methods for studying the stress-strain state; fracture mechanics and survivability of structures with cracks; vibration of non-conservative mechanical systems.

Well-known scientific schools have been founded and are successfully developing:

- strength of materials and structural elements under extreme conditions of thermomechanical loading (Academician of NASU G.S. Pisarenko);
- criteria of strength and patterns of deformation and fracture of materials and structures in a complex stress-strain state (Academician of NASU A.O. Lebediev);
- material fatigue and fracture criteria and statistical theories of the strength of materials under cyclic loading (Academician of NASU V.T. Troshchenko)
- Theory of vibration of non-conservative mechanical systems (Academician of NASU V.V. Matveiev).

## 4. PATHS OF DEVELOPMENT

The intensive development of the Kyiv School of the Theory of Structures took place mainly in the following research centers: Kyiv Polytechnic Institute, Institute of Structural Mechanics, Institute for Problems of Strength, and Kyiv Civil Engineering Institute (KCEI). The studies dealt with a wide range of issues, both general theoretical and applied ones.

### 4.1. Planar and spatial bar systems

The main objects of study in the first third of the twentieth century were truss and frame models of structures. The method of their analysis was perfected by solving a number of specific problems, not least the bridges, whose restoration problems in the 20s were dealt with by Ye.O. Paton.

Borys Mykolaiovych Gorbunov (1901–1944)

Olexandr Azariiovych Umansky (1900–1973)

The Kyiv School of the Theory of Structures paid much attention to the computation of strength of spatial bar systems. This important area of research began with the works of V.L. Kirpichov [252] and K.K. Syminsky [438], and was brilliantly continued in a series of studies by B.M. Gorbunov and O.A. Umansky [144, 145, 151, 160, 161, 469, 470, 479], the first of which were published when the authors were still students.

They graduated from the Kyiv Polytechnic Institute in 1925, graduation projects were carried out under the guidance of Ye.O. Paton.

Until 1930, both were engaged in the design of bridges of various types and carried out research work at the Institute of Structural Mechanics. At the same time, they taught at the Kyiv Polytechnic Institute and at the Kyiv Civil Engineering Institute of (KCEI), which was separated from the KPI in 1930.

Here O.A. Umansky headed the Department of Structural Mechanics created by him (1930-1933), and B.M. Gorbunov was a professor at the Department of Metal Structures (1930-1941).

In the monograph [161] published in 1932, new methods and ideas were introduced into the structural mechanics of bar systems (the "image method" of B. Mayor, the "method of the motors" of R.v. Mises, the method of traces, etc.), thoroughly revised and supplemented by the authors.

Whereas a plane system of forces can always be reduced to one force or to one pair of forces, in the spatial case, the system of forces is brought to a motor, which is two crossing vectors, namely force and moment (fig. 3). The fundamental importance of the operation of scalar multiplication of motors (screws), which unites all operations of vectoral algebra, essential for the structural mechanics, was clarified.

In addition, the approach of von Mises, based on the consideration of the displacements of the nodes of the pin-jointed trsuss system, was supplemented by the consideration of the displacements of the bars or disks, which made it possible to consider the bar systems of the frame type. The appearance of this book transferred the analysis of spatial trusses to a new, higher level. Its ideas also had a noticeable influence on the development of descriptive geometry. B.M. Gorbunov used this apparatus not only for spatial trusses but also for constructing theory of spatial frames analysis using the displacement method [156].

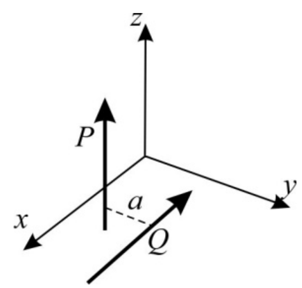


Fig. 3. Example of a motor

In [479] O.A. Umansky and B.M. Gorbunov showed that the virial of external forces (the sum of the moments of these forces, rotated 90 degrees, about an arbitrary point) is equal to the sum of the products of the forces in the bars and their lengths. Later, the concept of virial was used to determine the theoretical weight of trusses.

In a work published by O.A. Umansky in 1932 [476] the method of initial parameters was substantially developed and, apparently, for the first time, concentrated dislocation effects such as a fracture of the beam axis and local shear were taken into account. In those same years, he published a number of studies on the analysis of the work and the method of computing beams on elastic supports [475, 476, 479]. These studies later served as the basis for his monograph "Floating Bridges" [471] and was included in his two-volume "Special Course of Structural Mechanics" [474].

O.A. Umansky continued his studies of the kinematic properties of spatial frame structures [477] and established a theorem on the identity of the equilibrium conditions of a rigid body and the closure conditions of a spatial bar

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104

contour. He also proved that the computation of any bar system by the force method coincides with the calculation of some (mutual) system by the displacement method. He also developed the method of initial parameters as applied to structures with bars of varying cross section [478].

Studies related to the analysis of the work and the method of computation of the spatial bar systems were continued by D.V. Vainberg and V.G. Chudnovsky [485, 486, 498], who, among other things, considered the peculiarities of the behavior of cyclically symmetric frames in the calculation of strength [485, 486, 496, 498].

The use of cyclic symmetry in order to simplify calculations was continued in the later works of V.G. Chudnovsky [61, 64, 65].

David Veniaminovich Vainberg (1905-1973)

Volf Grygorovych Chudnovsky (1908-?)

Here, as in previous works, the basis of the approach was an implicit decomposition into a finite trigonometric series.

A detailed analysis of the work of spatial systems with any type of point symmetry based on the theory of group representation was subsequently performed by M.L. Buryshkin and V.M. Gordeiev and presented in the monograph [55].

The calculation of spatial trusses was often reduced to decomposition into flat faces. Difficulties in using such a technique arose in the calculation of torsion. Studies in this direction were carried out by M.D. Shmulsky [403] and A.M. Vasilenko [513].

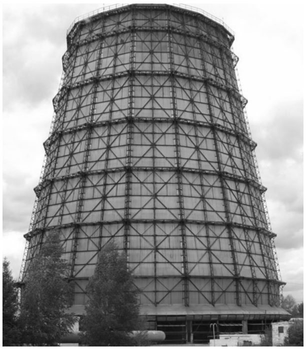
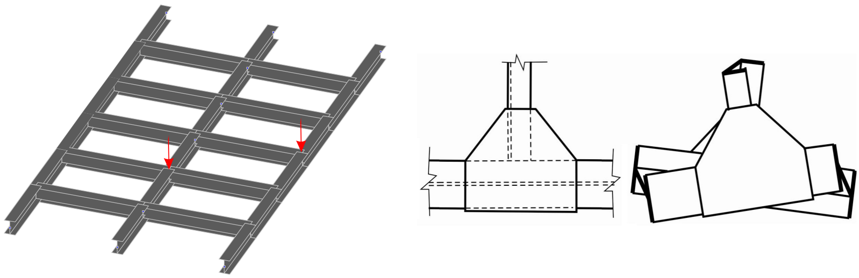


Fig. 4. Cyclically symmetric frame of a cooling tower

Another series of works was devoted to the analysis of structures composed of thin-walled bars. As a rule, thin-walled bars do not work alone, but as part of flat or spatial systems. In this regard, there were works where an attempt was made to solve the corresponding problems. The main problem here was the issue of compatibility of deformations in the joints of thin-walled elements. This question is easily solved only for a continuous thin-walled bar, in which the deplanations of the end cross sections of the joined elements on the intermediate support coincide.

Apparently, the first who tried to solve the problem of calculating a frame composed of thin-walled bars was B.M. Gorbunov, who proposed a method for calculating plane frames [148] under spatial loading directed perpendicular to the plane (Fig. 5(a)). Here, as in the subsequent works of B.M. Gorbunov with O.I. Strelbitska [152, 157, 158, 159] the issues of calculating thin-walled wagon frames were covered. The hypothesis of absolute stiffness of the joint gusset in its plane (Fig. 5(b)) was used, which ensured the equality of the end cross-sections deplanations of all the bars converging at the node.

Oleksandra Ivanivna
Strelbitska
(1905-1980)



(a) (b)

Fig. 5. Frame of thin-walled profiles: (a) - scheme, (b) - node deformation

Many attempts have been made to build a sufficiently universal algorithm for calculating arbitrary thin-walled bar systems, and here the main problem was the formulation of boundary conditions at the ends of the thin-walled bar. In some works, it was assumed that at the end of the bar the deplanation is either completely absent or does not encounter any obstacles. One of the first works of this direction was the article [427]. Spatial (in particular, cyclically symmetric) systems of thin-walled bars were considered in this work under the assumption that the bar nodes are either infinitely rigid, and the deplanation of the end sections of all the bars converging at the node is zero, or the design of the node is such that deplanation is not restricted.

In other studies, the hypothesis of equal deplanations at the ends of all thin-walled bars converging at a node was used. In the general case, the failure of the hypothesis about the equality of the deplanations of all end sections adjacent to the node was demonstrated in [351]. It was demonstrated using simple examples that the deplanations of the end sections of all elements converging in a node do not coincide, and their values depend on the structure of the node, the deformation of which has a noticeable effect on the behavior of the structure.

Much attention was paid to various kinds of improvements in classical methods of structural mechanics. An example here is the work of M.V. Kornoukhov [272], where the displacement method was generalized taking into account shifts in wide bars (Timoshenko bars). Other improvements to the classical displacement method have been proposed [166, 346].

The issues of developing effective methods and algorithms for solving problems of structural mechanics were considered. Before the computer era the problems of this kind, aimed at reducing the volume and increased convenience of calculations, were at the center of attention of specialists in structural mechanics. So, in the works of M.V. Kornoukhov [274] and O.A. Umansky [473] the problem of solving systems of three-term equations related to problems of structural mechanics was analyzed in detail. Starting from the research of I.Ya. Shtaierman [408], various schemes for an iterative method for solving problems were considered as an option for simplifying calculations. A series of works of this kind was later published by P.M. Sosis [423, 424, 426]. Later, the problem of choosing computational algorithms became acute in relation to nonlinear problems of structural mechanics, where both iterative and step methods were used [129, 257, 326].

**4.2. Stability and dynamics of structures**

The scientific tradition of stability analysis, established by S.P. Timoshenko, found its successors in the walls of the same Polytechnic Institute in the person of I.Ya. Shtaierman and O.M. Dynnyk. At the beginning of their research, they dealt with the stability analysis of a single bar (straight or curved) [411]. A typical example is the stability of the arches.

Oleksandr Mykolaiovyc h Dynnyk (1876 -1950)

Illia Yakovych Shtaierman (1891-1962)

A large amount of research here is made by O.M. Dynnyk, who published the results of his work performed in the 1930s and 1940s, in a monograph [88]. He calculated arches of circular, parabolic, sinusoidal and chain shape under various loadings. He also considered loss of stability of a circular ring and an arch beyond the elastic limit.

In particular, O.M. Dynnyk drew attention to one significant circumstance that escaped Timoshenko's attention, who was investigating the behavior of a shallow sinusoidal arch under the vertical load. The fact is that in a certain range of parameters of a shallow arch, the loss of stability of its equilibrium occurs not after reaching the limit point on the equilibrium state curve, but somewhat earlier by the bifurcation criterion.

The out-of-plane loss of stability of the arches was studied by I.Ya. Shtaierman [410]. He also began to study the stability of pipes and shells [405]. But all these works dealt with problems of a "local type" where important, but still isolated elements of a complex constructive system were considered. Methods for testing the stability of such systems began to be studied later.

A brilliant cycle of works devoted to the stability of bar systems was performed by M.V. Kornoukhov and his team [268, 269, 277, 278]. It should be noted that the formulation of similar problems was stimulated by the demands of practice, in particular, by the research of the high-rise construction of the Palace of Soviets of the USSR, which was designed in the prewar years (Fig. 6).

The idea of applying the method of displacements to the problem of the stability of bar systems was proposed by M.V. Kornoukhov in 1937 [276]. Its meaning was that the coefficients of the canonical equations of the displacement method were calculated using formulas that reflected the dependence of the reactions on the value of the longitudinal force in the bar under consideration. This approach became subsequently generally accepted and is used today.

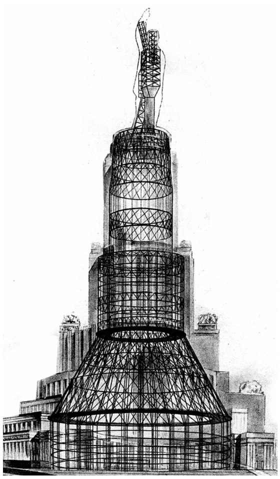


Fig. 6. The skeleton of the Palace of Soviets (draft)

M.V. Kornoukhov actively promoted the method of calculating "stable strength", which consisted, in fact, in the calculation of the system according to the deformed scheme [269, 275], but it was not widely adopted. In the modern formulation, the computation according to the deformed scheme, like the “P-Δ method”, began to be advanced in the Eurocode.

Kornoukhov’s works were comprehensively presented in his monograph [276] (it was awarded the State Prize of the USSR in 1950), where, in addition to description of the stability analysis method, a number of general principles of the theory of stability was pointed out. It was indicated how to replace one load with another, equivalent to the first, what simplifications of the design scheme are permissible, how to compare the safety factor at combined loading with those at separate components of the loading. One of the results arising from qualitative analysis is the fact that a “culprit” in the loss of stability of the entire structure is often one element or a small group of elements. In this connection, M.V. Kornoukhov introduced the concept of states of stability. But only 50 years later in the work of A.V. Perelmuter and V.I. Slivker [350] the criterion for determining the type of bifurcation (constrained or forced) of the bar or any part of the structure and the method for calculation based on the energy approach were presented.

Mykola Vasyliovych Kornoukhov (1903 -1958)

In [270] M.V. Kornoukhov studied the mechanism for the loss of stability of the two-bar truss (von Mises truss) in detail, and pointed out the possibility of loss of stability without passing through the bifurcation point.

The complex of important practical problems of structural stability is presented in the monograph by I.Ya. Shtaierman and A.A. Pikovsky [413], intended more for practicing engineers than for researchers. The authors analyze in detail the ideology of calculating systems with compressed elements, paying particular attention to the limiting value of the carrying capacity of such elements. Here, by the way, it was proposed to divide the total safety factor into two parts: “load dependent” and “structure dependent”, i.e. ideas that later formed the basis of the method of limit state design.

Peculiar problems were solved when calculating the stability of thin-walled bars. Here one can point to theoretical and experimental studies of M.I. Dlugach [89, 91] and L.N. Stavraki [427, 428, 429], the latter also investigated the stability of frames made of thin-walled profiles [429]. He obtained the value of the torsional stiffness coefficient of a compressed thin-walled bar and used it to construct a method for calculating the spatial framework of such bars. In this case, the stability of cyclic frameworks and dome like structure consisting of hinged bars was investigated. The latter problem was studied in detail in the work of P.S. Polyakov [366]. A detailed analysis of possible buckling modes of single-tier spatial frames was carried out by L.N. Padun-Lukyanova [314]. Multiple buckling modes of cyclically symmetric frames were investigated by M.Ya. Borodiansky [52].

Studies on the theory of stability have always been the focus of the Kyiv school of the theory of structures. They concerned both the fundamental questions of the theory and specific classes of structures.

In particular, in the works of O.M. Guz and his students (see, for example, [227, 228, 230]) stability problems of bars, plates, and shells were considered in a three-dimensional formulation. It was proved that stability theories based on the Kirchhoff–Love hypothesis and assumption of sectional rigidity are asymptotically exact.

The three-volume monograph [347, 349] is devoted to the presentation of stability theory based on the general equations of mechanics of a deformable solid, when the transition to applied theories that use hypotheses of a static or kinematic nature is realized by an accurate formal transformation of the corresponding general equations. At the same time, effects were discovered that were skipped by the usual approach based on some obvious static-geometric representations.

Among the stability problems that consider specific types of structures, one can point to I.Ya. Amiro and V.O. Zarutsky cycle of works devoted to the stability of ribbed cylindrical shells [2, 4, 503]. Nowadays the works of G.D. Gavrilenko [126, 127] can be considered as the continuation of these studies. The studies of the stability problems of systems with unilateral constraints [18] as well as frame systems with elements of varying cross-section are also worth mentioning here.

Igor Yakovych Amiro (1921-1998)

Volodymyr Oleksandrovych Zarutsky (1932 – 2011)

The use of the approach, that was based on the dynamic stability criterion, was proposed in the works of V.G. Chudnovsky [60]. Nevertheless, this approach turned out to be more in demand when considering the dynamic problems of systems with compressed bars.

Dynamic problems were considered by S.P. Timoshenko in connection with studies of bridge vibrations [450]. Experimental studies of bridge dynamics were carried out by the Kyiv Bridge Testing Station, which was organized by Ye.O. Paton in 1920.

The application of the displacements method to the dynamicproblems was proposed by M.V. Kornoukhov [273] and V.G. Chudnovsky. The latter focused on the analysis of vibrations of compressed systems. His studies on this problem were presented in detail in the monograph [62]. In addition, V.G. Chudnovsky noted that the well-known Bubnov problem of the critical stiffness of an intermediate elastic support, which translates its work into a class of absolutely rigid, is also generalized to the oscillation frequencies [59]. Vibrations of circular arches were considered by D.V. Vainberg [483, 495]. S.V. Serensen dedicated his work [391] to the dymaic analysis of multi-storey frames.

The dynamics of bars and bar systems has been the subject of research by G.S. Pisarenko, whose fundamental research concerned the theory of vibrations of elastic systems accounting energy dissipation in the material [355, 357] (awarded the M.M. Krylov Prize of the Academy of Sciences of the Ukrainian SSR).

His further work is related to the study of the strength of materials and structural elements in extreme conditions. The same subject became the main one in the Institute for Problems of Strength of the Academy of Sciences of the Ukrainian SSR created on the initiative of G.S. Pisarenko. He headed this Institute for over twenty years.

Georgii
Stepanovych
Pisarenko
(1910 -2001)

The problem of the dynamic action of a shock load has been the subject of many studies, beginning with the solution of S.P. Timoshenko, based on the accunting of local shock deformations in accordance with the approach of Hertz. Yu.V. Blagoveshchensky and D.V. Vainberg suggested using the series method to solve the basic equation of S.P. Timoshenko [51]. In the monograph by M.O. Kilchevsky [246] the dependencies obtained by I.Ya. Shtaierman were used instead of Hertz's relations. These dependencies related to cases of tighter touch of bodies. In addition, M.O. Kilchevsky considered the process of not only elastic, but also elastoplastic deformation of the contact zone [243].

**4.3. Elastic plates and shells**

The further development of fundamental works in the field of analysis of the structures behavior was mainly connected with the development of the theory and methods for calculating plates and shells. The beginning of such research was laid by the works of I.Ya. Shtaierman performed at the Kyiv Polytechnic Institute. As early as 1924, he considered an axisymmetric problem in the theory of calculating a spherical shell [409]; he was the first to suggest a method of asymptotic integration of the equations for a shell of revolution [407]. The practically important problem of calculating a cylindrical tank with a wall of variable thickness [406] was also solved, and a simple to use method of analogy between the dome and the arch on an elastic foundation [404] was developed.

One of the first works carried out at the Institute of Structural Mechanics of the Academy of Sciences of the Ukrainian SSR and devoted to the calculation of shells was the study of A.L. Goldenveizer [134] carried out in 1935. This famous scientist continued his work later in Moscow.

The approach V.G. Chudnovsky, presented in [58], was based on the synthesis of structural mechanics methods for bar systems and the theory of elasticity. It treated the ribs as a frame lying on an elastic foundation created by the shell itself. Later this method was transferred to the calculation of the ribbed domes according to the moment theory [65].

In a series of works of M.O. Kilchevsky [244, 245, 248] a generalized method of reducing the three-dimensional problem of the theory of elasticity to a two-dimensional one was proposed and developed. A number of variants of the theory of shells were proposed, in relation to which the ordinary theory can be considered as a particular case. The formulation of boundary value problems of the theory of shells in the form of integral and integro-differential equations was given and various ways of composing these equations are developed as well. In particular, for the first time it was proposed to use the potential method for solving problems of shell statics [242], which was later developed in the works of D.V. Vainberg and O.L. Sinyavsky [489, 490].

Mykola Oleksandrovych Kilchevsky (1909 - 1979)

A number of studies by D.V. Vainberg [482, 484, 491, 493, 494] was devoted to the plate calculation. In the monograph [484], awarded the Academician Galerkin Prize, the plane problem and the problem of plate bending are considered as one generalized boundary problem of the theory of analytic functions. This made it possible to obtain a number of new results for plates consisting of concentric rings, as well as for the strip problem.

The theory of calculating circular plates of constant and variable thickness was developed by A.D. Kovalenko, Ya.M. Grigorenko and their students, who obtained exact solutions to a number of problems on the stressed state of circular plates and shells of rotation of variable thickness [196, 200, 280, 281, 282].

The method of calculating plates and shells with discontinuous parameters (ribs, step change in thickness, kinks, etc.), based on the use of impulse functions, was proposed by D.V. Vainberg and I.Z. Roitfarb [487].

The probability of the shells loss of stability were the subject of research in a series of works performed by M.V. Goncharenko at the Taras Shevchenko Kyiv State University [136, 137, 138, 139, 140].

The solution of a large number of problems on the calculation of plates of various shapes and with various boundary conditions was obtained by the grid method, the development of which was carried out in the works of P.M. Varvak and his followers [506, 507, 508, 510, 511].

Petro Markovych Varvak (1907 - 1979)

But the particularly rapid development of research on the calculation of plates and shells began when computers became a research tool, since the transition to computer analysis required a number of improvements in the theory related to formalization and algorithmization, and also revealed new possibilities in formulating solved problems (see below).

### 4.4. Inelastic behavior of structures

At the Institute of Structural Mechanics, beginning in the 1930s, new scientific directions for that time began to develop - an analysis of the behavior of structures beyond the elastic limit and the assessment of their ultimate bearing capacity. This analysis was based on a number of experimental and theoretical studies carried out under the guidance of M.D. Zhudin [530, 532, 533]. A distinctive feature of this cycle of work was its focus on the study of the work of specific constructive forms. For example, the behavior of continuous steel beams beyond the elastic limit under the action of repeated and moving loads was studied [531, 535]. The continuation of these works was an analysis of the behavior of the trussed beams, made by V.V. Trofimovych [459, 460] and the research of L.P. Kunitsky [288] on the assessment of the influence of shear force on the reduction of the maximum load acting on the statically determinate and statically indeterminate beams. Experimental work [534], adjacent to the same cycle, was aimed at assessing the bearing capacity of columns of the high-rise structure of the Palace of Soviets of the USSR, which was designed in the pre-war years.

Mykola Dmytrovych Zhudin (1891 - 1968)

The oblique (biaxial) bending of rectangular beams beyond the elastic limit for the case of perfectly plastic material was considered by B.M. Gorbunov and V.G. Chudnovsky [155], and somewhat later by O.I. Strelbitska. She also investigated the limiting state of a thin-walled profile under constrained torsion, including the dependence between torque and bimoment [431].

The study of the behavior of elastoplastic structures under alternating loading revealed the need for an analysis of adaptability. The works [287, 291] were devoted to this problem.

The work begun by M.D. Zhudin was continued by O.I. Strelbitska, who studied the inelastic behavior and the limiting state of thin-walled bars and plates [430, 433, 434]. Thus, the previously initiated studies of frames made of thin-walled bars were continued, but already concerned their behavior beyond the elastic limit. In this case, the main attention was paid to the development of models for the limiting equilibrium of thin-walled bars and their experimental testing. In particular, the ultimate load for a complex stress state (bending, complicated by torsion, etc.) was investigated. Adjacent to the same problems are the studies of S.A. Palchevsky [315, 316].

These studies, like the works of M.D. Zhudin, were focused on identifying the limits of the bearing capacity of steel structures for which the assessment of the reserve of strength and stability is very important [533]. An approximate method of calculation based on the use of an idealized Prandtl diagram and the hypothesis of the instantaneous appearance of plastic hinges has proven useful here. The validity of this approach was confirmed by a number of experimental studies performed by Ye.O. Paton and B.M. Gorbunov [321], B.M. Gorbunov [154], M.D. Zhudin [532].

But later a similar problem arose acutely in relation to structures made of reinforced concrete. Such studies were carried out in the Research Institute of Building Structures of the State Building Committee of the USSR (Naukovo-doslidny instytut budivel□nykh konstruktsiy, NDIBK) by A.M. Dubinsky [102] as applied to the limit equilibrium of flexural plates, and successfully continued in a series of works by A.S. Dekhtyar and O.O. Rasskazov [73, 371] as applied to the assessment of the carrying capacity of shells.

But for the calculation of concrete and reinforced concrete structures the most significant is the consideration of the possible formation of cracks and the assessment of the role of creep. Here, the fundamentally important results belong to I.I. Ulytsky, who in a series of experimental studies, analysed a number of features of the creep process (the influence of environmental humidity, the role of thinness, etc.), which made it possible to justify proposals for dvelopment a creep measure [466, 467, 468].

Yosyp Ioakhymovych Ulytsky (1912-1965)

Yakiv Davvdovvch Livshvts (1907 – 1984)

Accounting for the effects of shrinkage and creep on redistribution of forces in statically indeterminate systems has been the subject of research by Ya.D. Livshyts [296] and many scientists (see, for example, [7, 104, 135, 285]). Correct accounting for the formation of cracks was performed in numerical studies [298, 380, 529]. A detailed analysis of the elastoplastic behavior of reinforced concrete structures has become the topic of recent work [117].

## 5. THE ERA OF NUMERICAL ANALYSIS

The problem of solving complex design problems has always been a "sore spot" of the theory of structures. Among the attempts to solve it, one can name the replacement of the solution of the corresponding resolving equations by a model experiment. Here, first of all, one should mention the polarization-optical method for studying stresses (the method of photoelasticity) - an experimental method for studying the stress-strain state on transparent models of optically sensitive materials. At the Taras Shevchenko Kyiv State University such researches conducted V.I. Savchenko and his students [387, 388, 389], whereas in NDIBK they were performed by B.M. Barishpolsky [9, 497]. In the early 60s of the twentieth century, an energetic attempt was made to solve the aforementioned problem through the use of electrical modeling systems EMSS, developed under the direction of G.Ye. Pukhov [367, 368].

However, in mass practice, such approaches have not been rooted; they have been driven out by rapidly developing numerical methods, which were based on the use of an electronic computer.

**5.1. Computer technology and numerical methods**

The transition to machine-based problem solving technology was preceded by a relatively short period when the main computational tool for engineering calculations was desktop electric arithmometers of the Rheinmetall type serviced by the operators of computing stations. To work with such computing stations, a special technology and language of communication was created, allowing formalizing the sequence of computational procedures to which the problem solving algorithm came down. Here, the initiator was P.M. Sosis [421].

Already in the first steps it was found out that it was necessary to algorithmize some stages of calculation. In particular, there was a problem of a formalization of description of a complex flat or spatial system, its topology, reflected structure composition, and the basic operations of static computation of discrete systems.

Petro Moiseiovych Sosis (1918-1967)

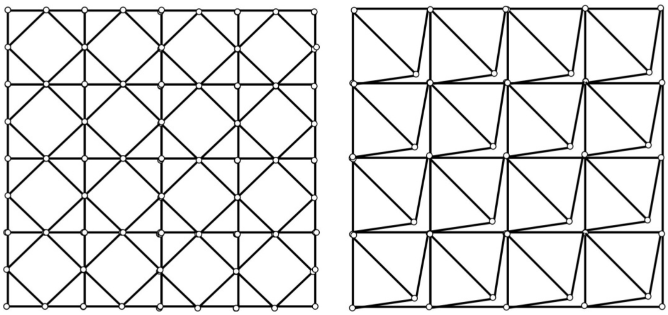
For a hinged trussed framework with an arbitrary topology A.V. Perelmuter suggested, apparently, for the first time to use the “node-bar” incidence matrix [334], and it turned out that the difference and summing operators of L.G. Dmitriev [100] were realized.

And it is quite clear that the finite difference method has become the natural way to solve the problems of calculating plates, shells and systems made up of them. A complication in its use was the need to move from the central differences to the "left" or right "differences when formulating the boundary conditions. One of the means of circumventing this complexity was the creation of bar models for which the equilibrium equations were equivalent to finite difference equations [99, 95, 305]. The authors of these papers justified the discrete model by the coincidence of its equilibrium equations with finite-difference representations of differential equations of a continuous medium.

So, for the biharmonic equation of the plane problem written through the Erie function $\varphi(x,y)$, when using the grid showed in fig. 7(a), a typical difference equation is as follows:

$$20\varphi_{0,0} - 8(\varphi_{1,0}+\varphi_{-1,0}+\varphi_{0,1}+\varphi_{0,-1}) + 2(\varphi_{1,1}+\varphi_{-1,1}+\varphi_{1,-1}+\varphi_{-1,-1}) + \varphi_{2,0} + \\ +\varphi_{-2,0}+\varphi_{0,2}+\varphi_{0,-2} = 0.$$

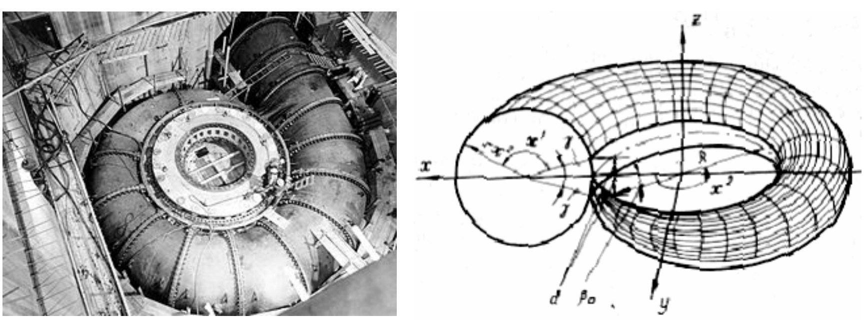
This equation can be interpreted as the canonical equation of the force method, written for a statically indeterminate truss, in which two variants of the structure are presented in Fig. 7, and sections of elements are selected in a special way. The advantage of such a simulation was that the models did not require the use of fringe points.



(a) (b)

Fig. 7. Simulation of a finite-difference equation with a bar lattice

In the works of the team of the Problematic Research Laboratory of Thin-Walled Spatial Structures (Problemna Naukovo-Doslidna Laboratoriya Tonkostinnykh Prostorovykh Konstruktsiy, PNDL TPK) of Kyiv Civil Engineering Institute (KCEI), conducted under the direction of V.I. Guliaev, the finite-difference method was successfully used to study shells of a non-canonical shape. For example, studies of the spiral shell with a variable radius of curvature shown in Fig. 8 were conducted [207], as well as shells with rapidly changing geometric parameters, shells of variable thickness, etc. [210, 212].




(a) (b)

Fig. 8. The spiral chamber of a hydraulic turbine:
(a) - general view, (b) – computational structural model

A radical tool turned out to be the transition to a variational formulation of the difference method, proposed for the first time in the studies of the team of the PNDL TPK of KCEI [501]. The usual way to obtain grid equations is to apply the finite difference method to differential equations and to boundary conditions. However, such a path in some cases (corners, contact conditions, mixed boundary conditions) leads to the fact that the matrix of systems of equations does not always turn out to be symmetric, which is inconsistent with the principle of reciprocity and complicates the problem computationally.

If a variational method of deriving grid equations is used, based on replacing the expression for the total potential energy of a system with a certain algebraic form according to certain rules of numerical differentiation and integration, the resolving function turns into a collection of its discrete values at the nodes of the region, and the functional reduces to a quadratic form of this values.

The grid equations are obtained from the expressions for the components of the gradient of the elastic potential at each grid point, while the symmetry of the coefficient matrix is guaranteed.

Many practically important problems were solved using the finite difference method in a variational formulation, (see, for example, [84, 93, 94, 502]).

With the active participation of O.A. Kyrychuk, studies were conducted aimed at ensuring the reliability of a fuel tank with a protective capacity at the Ukrainian Antarctic station Academician Vernadsky [289, 290]. Obtained results made it possible to formulate the basic requirements for the further safe operation of the tank in extreme Antarctic conditions.

In the 70-80s of 20th century in Kyiv a variant of the finite difference method was developed under the guidance of Professor Ye.O. Gotsuliak. It was the so-called curvilinear grid method, in which the problem of accounting rigid displacements was solved [195]. The method was effectively used to solve nonlinear problems of deformation and stability of complex shells [194].

Oleksandr Adrianovych Kyrychuk (1948 – 2018)

Yevgenii Oleksandrovych Gotsuliak (1942-2011)

**5.2. Finite element method**

But the true revolution in the problem of numerical calculations came with the introduction of the finite element method (FEM). This method was used and developed by many scientific teams of Kyiv (KCEI, Institute of Mechanics of the Academy of Sciences of the Ukrainian SSR, UkrRDIsteelconstruction, Kyiv Zonal Researche Innstitute for Experimental Design (Kyivsky zonalny naukovo-doslidny instytut eksperymentalnoho proektuvannya, KyivZNDIEP), etc.).

In the PNDL TPK of KCEI O.S. Sakharov developed the moment scheme of the method (MSFE), which formed the basis of a system of software for a computer, and was actively used to solve a wide variety of problems including plates, shells and three-dimensional bodies computation [38, 384]. This scheme was actively developed and adapted to solving problems of statics and dynamics of systems of various types, including non-linear problems [38, 39, 261, 379].

One of the advantages of the MSFE is the zero response to the rigid displacement of the element. Some of the other variants of the finite element method did not always correctly take into account the problem of rigid displacement when calculating shells. In addition, for many of them, another negative property of the stiffness matrix, called "shear locking," was observed, when bending thin plates and shells modeled with three-dimensional elements. In this case, errors associated with the manifestation of fictitious shear deformations significantly increase.

In particular, these problems become significant when curvilinear coordinate systems are used, which are introduced in order to better describe the geometry of bodies of complex shape. Then the components of the deformations depend not only on the derived displacements, but also on the displacements and turns of the element as a whole.

In order to eliminate these deficiencies of the MSFE, techniques [261, 377, 379, 385] were developed, allowing to take into account the basic properties of rigid displacements for isoparametric and curvilinear finite elements. Their essence lies in the fact that when recording the conditions for the connection of deformations with displacements, those expansion terms of deformations are rejected in a series that react to rigid displacements and to fictitious shear deformations. In this case, the exact equations of the connection of deformations and displacements are replaced by approximate ones.

The moment scheme was used mainly in the work of the PNDL TPK of KCEI for solving problems, the design scheme of which was represented by three-dimensional finite elements. At the same time, in the works of V.M. Kyslooky and V.K. Tsykhanovsky the nonlinear deformation of cable-stayed systems, as well as soft and hanging shells was investigated using the moment scheme (Fig. 9) [43, 258, 263].

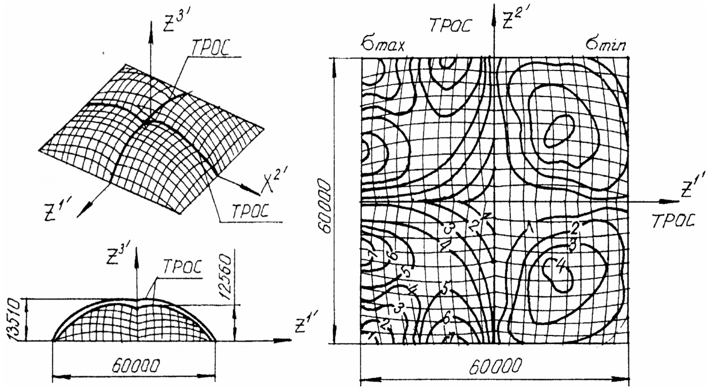
Volodymyr Mykytovych Kyslooky (1940-2017)

Valentyn Kostiantynovych Tsykhanovsky (1937 – 2012)

Viktor Volodymyrovych Kyrychevsky (1945 – 2006)

In the works of V.V. Kyrychevsky, this approach was used to solve problems related to nonlinear deformation and fracture of structures made of elastomers [249, 250].

Other teams preferred to use the classical FEM scheme, where the library of finite elements contained one-dimensional (bar), two-dimensional (plates and shells) and three-dimensional elements (see, for example, [187, 188, 190, 306]).



(a) (b)

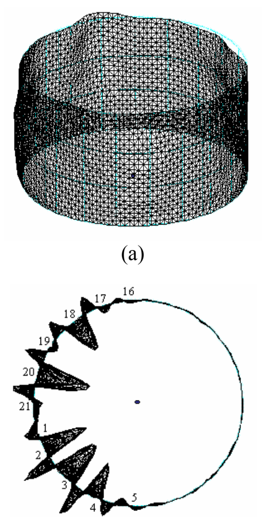
Fig. 9. The results of the calculation of the air-supported soft shell with cables [43]:
(a) – deformed discrete model of the middle surface, (b) – countors of principal stress

The finite element method in various modifications was used as the basis for the development of software systems and application packages that were developed at KCEI (the STRENGTH-85 system), KyivZNDIEP (GAMMA-2), UkrNDIproekt, Giprokhimmash, Research Institute for Automation Systems in Construction (Naukovo-doslidny instytut avtomatyzovanykh system u budivnytstvi, NDIASB) (programs of the LIRA family), UkrRDIsteelconstruction (PARADOX, PARSEK, SUDM), Research and Production LLC SCAD Soft (SCAD Office) and in other Kyiv organizations. In the 60s of the twentieth century, Kyiv became the de facto "capital of the FEM" of the Soviet Union, it remains the main center of development of this type even today, when the SCAD and LIRA software systems are the main tool for designers in the CIS countries.

The development of the finite element method took place in different directions: the library of finite elements was expanded, including at the expense of incompatible elements, for which convergence was proved [110, 187, 240], elements of multilayer plates and shells [360], and others. An attempt was made to create a finite element system [181, 182] that has a number of attractive features (the interpolating function realizes the minimum potential energy of the element deformations, all the major minors of the stiffness matrix and all its eigenvalues have a minimum value, etc.). A finite element model was implemented based on the moment theory of elasticity (Cosserat's body), which, according to the authors, made it possible to better represent the behavior of the structure in stress concentration zones [310].

The original scheme of the finite element method was proposed by Ye.O. Gotsuliak. The peculiarity of this scheme was the vector representation of the displacement function in the general curvilinear coordinate system. When constructing the stiffness matrix of a curvilinear finite element, the vector approximation of the displacement function was represented by Maclaurin's series, whose coefficients were the values of the desired vector function and its derivatives in the center of the finite element. The proposed approach was later extended to geometrically nonlinear problems of deformation and stability of thin shells with shape imperfections (Fig. 10) [300]. Figure 10(a) shows a finite element model of a fuel tank with real shape imperfections, and figure 10(b) shows a picture of a deformed state in the cross section of the shell corresponding to the stationary point of the equilibrium curve.

Certain difficulties in justifying the FEM arose for the case of using incompatible finite elements. The shape functions for these elements do not belong to the energy space and the standard method of proving "convergence in energy" does not work here. A rather general method for studying the convergence of incompatible finite-element systems, as well as a method for constructing convergent incompatible elements, was indicated only in 1981 by I.D. Yevzerov [108, 109].




(a)

(b)

Fig. 10

Many developments do not use curvilinear finite elements of shell theory instead of which flat elements are used (the shell is modeled by a polyhedron). The refusal to use curvilinear elements, which gave a number of simplifications, was justified by the evidence of their convergence [111, 112].

A definite alternative to the finite element method is combined approaches to solving multidimensional problems, in which the dimension of the original equations is reduced analytically at the first stage, and numerical methods are used at the second stage to solve the reduced equations. One of such combined methods is the potential method used by D.V. Vainberg and O.L. Syniavsky [489] for shells calculations, and in the works of Yu.V. Veriuzhsky [514, 515] for solving of a fairly wide class of problems.

The investigations of G.B. Kovnerystov [283, 437] were devoted to the contact problems, which were solved numerically by means of integral equations.

Oleksandr Leoni-dovych Syniavsky (1937-2018)

Yurii Vasyliovych Veriuzhsky (1937-2017)

Pavlo Pavlovych Voroshko (1940-2014)

Georgii Borysovych Kovnerystov (1929 – 1992)

To determine the stress-strain state of various structural elements, as well as solving problems of fracture mechanics, P.P. Voroshko applied both the finite element method [526] and the method of boundary integral equations [525].

In the works of V.K. Chybiryakov the method of finite integral transformations was modified for the first stage, while different effective numerical methods were used to solve the reduced equations at the second stage. This method was widely used by the author and his students for solving static and dynamic problems [56, 57].

Other methods for solving spatial problems were also developed [293, 294, 517].

Instead of the transition from the system of differential equations to algebraic equations, which is characteristic of the finite element method, a transition to a system of ordinary differential equations is possible. Close to this idea is the method of lines (differential-difference method) – this is a very effective method of reducing the dimension of the original boundary value problems.

L.T. Shkeliov [401, 402] extended the method of lines to the stress-strain analysis of plates of arbitrary shape.

It can be noted that a significant role in the development of methods for calculating spatial structures of the shell type was played by problems connected with the analisys of a number of original engineering objects that were developed and put into practice by the team of researchers of the KyivZNDIEP Institute (Fig. 11).

(a) (b)

Fig. 11. Shell roofs: (a) - airport terminal in Boryspil, (b) - furniture house in Kyiv

**5.3. Semi-analitical finite element method**

Significantly increase the efficiency of the FEM allows its integration with the method of separation of variables. This approach is called the semi-analytic finite element method (SAFEM). Its essence consists in decomposition of the unknown quantities along one coordinate in a certain system of continuous smooth basis functions in combination with finite element discretization along the other two.

For a long time, the Research Institute of Structural Mechanics of the Kyiv National University of Construction and Architecture conducted research aimed at expanding the scope of SAFEM to simulate the processes of deformation of spatial inhomogeneous bodies of complex shape for cases of static and dynamic loading [21, 24], which required a number of important issues. Thus, the issues of combining SAFEM with a moment scheme of finite element (MSFE) [222] were considered which made it possible to create a unified approach to modeling the deformation processes of massive, thin-walled and combined structures. In contrast to the well-known approaches, where SAFEM was used only for homogeneous rotation bodies or prismatic bodies with hinged boundary conditions at the ends, the method is adapted for calculating non-uniform non-canonical bodies: cyclically symmetric [219, 220, 381, 383] and prismatic curvilinear bodies [222, 224]. In addition, modeling of arbitrary boundary conditions at the ends of bodies is provided [223].

On the basis of the new types of created finite elements, the techniques for physically and geometrically nonlinear problems analysis [21, 221, 222, 224] as well as dynamic processes [23, 26, 29, 226] have been developed. Also the techniques for modeling crack development from the point of view of fracture mechanics [28, 225] were obtained.

Some examples of solved problems are illustrated in Figures 12, 13.

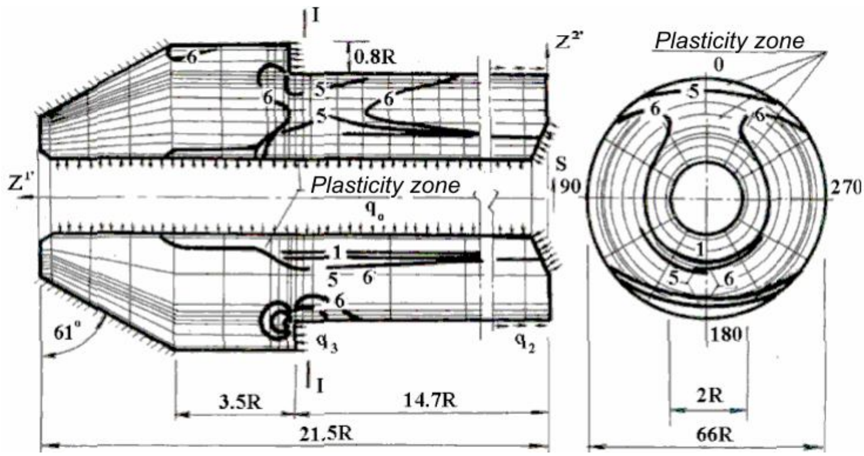



Fig. 12. Nonaxisymmetric nonlinear deformation of the high-pressure valve

### 5.4. Numerical analysis of shells

The application of FEM has significantly expanded the class of researchable problems. The subject of research was the non-classical problems of the theory of plates and shells, where non-canonical-shaped structures, complex boundary conditions, multilayer shells, etc. were considered. Great attention was paid to shells with holes and cutouts, as well as shells with different types of reinforcements [30, 33, 198, 259]. Tissue shells were also considered [167, 172]. A large complex of studies was associated with the analysiss of the behavior of multilayer shells [19, 37, 372, 375].

The stress-strain analysis of layered shallow shells, the material of which is orthotropic, is described by a system of differential equations in partial derivatives of the tenth order with variable coefficients and corresponding boundary conditions. The solution of such a problem involves considerable computational difficulties. Therefore, to solve it, a numerical-analytical approach based on reducing the two-dimensional boundary-value problem to systems of ordinary differential equations using the spline-approximation method in one of the coordinate directions was proposed. The obtained one-dimensional boundary-value problem was solved by a stable method of discrete orthogonalization [204, 205, 524]. In this way, a large number of static and dynamic problems were solved [196, 202, 203].

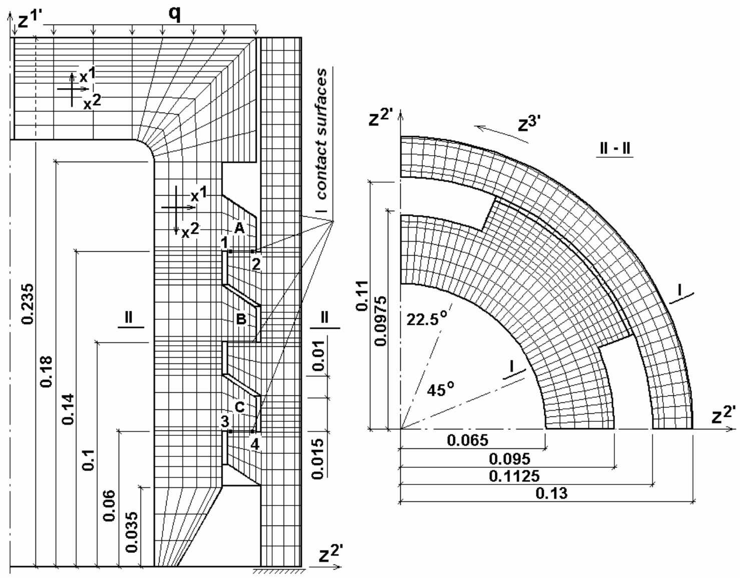



Fig. 13. Dynamic interaction of parts, heterogeneous in the annular direction

A team of authors from the Kyiv Automobile and Highway Institute (V.G. Piskunov, V.Ye. Veryzhenko, A.F. Ryabov and others) developed the theory of multilayer plates and shells, which was based on the principle of using kinematic hypotheses common to the entire multilayer package [357, 358, 359, 370, 519].

Modern trends in the development of structural mechanics and the practice of designing thin-walled shell structures lead to the development of refined numerical methods for the study of nonlinear deformation and stability of shells of various types.

Under the guidance of Professor M.O. Solovei at the Kyiv National University of Construction and Architecture numerous studies of nonlinear deformation, stability and post-critical behavior of a wide class of thin elastic

Vadym Georgiiovych Piskunov (1934-2016)

Mykola Oleksandrovych Solovei (1946–2014)

inhomogeneous shells of various shapes and structures under the action of thermomechanical fields were conducted [41, 42, 259].

The two-dimensional theory of plates and shells, as well as the one-dimensional theory of bars, was not used, and the entire analysis was carried out from the standpoint of the three-dimensional theory of elasticity. The need for a three-dimensional approach to the computation of thin shells is caused by the fact that real shell structures are often designed as heterogeneous systems: smooth and step-variable thickness, with kinks, reinforced ribs and pads, weakened holes, grooves and channels, faceted, multi-layered. The problem of choosing a design model (or a combination of several) for a fairly accurate approximation of design sections with different geometric and physical features was solved as follows.

Two types of its features were considered: a) geometric features in the form of continuous and step-variable thickness; b) structural inhomogeneities of the material along the thickness and in the plan in the form of a combination of multilayer packets. Each layer of material may be anisotropic and different. Thus, thin multilayer shells of variable thickness and of complex geometric shape are considered as three-dimensional bodies that can be supported by ribs and overlays, weakened by grooves, channels and holes, and have breaks in the middle surface (Fig. 14) wlile Fig. 15 shows examples of calculated structures of complex shape.

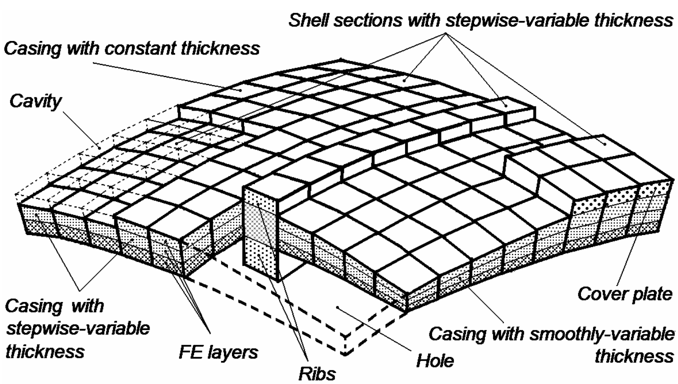



Fig. 14. Shells heterogeneity

A non-classical kinematic hypothesis of a deformable straight line is used - a straight line in the thickness direction, which contracts or elongates, remains straight after the deformation of the shell. This line is not necessarily the normal to the midsurface of the shell. The layers of the shell are rigidly interconnected in a monolithic package and are jointly deformed without slipping and tearing along the surfaces of contact on which the condition of equality of the components of the displacement vector is satisfied. A model of an elastic nonlinearly deformable continuous mediua is used. More specifically, the components of large displacements and small deformations are linear functions of displacements.

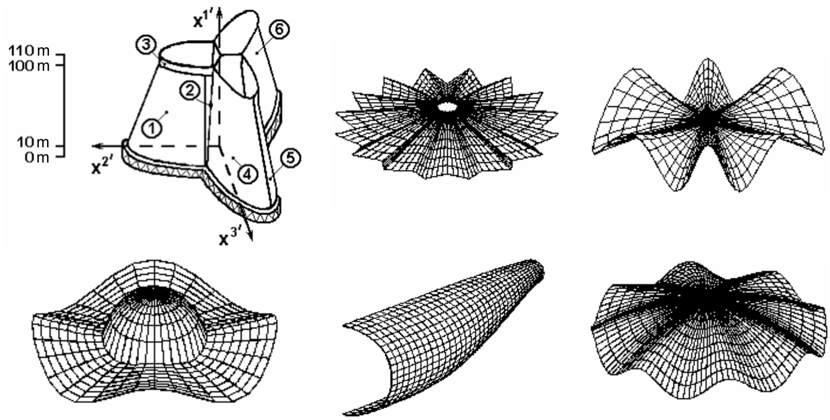



Fig. 15. Examples of thin-walled shell structures of complex shape

The difficulties of taking into account the joint work of structural elements of various dimensions in a heterogeneous shell are overcome by modeling sections of stepwise variable thickness with the same type of spatial finite element. In the formation of resolving equations, the procedures for determining the necessary values of the element parameters are used, which allow to accurately describe the geometry of structural elements and take into account their joint work as three-dimensional bodies.

An assessment of the curvature K = $2a^2/(Rh)$ effect on the stability of a smooth, hinged panel of constant thickness is presented in Fig. 16 as an example.

The shell under consideration was heated linearly over thickness (the outer surface cooled and the inner heated by the same value T, °C). Equilibrium modes are stable up to the top and after the bottom critical points of the diagrams. The panels at K ≤ 16 do not lose their stability. It was established that the “shallow” panels (K ≤ 30) deform and lose their stability according to the “f” mode. The modes of deformation of “non- shallow” panels (K> 30) during the loading process change and become more complex (modes “a - e” for K = 48).

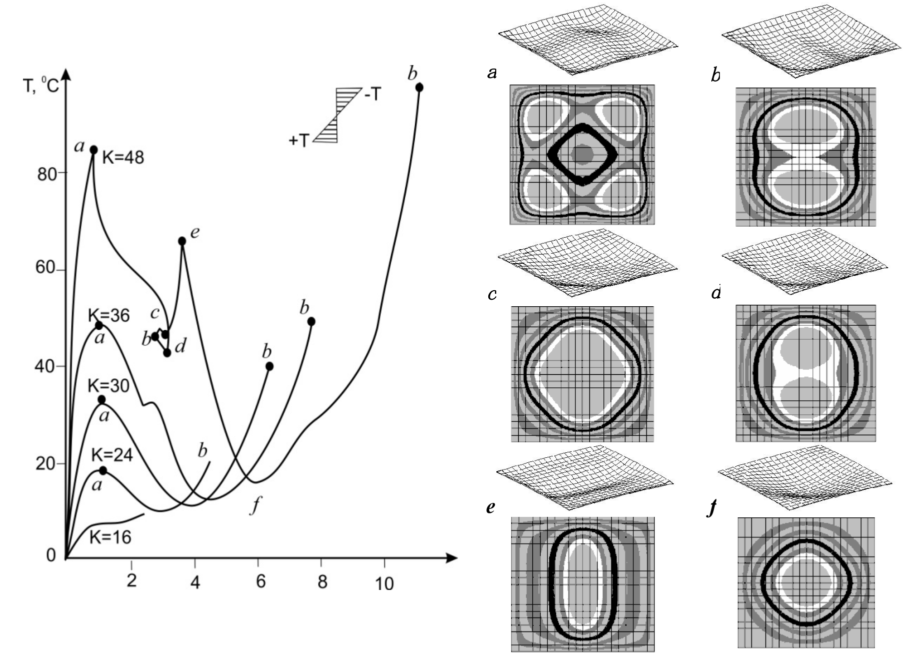



Fig. 16. Shell stability under thermal loading

Currently, this topic has been extended to the problems of natural vibrations of inhomogeneous shells, taking into account the prestressed state, when the modal characteristics of the object are calculated at each step of thermo-force loading [10].

Note that not only the stability of the shells was developed in the framework of numerical studies using computers. Some characteristic tasks were investigated, the solution of which without the use of computers was previously inaccessible. These included such essentially nonlinear problems as the stability analysis of systems with one-way connections [16, 32], the spatial stability problem of the skeleton of high-rise buildings [364], a subtle analysis of the role of ribs [3], and many other problems that arise in design practice.

**5.5. Computational problems**

One of the most laborious tasks arising in the process of calculating structures is the solution of large systems of linear and nonlinear equations. The limitations of the random-access memory and the low speed of exchanges with external memory, characteristic of the first computers available for use, made it necessary to focus on the use of iterative methods for solving the problem [264, 499, 500]. Various variants of the descent method were tested and developed: the gradient method, the ravine method, the upper relaxation method, etc. The gradient descent method was used to find the minimum eigenvalue in the stability problem for rectangular plates with notches [376].

Only gradually, around the end of the sixties, were the first successful results of applying direct methods for solving large systems of linear equations demonstrated. For example, it was pointed out that the Gauss block method was effectively used with a minimized number of references to external memory [232]. It was especially effective in solving problems with a large number of loading options or in combination with iterative algorithms that require multiple solutions of equations with a constant matrix and variable right-hand sides (for example, in the method of elastic solutions).

Two circumstances played an important role in promoting direct methods: taking into account the tape structure of the coefficient matrix, which is realized with a certain numbering of unknowns, and effective discipline of exchanges with external memory [18, 422, 423]. Naturally, the development of computational algorithms, leading to the appearance of a successful structure of the coefficient matrix, was the subject of special studies [5, 186].

Starting from the 90s, direct methods, which take into account the sparse structure of the matrix, have gained great popularity. Due to efficient ordering algorithms, which significantly reduce the fillings in the factorization process, it is possible to significantly reduce the size of the factorized matrix and the duration of calculations [121, 122].

In modern commercial and scientific FEM programs when solving large-scale problems, the most widespread are the subtle implementations of the subspace iteration method and Lanczos method. The method of iterating a subspace has established itself as a very reliable, although being not the fastest method. The block version of this method is especially effective when using shifts [5].

The generalized conjugate gradient method based on aggregate multilevel preconditioning [113] combines the advantages of the conjugate gradient method, aggregate preconditioning, and the use of a shift strategy.

Serious computational difficulties were overcome in solving the problem of the inelastic behavior of reinforced concrete structures. Two different methods of solving it were considered. In the first method, the load parameter is the leading one. The algorithm [118] uses the implicit Newmark method when balancing nonlinear iterations by the Newton-Raphson method. In it, the leading parameter is the load. When solving many problems of statics, this approach leads to the fact that when "slipping" the limit point of the equilibrium state curve (the "load-characteristic displacement" curve), the Newton-Raphson method diverges. Therefore, an algorithm for solving a nonlinear static problem was applied, in which the leading parameter is the length of the perpendicular to the tangent to the equilibrium curve.

The procedures for the formation of a matrix of tangential stiffness, its factorization, direct and inverse substitutions, and finally, calculations of the vector of internal forces are performed by more than 99% of the computational work, so they demanded maximum acceleration. The stiffness matrix factorization and direct / inverse substitutions are determined by the choice of the solver [115, 116, 118]. The peculiarity of these tasks is that the duration of the aggregation procedure of the tangential stiffness matrix, as well as the procedure for determining the vector of internal forces are values of the same order as the duration of solving a system of linearized algebraic equations, since the calculation of finite element stiffness matrices is associated with the calculation of a large number of integrals. Therefore, schemes were developed for parallelizing these algorithms.

The appeal of specialists in the theory of structures to the construction of computational algorithms proved to be useful and fruitful. As an example, we can refer to the idea of using superelements, proceeding from mechanics, which then received independent development in computational mathematics in the form of a block multi-frontal method for solving equations [118]. At the same time, the very idea of the simultaneous formation and solution of resolving equations (frontal method) was also proposed by specialists in structural mechanics. Other issues of the computational plan were also solved, in particular, methods for improving conditionality were found [118, 373], a system for monitoring results was created. The discovered mechanical interpretation of the computational operations turned out to be useful, in particular, the fact that the step of Jordan exceptions corresponds to the installation / removal of the connection [170].

However, not only the problem of solving resolving equations was the subject of research. For example, the algorithmization of stability problems with the massive use of complex spatial design schemes required more precise answers to a number of fundamental questions. And here some important results were obtained: it was shown in [348] that the habitual use of the concept of computational lengths cannot be applied in some spatial problems, and in problems solved taking into account the sequence of construction creation it is necessary to clarify the very issue of stability [331, 340]. As applied to spatial problems, it was necessary to more strictly substantiate the possibility of using the well-known method of checking the location of a given loading parameter in the spectrum of critical values [419].

### 5.6. Numerical solution of nonlinear problems

Computer modeling has become practically the main method for solving nonlinear problems, for which complicated performances have become real, both from the point of view of the geometry and structure of the analyzed objects, and taking into account the complicated loading conditions and the influence of the external environment.

Naturally, the transition to non-linear problems required clarification, specification and improvement of the usual approaches. It was necessary to develop algorithmic numerical methods for nonlinear analysis, which can be implemented on a computer, and allow for a global study of resolving nonlinear differential equations, identification of singular points, construction of branching solutions and analysis of their stability.

To overcome the basic mathematical difficulties in solving static problems, in [208] it is proposed to use the method of continuation with respect to the loading intensity parameter, based on the transition to the Cauchy problem, and the numerical analysis of the obtained problem. And in the neighborhoods of singular points where the problem degenerates and vanishes the Jacobian of a system of nonlinear algebraic equations (critical value), an algorithm for approximate branching is used. In the study of the stability of T-periodic processes in [209], a synthesis of the procedure for the continuation of the solution by the leading parameter, the Floquet theory, and the methods of the theory of branching was proposed. An example of such an analysis from [213] in the form of the dependence of the amplitude of vertical oscillations of the node of the von Mises truss on the loading intensity is presented in Fig. 17.

When passing through the limiting points, the method of changing the leading parameter was used, later a method was used that applies the length of the arc of the state curve as the continuation parameter [35]. This markedly simplified the passage of a turning point in the study of equilibrium states or regular periodic motions. The continuation technique makes it possible to find regions of stable and unstable periodic solutions using the eigenvalues of the monodromy matrix on the basis of the Floquet theory.

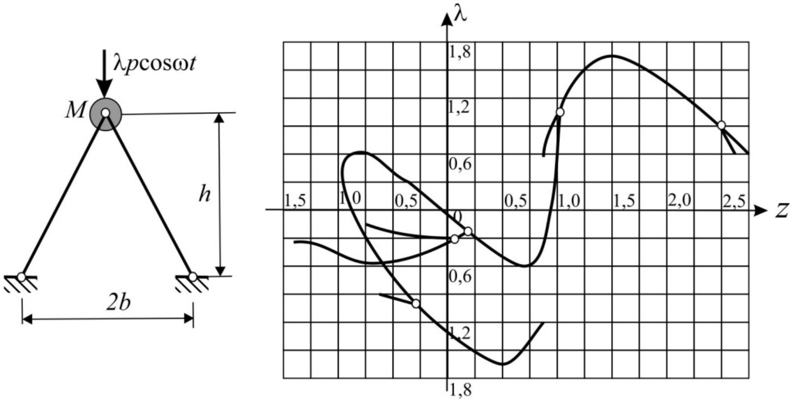



Fig. 17. Dynamic loading of the von Mises truss

The methods developed in [208, 209] were widely used to solve a number of specific problems that were performed at the PNDL TPK of the KCEI (see, for example, [213] or [216]).

In the monograph by V.I. Guliaev, V.V. Gaidaichuk and V.L. Koshkin [214] nonlinear problems of statics and dynamics of flexible bars were considered. The behavior of flexible rectilinear and curvilinear bars, subjected to arbitrary static and dynamic disturbances caused by force and kinematic excitation of harmonic oscillations, portable, relative and Coriolis inertial forces, gyroscopic forces of rotating rotors, nonconservative forces of interaction with external and

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104

internal flows of liquid and gas was studied. An example is the analysis of the behavior of helical coil springs under static loading (Fig. 18).

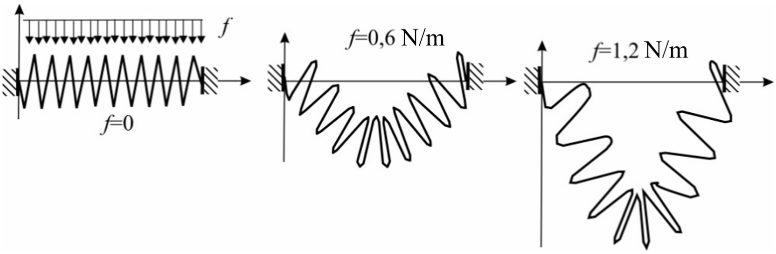


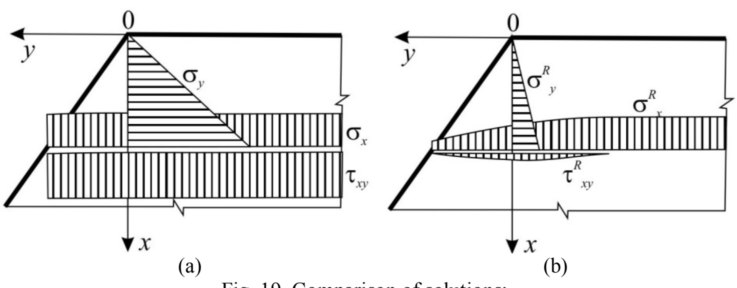

Fig. 18. The deformation of the coil spring

Later V.I. Guliaev, V.V. Gaidaichuk and their students considered a number of practical important and complex tasks related to the blades of helicopters and wind turbines, as well as the operation of drill strings for ultra-deep oil and gas production (see, for example, [215, 217, 218]). In this case, some effects and phenomena that had not been studied before were discovered.

When performing the stress-strain analysis of plates and shells, first of all, attention was paid to their work during large deflections (see, for example, [179, 199, 201, 295]). Nonlinear problems with physical nonlinearity of various origins were also studied (see, for example, [22, 103, 176, 180, 191, 252, 284, 435]).

Speaking about the achievements of the Kyiv school in solving nonlinear problems of the theory of structures, it should be noted that it was in the Kyiv Institute of Hydromechanics of the Ukrainian Academy of Sciences where E.Y. Rashba opened a new class of nonlinear problems that later became called genetically nonlinear ones [369]. This work was the first study in which the mechanical problem of building up a solid in the field of mass forces was solved. Quasistatic elastic stresses in an infinitely extended wedge, continuously increased by not strained but heavy layers were determined in it. The resulting solution is markedly different from the classic one, which is clearly shown in Fig. 19.

Emmanuil
Yosypovych Rashba




(a) (b)

Fig. 19. Comparison of solutions:
(a) - with instant erection, (b) - taking into account gradual build-up

Nonlinearity of this type is especially pronounced in massive structures such as dams. Later another employee of the Institute of Hydromechanics L.I. Dyatlovytsky actively engaged in the computation of such objects after the professor E.I. Rashba [84, 85, 86].

It should be noted that in recent years, genetic nonlinearity is almost always taken into account when designing high-rise buildings, bridges and other complex structures, where the effect of changes in the calculation structural model is noticeable [342].

**5.7. Numerical methods of probabilistic calculation**

Only a few problems of probabilistic analysis and statistical dynamics of structures can be solved analytically. Numerical studies of such problems were performed in the department of statistical methods in structural mechanics at the PNDL TPK of KCEI (later - Institute of Structural Mechanics of Kyiv National University of Construction and Architecture, KNUCA) under the guidance of Ye.S. Dekhtiariuk and dealt with various problems.

Yevgenii Semenovych Dekhtiariuk (1935-2012)

Statistical processing of the results of field tests was the subject of research in the works [48, 123, 518]. On the basis of the developed algorithms, a software system was created that focused on processing experimental data concerning the parameters of the mechanical behavior of aircraft.

The development of a generalized statistical testS method (Monte Carlo method) and its use for solving problems in the theory of elasticity are presented in [79, 303, 304]. Selective realizations of random external loads were modeled on the basis of the fast Fourisr transform [302]. In this way, problems of stationary random vibrations of mechanical systems were solved in linear and geometrically nonlinear formulations, and the probability of failures was also studied [304].

In problems of statistical dynamics [14], the main attention was paid to the analysis of dynamic stability and the study of simple and combination resonances [12, 13, 77, 80, 81, 82, 233].

**5.8. Cable-stayed structures**

A peculiar class of nonlinear problems is associated with the calculation of cable-stayed and cable-stayed-bar systems, which began to be very actively studied in the sixties of the twentieth century. Interest in this problem was initiated by the translation of the book by F. Otto “The Suspended Roof”, which appeared in 1960, and was stimulated by the real problems of cable-stayed systems design that were solved in UkrRDIsteelconstruction, KyivZNDIEP and NDIBK.

The first works performed in Kyiv belonged to V.M. Gordeiev [168, 171, 173] and L.G. Dmitriev [96, 97, 98]. The basic resolving equations of statics and dynamics for cable-suspended roofs having the form of a grid located on the surface of negative Gaussian curvature (Fig. 20) were obtained. In the work [167], the mesh was so thick that it was already a tissue membrane. Similar was the formulation of the problem in works performed in the PNDL TPK of KCEI [257, 262, 377], where the main resolving equations were obtained and analyzed for statics and dynamics of cable-stayed networks. The approach to static calculation as a special case of dynamic analysis made it possible to develope an effective scheme for solving nonlinear problems of statics using a modified dynamic relaxation, called the method of discrete breaking [257].

The problem of calculating the cable system should be solved in a geometrically nonlinear formulation. An exception can only be a calculation for the equilibrium load, and then only if the elastic displacements it causes are sufficiently small. With regard to cable-stayed networks, the possibility of linear formulation was examined and justified by V.M. Gordeiev [171, 172], who indicated the limits of applicability of such an approach and solved a number of important practical problems.

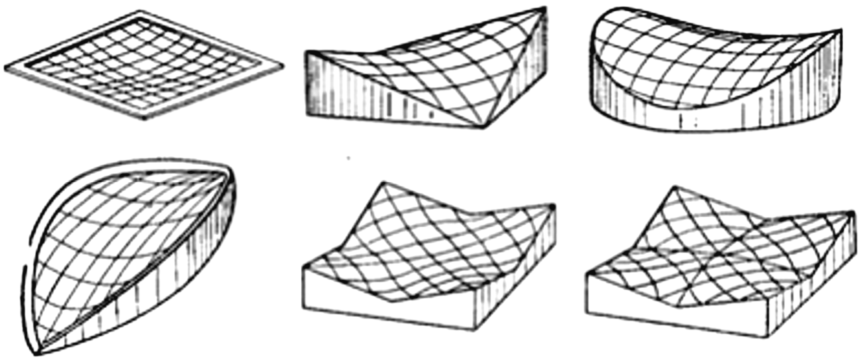


Fig. 20. Cable-stayed networks

Static-kinematic analysis of cable-stayed systems was investigated by L.G. Dmytriiev [96], while the fundamentally important role of the possibility of creating a prestress for instant-rigid systems to which cable networks belong, was established, and also the ranking of properties associated with the possibility of creating a prestress was indicated. It is indicated that the prestress corresponds to the constraints that prevents the skewing of the bars.

Leonid Georgiiovych Dmytriiev (1931–2018)

Oleksandr Vasyliovych Kasilov (1933 - 1988)

In another work, L.G. Dmytriiev presents cable-stayed networks of not only a simple rectangular structure, but also systems of a more general form (triangular, hexagonal, etc.) [97]. Extensive information on the structural solutions of cable-stayed coatings, including the author's development, is presented in a monograph written by him together with O.V. Kasilov [99]. The work of the team, headed by

L.G. Dmytriiev and O.V. Kasilov, on the development and implementation of cable-stayed structures in the construction were awarded the State Prize of Ukraine.

L.G. Dmytriiev proposed an iterative process for calculating cable-stayed networks, at each stage of which the magnitudes of displacements are refined until the unbalanced part of the load found from the static conditions is sufficiently small [101]. Subsequently, L.G. Dmitriev applied another fast converging process to the calculation of networks, based on the separation of two types of geometric nonlinearity), which make it possible to use the computer efficiently [100].

A special case is associated with the calculation of vertically arranged flat cable-stayed networks, which served as a supporting structure for a short-wave antenna (Fig. 21). The tension of the elements of such a network was provided by a system of counterweights and were known.

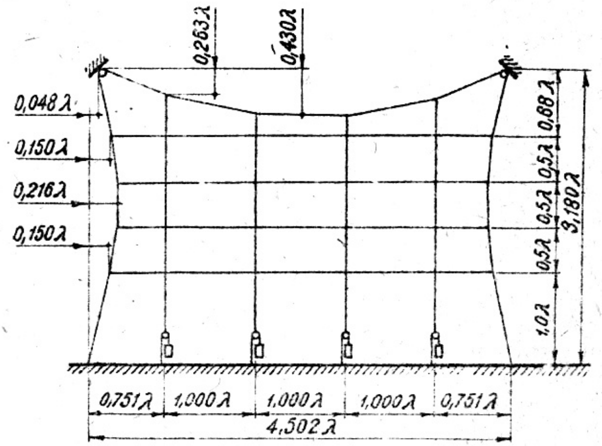



Fig. 21. Layout of antenna network for radio waves of λ length

For such networks, a kind of structural mechanics has been built, where the relative cable tension - the projections of the force in the thread onto the network plane related to the length of the thread section - play the role of stiffness. This structural mechanics is linear with respect to the displacement of network nodes out-of-plane and with respect to the angle of inclination of cables to the network plane. These values are linearly dependent on the load. But equilibrium equations alone, as a rule, are not enough for constructing these diagrams. As a result, statically indeterminate problem arise that can be solved by the force method and the displacement method.

An independent and difficult task is the problem of forming spatial cable-stayed networks. In such a network, the length of the cables must be chosen so that the network, even in the absence of a load, is taut and has a functionally determined shape.

Most cable-stayed networks are designed so that the cables in the nodes are not interrupted, and during the installation period they only touch the crossing points and slide one over the other. The network acquires its form at this particular time.

To solve the corresponding problem, a kind of technique was proposed [165, 1884, 185], in which a nodal insert was assigned to the crossing point of two cables (Fig. 22) without interfering with the mutual displacement of the cables. Each insert is characterized by three spatial and two cable coordinates, which are unknown when performing the calculation. The spatial coordinate is the Cartesian coordinate of the center point of the insert in the general coordinate system, the cable coordinate is the distance from the beginning of the cable to the axis of the insert. The system of nonlinear equations for the mentioned unknowns was solved by the step iteration method.

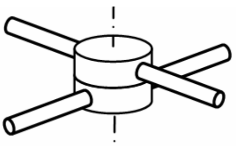


Fig. 22. Nodal insert

Fig. 23 shows the result of the choice of the shape of a radial cable network with a rigid external and flexible internal contour, to which the radial cables are attached so that their ends can freely slide along the contour. The shape of the inner contour and the coordinates of attachment of the radial cables to it are defined flawlessly.

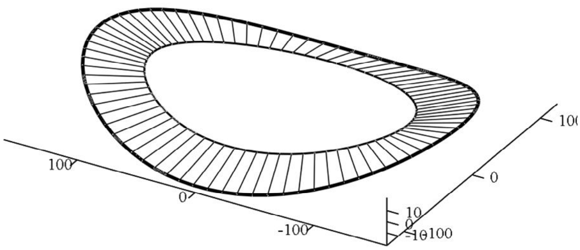



Fig. 23. Radial cable network with flexible internal contour

Suspended roofs have often been considered from the point of calculation of very flexible membrane-type shells (see, for example, [263]). It should be noted here that when it comes to awning structures where the fabric is used, it is necessary to take into account that the fabric shell not only does not perceive bending moments (and this is similar to a membrane), but also does not perceive a shear force. This effect was taken into account in [167, 172], and is not taken into account, for example, in [43]. Thus, tent constructions were excluded from the consideration, and examples of calculations of soft shells belonged to constructions made of rubber-material.

For such structures, as well as for thin membranes, the problem of transition to a uniaxial stressed state in the zones of fold formation is important (Fig. 24). To solve this problem, two main methods were used: modeling the zone of fold formation by a constructive anisotropic material [43, 395], or modeling with a material that did not perceive compressive stresses, i.e. peculiar unilateral constraint [179].

The methods of static calculation and stability testing were developed for the cable-bar systems, to which the masts, bridges, etc. belonged (Fig. 25). The canonical equations of the force method, the displacement method and the mixed method were generalized for such structures [169, 328, 333]. The dynamics of such systems is discussed in [258]. At the same time, the possibility of disconnecting from the work of the cable element, which turned out to be compressed at some point in time, and its incorporation into work at restoration of tension, was taken into account.

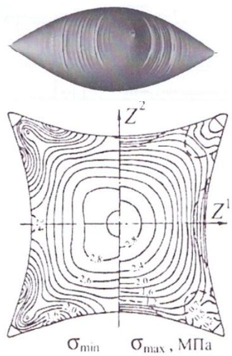
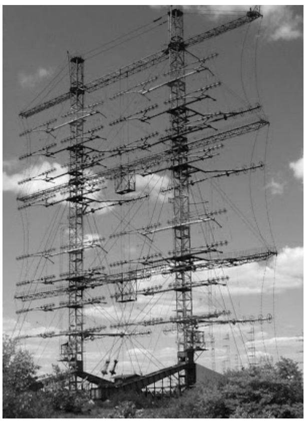



Fig. 24. Results of study of pneumatic pillow (the folded area is limited by the dotted line)



(a)

(b)

Fig. 25. Cable-bar systems: (a) - transmitting antenna, (b) - cable-stayed bridge

An important engineering problem related to the regulation of forces in such structures as applied to cable-stayed bridges was solved in the works of G.B. Fuks and M.M. Korneiev [125, 267], who considered regulation as a process of creating prestressing in a weightless design scheme by forced deformation of the design model being manufactured.

In parallel with the works on the theory of cable and cable-bar systems, studies have been developed on the methods for calculating flexible ropes of finite stiffness which were conducted by V.M. Shymanovsky [393, 396, 400].

Geogii Borysovych Fuks
(1927-2008)


Vitalii Mykolaiovych Shymanovsky (1928- 2000)

A natural generalization of the design model of the cable-bar system, in which there are non-perceiving compression cable elements, is a system with unilateral constraints, to the analysis of which was devoted a series of works of a pioneer nature. Thus, in [183] it was proved that the problem of static calculation of systems with unilateral constraints can be represented as a quadratic programming problem. The ways of formulation of such a problem in forces and in displacements were indicated in this paper. Methods of static calculation and verification of stability, as well as properties of possible solutions, are analyzed in detail in [330]. Algorithms for solving the problems of stability and vibrations of deformable systems with unilateral constraints, and the solution of a number of such problems are presented in the book [18].

In order to reduce the dimension of the original nonlinear equations and to decrease the complexity of the nonlinear problems of shell statics and dynamics, a reduction of the basis was proposed. This reduction was based on the assumption that the desired form of equilibrium of the entire construction with a sufficient degree of accuracy can be represented as a linear combination of several specially selected functions [78, 192, 260, 286].

**5.9. Optimal design**

The desire to optimize design has always been present in project practice and served as a motive for a number of research work. One of the first in this series is the work dedicated to the search for the regularities, which the constructed structures of different types possess. Such a systematic comparison of various schemes of bridge structures with the aim of choosing the best solution was carried out by many researchers, including Ye.O. Paton [320].

This direction of research to some extent is not theoretical but experimental, since it can be assumed that every real construction is a unit experiment, although not specifically designed to investigate the regularity of the behavior of the whole set of similar structures. However, the multiplicity of this kind of experimental data allows them to be used to identify hidden patterns or to formulate a problem for conducting special investigations [92, 318, 323].

The problem of optimization is often considered as part of a work devoted to the analysis of behavior and recommendations for the design of structures of a particular type or destination. As an example, it can be noted that a significant part of the book by Ye.O. Paton and B.M. Gorbunov [321] is devoted to the problems of choosing the best constructional solution (optimal height and magnitude of spans, the best grid scheme, etc.)

The simplest problems associated with the optimal layout of the cross section of the beam, or with choosing the shape of the beam of equal resistance were considered. Here, the work of I.Ya. Shtaierman [412], can be marked. He, unlike other authors, as a beam of equal resistance chose one, for which the greatest tangential stresses were identical along the entire length (the third strength theory).

Starting from the middle of the 20th century mathematical methods of optimization began to be used to solve such problems: linear and nonlinear mathematical programming, theory of optimal control, nonclassical variational calculus, and so on. Most often, tasks of optimizing objects of a certain type were considered. Thus, for the tower structures using the Pontryagin maximum principle, the optimum shape of the tower structure belts was found [178]. For the spatial structural roofs the optimal parameters of the design solution were found with all costs included, accounting heating costs and other current costs [49, 178, 398, 523]. With the use of computer technology and in a refined formulation, optimal design of crane beams [139] was considered, when not only the cost of materials was taken into account, but also the laboriousness of constructions production. Interesting results were obtained here, showing the differences in the design, depending on the ratio of the value of materials and labor resources.

Optimization problems were solved, as a rule, in relation to structures of a particular type. The problems of finding the optimal configuration of the shell surfaces [20, 68, 70, 505, 509], optimal reinforcement of constructions from reinforced concrete [74, 76, 133, 527] and others were considered. Searching for an optimal shape was most often considered in the form of searching for optimal parameters when choosing from a certain family of functions (for example, surfaces of a transfer or surfaces developed from a certain initial form by two-component geometric transformations [71]).

It is known that when calculating for single load, the construction of the minimum weight is statically determinate. However, most of the real problems are related to the case of multicomponent loads and there are significant complications. In the series of works by I.D. Glikin and A.I. Kozachevsky [130, 131, 132, 133], an iterative step-by-step approach to the optimum with small changes in the parameters of the problem (sections of the elements or their reinforcement) was proposed for optimization at many loads. It leads to the necessity of solving the problem of linear programming at each step of the iteration. In the general case, such a process leads to a local minimum. An alternative approach is to find the optimal solution taking into account the possibility of creating a pre-stress. The search for such a solution, as the task of finding the Chebyshev point of the system of inequalities describing the limitations of strength, was proposed in [335].

The practice of solving specific optimization problems has shown that search for an exact solution, especially for multi-extreme problems, is associated with large computational and fundamental difficulties. Therefore, in such cases, the problem was coarsened and some approximate solution was searched for.

It should be noted that into the concept of an approximate solution to the optimization problem mathematicians and engineers often put not one and the same meaning. From the mathematical point of view, the discrepancy with the exact solution is estimated by the difference in the coordinates of the points of the approximate and exact solutions, whereas from the engineer's point of view, the deviation from the global minimum of the objective function should be considered as a solution assessment. With a small deviation, the acceptability for the practice of such an approximate result is not in doubt.

This idea apparently was first expressed and broadly considered by V.M. Gordeiev, who suggested instead of finding the point of an extremum to seek out, and then analyze in more detail all the set of solutions adjacent to this point [162, 174, 397, 398]. The fact is that most real optimization problems have a "smooth extremum", i.e. even a noticeable deviation from the ideal solution does not change the value of the objective function much. So it gives the opportunity to take into account additional difficult to formalize conditions (for example, the discreteness of certain parameters) without leaving the set of solutions close to the optimal in the space of design parameters.

To solve the problem in such a statement, a specially developed method of uniform reserves proved to be well-adapted. This new method is the development of known methods of the interior point and the method of centers for solving mathematical programming problems [398]. In addition, approximations of the solution region close to the optimal by n-dimensional ellipsoid (n is the number of design variables) were used.

The cycle of studies related to the optimization of metal structures, including metal cable-stayed bridges, was performed by V.V. Trofymovych and V.O. Permiakov [352, 353, 461, 463, 464]. Among others, problems were solved, in which the prestress value of the system and the cross-section of its elements were chosen, as well as the problems of finding optimal configuration of the lattice structure. Linear programming was used as a research tool.

More complicated formulations of optimization problems were solved by methods of nonlinear mathematical programming, while using the genetic algorithm in combination with descent methods. For example, the construction of a wind power plant was optimized. Here, based on the minimizing the cost of kilowatts of the output power, the parameters of the bearing structure as well as the diameter of the wind wheel were optimized [325].

As for reinforced concrete structures, here the problem formulation often involved optimizing only of structural reinforcement. It is also assumed that the formwork dimensions of the structure are specified and are not subject to change (i.e. stiffness parameters are practically known). When optimazing shell structures, additional assumption was used according to [68, 69, 70, 74, 527]. This assumption is that reinforcement should correspond to the maximum load, which is determined on the basis of the kinematic model of the method of limiting equilibrium. The account of

Viktor Volodymyrovyc h Trofymovych (1926–2004)

Volodymyr Oleksandrovych Permiakov (1938–2007) redistribution of internal forces due to the inelastic behavior of the material in the optimization problem of reinforcement was carried out account in [40].

In view of the relatively low consumption of reinforcement in shell structures, the optimization effect is relatively small. More interesting results are related to finding the optimal surface of the shells. Studies of this type of were performed for the shells of revolution [67, 371] and shells whose shape was a transfer surface [505].

The description of optimization methods applied in structural mechanics, including problems with incomplete load information or multicriteria optimization problems, was presented in the monograph by V.I. Guliaiev, V.A. Bazhenov and V.L. Koshkin [211]. In contrast to the canonical optimization problems formulated in finite-dimensional spaces and solved by methods of mathematical programming, the main attention was paid to optimal design problems in functional spaces. As illustrative examples, the problems of minimizing the mass of thin conical, spherical (Fig. 26) and cylindrical elastic shells under strength and geometric constraints were solved. Optimal shell design taking into account geometrically nonlinear behavior under stability constraints was considered as well.

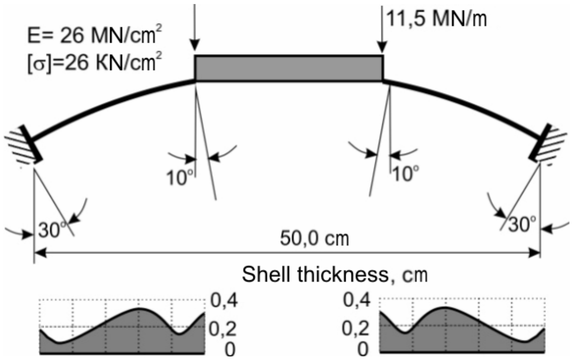



Fig. 26. A spherical shell with optimal mass distribution

Also the problem of choosing the optimal parameters of the passive vibration protection system was solved. Vibration dampers of simple systems subject to the harmonic loading of a changing frequency were considered. Later, the problem was generalized to the case of optimization of the pendulum damper during random loading [17], as well as to search for the optimal parameters of the pendulum damper of high-rise structures vibrations under the influence of wind pulsations [15].

In conclusion of this section, we once again mention the problem of determining the disadvantageous combination of forces acting on an elastic system. This problem, on the one hand, belongs to the classical directions of structural mechanics, such as, for example, the problem of loading lines of influence, and on the other hand, it turned out to be in a certain shadow. Interest in


Georgii Vakhtangovich Isakhanov (1921–2012)

it was revived during the transition to algorithmization of calculations, and here, the authors of [6] made a serious breakthrough in their time, consisting in using the idea of describing the logical relationship of individual load cases in the form of a directed graph. This idea was subsequently developed in [175] and found its completion in [163]. In the latter of them, a rather general concept of calculation is built, based on the need to search for a disadvantageous state, and this search does not complete the calculation, but organizes it from the very beginning. Here the work [131, 436], dedicated to this topic, also could be mentioned.

Usually, the basis of calculations, searching for an unfavorable combination of loads, is the use of the superposition of the contributions of individual load cases, i.e. it is assumed that the calculation relates to a linear system. An attempt to solve the problem of finding unfavorable loading, based on the application of the theory of optimal control, was undertaken in [329], but it did not find wide application, since the complexity of the calculations turned out to be excessive. The problem is still waiting to be solved.

## 6. SCIENTIFIC SCHOOL OF THE STRUCTURAL MECHANICS OF THE KYIV NATIONAL UNIVERSITY OF CONSTRUCTION AND ARCHITECTURE

In 1961, at the Kyiv National University of Civil Engineering and Architecture (then the Kyiv Civil Engineering Institute, KCEI), on the initiative of Professor D.V. Vainberg the Research Laboratory of Thin-walled Spatial Structures was established. It was transformed in 1966 into the Problem Research Laboratory of Thin-walled Spatial Structures (PNDL TPK).


David Veniamynovych Vainberg (1905–1973)

Professor D.V. Vainberg headed the Department of Structural Mechanics and PNDL TPK until his death in 1973. From 1974 to 1989 the head of the department and the supervisor of the laboratory was Professor G.V. Isakhanov. From 1989 to the present, the Department of Structural Mechanics and the laboratory, subsequently transformed into a research institute, is headed by a full member of the National Academy of Pedagogical Sciences of Ukraine, Professor V.A. Bazhenov.

The Scientific and Research Institute of Structural Mechanics (Naukovo-doslidny institute budivel’noi mekhaniky, NDIBM), established on the basis of PNDL TPK in 1991, conducts fundamental and applied research on the theory and methods of calculating the strength, stability, and vibrations of complex spatial structures under the external influences of a different physical nature and developing on this basis a problem and object oriented software. The results of the work have practical application for solving the problems of strength, stiffness, stability, modeling of vibration processes and determining the bearing capacity of building constructions and structures. Individual parts of the constructions, as well as critical structural elements of machines operating in various industries, including gas and steam blades turbines, turbine rotors, nuclear reactor shells, shallow shells of roofs of underground and ground structures, bearing elements of towers and masts, bar systems, pressure vessels, elements of valves, damper devices and a number of other important objects also subject to close attention and calculation.

The result of the activities of the Scientific School was the training of a significant number of highly qualified specialists in the field of structural mechanics and mechanics of a deformable solid. In particular, during the existence of the Scientific School, following researchers defended their candidate dissertations:

| | | | |
|---|---|---|---|
| 1964 | V.M. Rakivnenko, G.B. Kovnerystov | 1965 | V.Z. Zhdan, I.A. Kolomiets |
| 1966 | F.O. Romanenko, O.L. Syniavsky, V.I. Guliaiev | 1967 | Yu.K. Chekushkin, A.V. Odynets |
| 1968 | O.S. Sakharov, V.P. Stukalov | 1969 | V.A.Bazhenov, V.M. Kyslooky, I.A. Bazylevych |
| 1970 | Jo.Z. Roitfarb, P.P. Voroshko S.M. Liubchenko | 1971 | Yu.V. Veriuzhsky |
| 1972 | V.M. Gerashchenko, R.K. Demianiuk | 1973 | V.K. Chybiriakov |
| 1974 | O.V. Shishov, D.R. Kolev, V.V. Kyrychevsky, Ye.O. Gotsuliak | 1976 | V.G. Kobiev, G.Yo.Melnichenko, S.Ya. Granat, P.P. Lizunov |
| 1977 | V.V. Gaidaichuk, G.G. Zavialov, Chu Viet Kyong | 1978 | O.I. Guliar, R.K. Bobrov, S.M. Chorny |
| 1979 | A.I. Vusatiuk, Spartak Mohammed Salem, A.S. Svystov | 1980 | M.O. Solovei, I.Ye. Goncharenko, O.A. Kyrychuk, Adnan Ali, Pemsing Krishna, Samsur Abdullah, Hoang Xuan Liong |
| 1981 | O.N. Beskov, V.Ye. Veryzhenko, O.I. Vynnyk, L.A. Vriukalo, O.L. Kozak, A.D. Legostaiev, B.M. Marzytsyn, O.I. Ogloblia, O.O. Kholodenko | 1982 | N.T. Zhadrasinov, O.Ya. Petrenko, V.V. Savytsky, V.V. Khymenko, V.K. Tsykhanovsky, O.V. Shymanovsky |
| 1983 | G.S. Kondakov, S.L. Popov, O.V. Savchuk, V.M. Chaban | 1984 | V.M. Karkhaliov, P.G. Melnyk-Melnykov |

| | | | |
|---|---|---|---|
| 1985 | O.V. Gondliakh, Dan Khyu Kun, O.A. Zverev, O.I. Korzh, T.I. Matchenko, T.L. Savchenko, M.K. Sysengaliiev, Fan Din Ba | 1986 | O.V. Glimbovsky, V.M. Yermishev, S.V. Zablotsky, Saudi Khasen, V.V. Chemlaiev |
| 1987 | K.Ye. Boyko, L.S. Ivanova, V.B. Kovtunov, V.L. Koshkin, S.G. Kravchenko, T.A. Kushnirenko, Kyonh Le Chunh, Ye.D. Lumelsky, Nhuen Shy Chan, O.I. Pylypenko, I.V. Polovets, S.V. Potapov, A.G. Topor, Khettal Takhar, V.M. Chyrva | 1988 | Yu.M. Appanovych, G.G. Burtsev, A.A. Grom, O.O. Odynets |
| 1989 | V.G. Borysenko, O.V. Bratko, O.B. Vasyliev, G.Ye. Zakharov, Ignas Aloyis Rubaratuka, V.Ye. Kravtsov, V.V. Lazhechnikov, O.Ye. Mayboroda, O.Yu. Mozharovsky, Yu.Z. Totoyev, O.B. Ushak | 1990 | V.S. Boyandin, G.L. Vasilieva, Yu.L. Dinkevich, O.V. Mirchevsky, Sadik Obanishola Mufutou, O.A. Fesenko, N.L. Filippova |
| 1991 | O.A. Bogutsky, O.V. Gerashchenko, A.M. Katsapchuk, Ye.E. Kotenko, O.P. Koshovy, O.B. Krytsky, Yu.S. Petryna, Said Ahmed Shah, Eneramadu Kelechi Obinna | 1992 | O.V. Belolipetska, Yu.V. Vorona, K.Ya. Golovatiuk, Yu.M. Dyadenchuk, T.G. Zakharchenko, I.O. Klimko, Obanishal Sadni, I.O. Serpak, Vazir Pad Shah, V.O. Yasinsky |
| 1993 | Gbenu Atiglo Raphael, V.O. Pokolenko, Xia I Puygen, P.P. Cheverda, Shu Ming | 1994 | V.K. Bondar, N.A. Valeieva, Jaber Chord, G.M. Ivanchenko, M.G. Kushnirenko, O.O. Lukianchenko, Temor Shah, Yu.O. Shinkar |
| 1995 | O.G. Kovalevska, V.O. Rutkovsky, Juan Carlos Inchausti | 1996 | Hablos Abd Razzak, G.L. Dmytriiev, Ayat Nouari |
| 1997 | N.A. Snizhko | 1999 | D.E. Prusov, Labu Mezian, Saidi Amin |
| 2001 | Yu.D. Geraimovych, S.O. Pyskunov, I.I. Solodei | 2002 | Genduzen Abdenur, Busettta Mubarek |
| 2003 | O.V. Kostina | 2005 | O.P. Kryvenko |

| | | | |
|---|---|---|---|
| 2006 | M.V. Goncharenko | 2007 | O.O. Shkryl |
| 2008 | M.S. Barabash | 2009 | V.P. Andryievsky |
| 2011 | S.V. Mytsiuk | 2012 | Yu.V. Maksymiuk |
| 2013 | M.O. Vabischevych, D.V. Bogdan | 2019 | A.V. Pikul, R.L. Strygun |

A considerable number of representatives of the Scientific School defended doctoral dissertations:

| | | | |
|---|---|---|---|
| 1975 | O.L. Syniavsky | 1978 | O.S. Sakharov |
| 1979 | V.I. Guliaiev | 1981 | Yu.V. Veriuzhsky |
| 1984 | V.A. Bazhenov | 1987 | V.Ye. Verizhenko |
| 1989 | V.K. Chybiriakov, P.P. Lizunov, O.I. Guliar, V.V. Kyrychevsky, P.P. Voroshko | 1990 | Ye.O. Gotsuliak, Ye.S. Dekhtiariuk |
| 1992 | V.V. Gaidaichuk, G.B. Kovnerystov | 1993 | O.A. Kyrychuk |
| 1994 | O.V. Gondliakh | 1995 | O.L. Kozak, O.I. Ogloblia |
| 1999 | V.K. Tsykhanovsky | 2004 | S.Yu. Fialco, Tran Duc Chinh |
| 2006 | Ya.O. Slobodian | 2008 | M.O. Solovei |
| 2009 | V.M. Trach | 2011 | S.O. Pyskunov |
| 2012 | G.M. Ivanchenko | 2013 | I.I. Solodei |
| 2018 | O.O. Shkryl | 2019 | Yu.V. Maksymiuk, Yu.G. Kozub |
| 2020 | M.O. Vabischevych | | |

In total, during the existence of the scientific school, 31 doctoral and 173 candidate dissertations have been defended.

The achievement of such a level of training of scientific and technical personnel was greatly facilitated by the corresponding publications of applicants in the collection of "Opir Materialiv i Teoriia Sporud".

An interdepartmental collection of scientific articles “Soprotivlenie Materialov i Teoriya Sooruzheniy” was created in 1965. Since 1998, the collection has been published under the title Opir Materialiv i Teoriia Sporud" (Strength of Materials and Theory of Structures: Collection of scientific articles) ISSN 2410-2547. The editorial board of the collection includes professors from universities in Poland, Vietnam, and the USA. 103 issues were published for the period 1965-2019. The collection publishes scientific articles that are prepared in Ukrainian, English and other languages and contain the results of basic research on topical problems of strength of material, structural mechanics, mechanics of a deformable solid, theory of structures, related applied problems of strength and reliability in mechanical engineering, construction and other industries of modern technology. It also highlights the issues of teaching structural mechanics, and provides information on new educational and scientific publications on the subject of the collection. The archive of collection

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104

issues is available on the *opir.knuba.edu.ua* website in compliance with the open access policy in the sense of the Budapest Open Access Initiative, which makes it possible to disseminate new research results in each industry and in each country.

The collection is indexed in the scientific and metric databases Web of Science (https://openscience.in.ua/ua-journals), Index Copernicus (https://journals.indexcopernicus.com/search/details?id=32331), DOAJ (https://doaj.org/toc/2410-2547), has an estimate using the Journal International Compliance Index criterion. JIC index = 0.173 (https://jicindex.com/journals/42-64). The collection is also presented in the Ukrainian abstract journal "Source" and in the abstract database " Ukrainian Science".

Since 2017, according to the decision of the Scientific Council of the Ministry of Education and Science of Ukraine, the collection has a special status of a professional publication for publishing the results of research carried out in scientific institutions of Ukraine due to state budget funding in the direction of “Mechanics”. The collection received category A in accordance with the Procedure for the Formation of the List of Scientific Professional Publications of Ukraine.

The collection goes to the leading libraries of Ukraine, in particular, goes to Vernadsky National Library of Ukraine (the full text of the collection is also available on the website of this library), the National Parliamentary Library of Ukraine, Vasyl Stefanyk National Scientific Library of Ukraine in Lviv and others.


From 2005 to 2012, a 6-volume edition of “Successes in Mechanics”, dedicated to the beginning of the 3rd millennium and edited by Academician O.M. Guz was being published in Kyiv. The edition was intended to familiarize the world scientific community with the latest achievements of Ukrainian science in the field of mechanics. The publication includes generalized review articles published in the journal Applied Mechanics by leading scientists who took an active part in the development of the corresponding areas of mechanics. Among others, the edition presented reviews prepared by representatives of the Scientific School of Structural Mechanics of KNUCA dedicated to solving problems of nonlinear continuum mechanics with the help of MSFE and SFEM [21, 38], as well as the study of nonlinear deformation and stability of elastic inhomogeneous shells under thermomechanical loads [42].

NDIBM collaborates with scientific institutions of the National Academy of Sciences of Ukraine - S.P. Timoshenko Institute of Mechanics, G.S. Pisarenko Institute for Problems of Strength, E.O. Paton Electric Welding Institute, with research institutions in the field of construction - State Research Institute of Building Constructions (NDIBK), the Research Institute of Building Production (NIISP), OJSC "V. Shimanovsky UkrRDIsteelconstruction", with industrial enterprises in the field of mechanical engineering - State Enterprise "Gas Turbine Research and Production Complex "Zorya-Mashproekt", State Enterprise Zaporozhye Machine-Building Design Bureau "Progress", Motor Sich JSC.

The results of research carried out at NDIBM were awarded the State Prizes of Ukraine in the field of science and technology:

1991 "Theory, methods of mathematical modeling and numerical analysis of the complex spatial structures processes of deformation";

2003 "Scientific research, development and implementation of low energy-intensive technologies and equipment in construction";

2013 "Development of the innovative model and terms for preparation of building industry specialists taking into account possibilities of modern materials and technologies".

## 7. JUSTIFICATION OF CALCULATION STRUCTURAL MODELS, RELIABILITY ANALYSIS

A variety of modern space-planning solutions and new constructive forms, which are realized at the same time, have created a new design situation, when it is often impossible to focus on traditional and well-tested structural models. At the same time, modern computer technology creates practical possibilities for using calculation structural schemes of a new type. In this regard, the theory of structures has encountered numerous problems associated with the verification and justification of structural models that were not previously studied in detail. A typical example is the structural scheme of a modern high-rise building with bearing structures in the form of a set of plate-shell components.

Applied methods of calculating multi-storey buildings, including those that use the structure-foundation-soil interaction model was developed at KNUCA, NDIBK, KyivZNDIEP (see, for example, [54, 265, 308, 386]). More general problems of the analysis of structural schemes are presented in detail in the monograph [345].

Naturally, the formulation of new complicated problems led to the need for experimental confirmation of the main theoretical provisions. For example, the cycle of experimental studies conducted at the Institute of Mechanics on models of finned cylindrical shells made it possible to obtain data on the effect of geometric imperfections [1, 528]. An experimental estimate of the bearing capacity of spatial roofs was performed by NDIBK [399, 504]. Here the study of a large model of the roof in Kyiv can be mentioned (Fig. 27).

The formation and development of welding in construction required the implementation of a large amount of experimental work to confirm the strength of welded structures. Such work was carried out at the E.O. Paton Electric

Welding Institute by V.V. Shevernitsky, V.I. Trufyakov, V.I. Makhnenko, L.M. Lobanov et al. [299, 301, 392, 465].

In connection with the evaluation of experimental data, studies were carried out to assess the reliability of calculation models and their adequacy to the problem being solved [105, 106, 107].

One of the mass construction objects is high buildings therefore the justification of their design models is an urgent problem. A series of works in this direction was done by D.M. Podolsky in KyivZNDIEP [361, 362, 365]. Among them, it is useful to note the work [365], in which the important problem of taking into account the incompleteness of the available information was formulated. The fact is that a decision on the adequacy of the structural model is made on the basis of this incomplete information. In addition, it is important to evaluate the influence of how the non-ideality of the structural scheme affects the calculation results and how one can predict the expected deviations from the ideal computational model [193, 312].

Fig. 27. Roof of bus fleet

Some of the problems, dictated by the demands of practice, put forward a number of fundamentally new challenges. They had a noticeable influence on the direction of the research carried out in the field of the theory of structures. As an example, we can point to the Chernobyl catastrophe, which caused and raised a number of issues unconventional for the theory of structures.

In particular, it became necessary to retrospectively assess the loads on structures, as well as to predict the behavior of damaged structures, often based on inaccurate data on the degree of their destruction [236, 414, 415, 516, 520, 521]. The purpose of these studies was to get an idea of the state of the structural elements that had become inaccessible for explicit observation. Much work in this direction was carried out by the NDIBK team with the involvement of a number of specialists from other organizations [124].

It was necessary to estimate the risk of possible collapse of structures [266, 309, 394, 522], for which calculations of damaged structures of the 4th power unit and elements of the Shelter Structure were performed and estimates of residual resource were made.

Many of the very diverse problems, including those in the field of the theory of structures, arose in the process of designing a new safe confinement (NSC), which was erected over the Shelter Structure (Fig. 28).

Fig. 28. Installation of confinement structures

Additional studies of possible extreme loads, such as tornadoes, clarification of the seismological situation, development of methods for calculating the effects of an avalanche that could occur when snow slides from an arch system with a span of 256 meters and a height of 108 meters were required (Fig. 29).

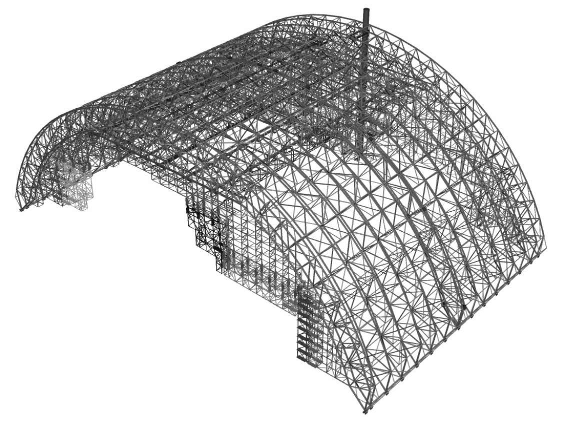


Fig. 29. Bearing frame of confinement (calculation model)

All this issues should be taken into account when designing the NSC [237, 238, 279, 313, 416]. The very problem of designing this grandiose and very responsible structure was also associated with a number of studies, in particular, the question of optimal shaping was studied [44], and the problem of the behavior of a large-span structure during an earthquake was analyzed taking into account the influence of asynchronous seismic excitations of supports on the dynamic response [31, 332].

Naturally, most of the mentioned studies were carried out on the basis of numerical methods that were implemented in the software, both of industrial type and specially designed for the analysis of the behavior of the NSC. Many of the techniques used in these calculations have a wider range of possible applications than just analysis of the strength of NSC structures [332].

In addition, this disaster has allowed realizing the existence of restrictions for the tendency to increase the unit capacity of objects. Explicitly, the concept of unit capacity growth was called into question, apparently for the first time by academician B.Ye. Paton [317] who noted that the ”…growth of unit capacity of machines, construction systems, structures and installations is often not accompanied by the same increase in their reliability, and this can lead to large-scale losses, such as, for example, during the Chernobyl disaster”. In the theory of structures, an analogue of this concept is the principle of material concentration. The limitations arising from this thesis of B.Ye. Paton were analyzed in [339].

**CONCLUDING REMARKS**

Above was a brief presentation of the 120-year history of studies on the theory of structures performed by scientists of the Kyiv school. It not only testifies to the serious contribution to the theory and practice of computational analysis of building structures, but also makes it possible to talk about some trends that determine the future development of this applied science.

Perhaps the main thing here is the conversion of all the tools of the theory of structures into a numerical form of analysis, based on the development of modern computing tools, the algorithmization of known and new approaches to solving problems and the rigorous justification of these algorithms. The development of effective implementations of the methods of numerical analysis, especially when solving nonlinear problems of statics and dynamics, is of great impotance as well.

The entire history of the development of structural mechanics [11, 34, 45] shows that one of the main paradigms is the desire for an increasingly detailed analysis of the behavior of structures and the use of detailed calculation schemes. The calculation models of modern structures contain thousands or even tens of thousands of elements, and this fact does not serve as an obstacle to their analysis when calculations are performed using computers. The problem is not the ability to perform the calculation, but the ability to analyze its results. Orientation to a detailed stress analysis leads to a paradoxical situation when the description of the analysis results becomes more difficult to comprehend. And here an important scientific problem emerges, consisting in the need to develope generalized characteristics of the stress state, allowing us to consider the features of the "behavior of the system as a whole."

The logic of the theory of structures development was aimed at taking into account factors that more accurately determine the stress and strain state. And as one of the urgent areas of research, the problem of taking into account the nonlinear behavior of the structural complex has been advanced. The complexity of non-linear analysis is due to the fact that one has to abandon a number of assumptions of classical structural mechanics and cannot use many familiar principles and theorems (the principle of independence of the action of forces, the theorem on reciprocity of displacements, etc.). These difficulties are not completely overcome at present. We can only say with certainty that the implementation of non-linear analysis with sufficient completeness and accuracy for practice is impossible without the use of computers, so methods adapted for computers are of paramount importance.

"The challenge of computerization" was adequately received by the Kyiv school of the theory of structures. Here was the computer capital of the construction industry, and the LIRA and SCAD software systems created and developed in Kyiv were and remain the main toolkit of construction designers in all CIS countries.

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*Стаття надійшла 28.05.2020*

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**KYIV SCHOOL OF THE THEORY OF STRUCTURES**

The paper presents a review of more than a century-long history of Kyiv school of the theory of structures. Particular attention is paid to the fundamentally new opportunities for the development of the theory of structures in the era of numerical analysis. The publication contains a wide bibliography.

**Keywords:** bar systems, stability, shells, structural mechanics, finite difference method, finite element method, calculation model.

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**КИЇВСЬКА ШКОЛА ТЕОРІЇ СПОРУД**

Стаття присвячена аналізу більш ніж вікової історії Київської школи теорії споруд. Особлива увага приділена принципово новим можливостям розвитку теорії споруд в епоху чисельного аналізу. Публікація містить широку бібліографію.

**Ключові слова**: стержневі системи, стійкість, оболонки, будівельна механіка, метод скінченних різниць, метод скінченних елементів, розрахункова модель.

УДК 624.04
*Баженов В.А., Перельмутер А.В., Ворона Ю.В.* **Київська школа теорії споруд //** Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 3-88. – Англ.

*Розглянута історія виникнення і становлення Київської школи теорії споруд. Особлива увага приділена принципово новим можливостям розвитку теорії споруд в епоху чисельного аналізу.*

Іл. 29. Бібліогр. 536 назв.

UDC 624.04
*Bazhenov V.A., Perelmuter A.V., Vorona Yu.V.* **Kyiv school of the theory of structures** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 3-88.

*The history of Kyiv school of the theory of structure birth and formation was considered. Particular attention was paid to the fundamentally new opportunities for the development of the theory of structures in the era of numerical analysis.*

Fig. 29. Ref. 536.

УДК 624.04
*Баженов В.А., Перельмутер А.В., Ворона Ю.В.* **Киевская школа теории сооружений //** Сопротивление материалов и теория сооружений. – 2020. – Вып. 104. – С. 3-88.

*Рассмотрена история зарождения и становления Киевской школы теории сооружений. Особое внимание уделено принципиально новым возможностям развития теории сооружений в эпоху численного анализа*.
Ил. 29. Библиогр. 536 назв.

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UDC 624.07+006.036

# STRENGTH ANALYSIS IN REGULATORY DESIGN DOCUMENTS AND COMPUTATIONAL SOFTWARE

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DOI: 10.32347/2410-2547.2020.104.89-102

**Abstract.** Modern building design standards have a long history. During this time, they have undergone a number of changes, but some of their provisions and recommendations, once proclaimed, remain unchanged. And although they do not meet the modern possibilities of computational analysis, but continue to exist due to the established tradition. In this paper, attention is paid to only some of the mentioned conflicts, which are related to the software implementation of regulatory requirements. The first of them is connected with two different parts in the process of design justification: calculation of the stress-strain state and verification of the accepted cross sections. It is noted that when the calculation model adopted for computer analysis of the structure does not correspond to the model that was meant when compiling the regulatory document, there may be contradictions or inaccuracies that cannot be resolved without decoding the approach adopted in the standards. Unfortunately, such a decoding is not provided in our rationing system. Another group of conflicts is connected with conducting the structural analysis taking into account geometrical and physical nonlinearity declared by standards. The matter is there are some problems that cannot be solved when using nonlinear calculation, for instance, dynamic analysis using eigenmode decomposition with the subsequent summation of modal reactions. The problem of choosing an unfavorable combination of loads is also in this list. In the final part of the article some proposals are formulated. This proposals aimed at eliminating contradictions between the desire to develop simple and understandable design rules and the ability of modern computer to solve problems without the use of dubious simplifications.

**Keywords:** load-bearing capacity, building codes, computer analysis.

**Introduction.** The experience of design activity in recent decades shows that the development of automation of engineering calculations has the most serious impact (unfortunately, both positive and negative) on the quality of justifications for design decisions. The level of detail and accuracy of calculation, which is now available to designers en masse, yesterday was still unattainable even for the most qualified organizations and professionals. At the same time, the availability of modern powerful computing systems creates a number of new problems. One of them is the growing number of inconsistencies between the capabilities of software systems, which are focused on a detailed analysis of the work of structures, and the requirements of regulations, which are focused on established experience.

Almost all modern tools of building design automation implement to some extent the requirements of existing regulations. At the same time the inclusion of regulatory requirements in software systems is not only a problem of their developers, but also a problem of a wide range of users. The point is that users have to understand which requirements for regulatory documents can and should be imposed on the relevant software, when deciding on its use. An almost complete disarray takes place here today. Some users would like everything to be implemented (including departmental, company and other detailed instructions), others would like the developers to allow them to decide which rules should be followed and which can be ignored, still others want detailed references to justify regulatory requirements and etc.

Of course, one can rely on the following principle. The implementation of regulatory requirements in the software must strictly adhere to the text of the regulatory document. In cases where this is not possible in general (examples of such a situation are given below), the program should refuse to perform the appropriate function in the part that does not adequately reflect norms, notifying the user. In this case, an accurate reflection of possible limitations of this kind in the program documentation should be a prerequisite.

Another set of problems is due to the fact that modern software systems focus on the use of universal provisions of such disciplines as the theory of elasticity, the theory of plasticity, structural mechanics, etc. while some provisions of the norms are based on simplified approaches, test results and experience of operation of existing structures. But being presented in the regulatory document, such provisions suddenly take advantage over scientifically sound and more accurate solutions, which do not appear in the codes only due to the complexity of calculations.

Almost all modern tools of construction design automation implement to some extent the requirements of existing regulations. Meanwhile there are certain problems of technical, legal and economic nature, which often arise due to the fact that the developers of regulations did not forecast the possibility (and necessity!) of their software interpretation.

**Two interpretations of the concept of "calculation of structures"**

The design justification of design decisions is a multi-stage process, in which, at least, two main parts should be distinguished: calculation of the stress-strain state (SSS) and verification of the accepted cross sections (or their reinforcement). Unfortunately, this fact is not emphasized and when talking about the calculation of structures is not always clearly stated what we are talking about.

At the same time from the point of view of rationing the differences here are fundamental: the calculation of SSS is the problem of structural mechanics and this process in principle should not be the subject of rationing, while checking the bearing capacity of sections is a conditional procedure aimed at achieving a certain degree of safety. The rationing, i.e. the establishment of certain requirements of society, is quite appropriate here.

Returning to the stage of the SSS calculation, we can say that only some "permitting procedures", which establish acceptable simplifications of the problem, can be controlled by the design code. It is important to note here that it is a question of allowable simplifications, instead of their obligatory application though in texts of regulatory documents this fundamental difference is not stipulated in any way. The question arises here about the inequality of the results of the simplified calculation performed in accordance with design standards and the possible result of a more accurate analysis.

It should be noted that modern software systems often have the ability to perform the structure calculation in much more detail and accuracy than required by regulations. Such details of the stress-strain state and such details of the behavior of the structure under load can be found, which were not taken into account by the authors of the normative document or, more often, taken into account in the design standards by applying some special coefficient of working conditions or other ways to take into account additional bearing capacity. Since these techniques are not deciphered in detail in the regulations, the corresponding feature may be taken into account twice: the first time in the framework of computer simulation and the second time in the regulatory verification, which is performed using the above additional coefficient. As a result (and this has happened many times) a project with a more thorough calculation justification will be less economical than a rougher calculation according to the standards.

The situation may be even more complicated when the normative document provides for a calculation procedure in which some empirical correction factors are used. A typical example is the standards for seismic analysis of structures [3], where the results of the response specrtum method are adjusted by the reduction factor $K_1$, which is introduced to take into account the plastic behavior and local damage. Since the degree of plasticization of structural elements and the amount of local damage is not specified, it remains unclear what to look for when using other methods of calculation (direct integration of equations of motion, deformation method of checking the ultimate forces, etc.).

Another example is the calculation for temperature effects. The fact is that the design standards of structures set the maximum distances between the temperature seams (see, for example, section 1.13.2 16 of the State Building Codes of Ukraine DBN B.2.6-198: 2014 [6]). Traditionally, it is considered that the calculation of temperature effects can be omitted when a compartment length does not exceed these limits. But it has been repeatedly detected that such a calculation leads to the conclusion of a significant overstrain of the load-bearing structures, which causes surprise and numerous discussions.

The discovered contradiction is due to the fact that the standard calculation models of force calculation do not take into account some flexibility of the nodal joints (for example, slippage of the base of the steel column on the foundation within the black holes for anchor bolts). Such shifts, which are absolutely insignificant under force loading, are decisive under the kinematical influence of the thermal deformation type. Their values may be compared with thermal elongations and they dramatically affect the stress-strain state. Here, the rules, which are based on many years of practical experience, are "smarter" than traditional analysis.

Thus, it can be stated that if the calculation model adopted for computer analysis of the structure does not correspond to the model that was meant when compiling the regulatory document, there may be contradictions or inaccuracies that cannot be resolved without decoding the approach adopted in the standards. Unfortunately, such a decoding is not provided in our rationing system.

**On regulation of calculation methods**

Although the science of "structural mechanics" can not set standards, if we keep in mind the methods and rules of calculation, but when it comes to choosing a calculation model, the question is not so clear.

The fact is that the design standards are a chain of trade-offs, where some inaccuracies in the calculation of some parameters (e.g., internal efforts in the system) are offset by safety factors embedded in other parameters (e.g., in the design strength). In addition, the method used by the authors of the rules can be based on a certain calculation model, and this model occurs to be specified in the normative document.

Traditionally, building design standards have focused on certain set of calculation schemes. Most often, these were plane bar systems loaded in one plane or in mutually orthogonal planes and operating in a uniaxial stress state. Spatial structures, especially of shell type, are considered much less often. However, they are almost standard when calculating using software. And here there is a certain imbalance of possibilities, when many cases, normalized for traditional calculations, are simply absent for the calculations of spatial systems.

As an example, let us mention the fact that the design standards for steel and reinforced concrete structures provide a material stress-strain diagram only for uniaxial state and there are no recommendations for assessing the performance of structures in 2D or 3D stress state. In this case, the normative documents on the design of reinforced concrete structures, which are calculated by a nonlinear deformation model, for example, are focused on checking the values of ultimate deformations, but such criteria are given only for uniaxial stress state. How they should be transformed with respect to the 2D stress state is completely unclear. After all, there is no theoretical justification for the use of deformation criteria here. Moreover, any theory of plasticity is based on the concept of the boundary surface in the stress space, whereas the concept of the boundary surface in the space of deformations simply does not exist.

Another problem concerns the interpretation of the results of the spatial calculation model analysis in accordance with the regulatory documents. So, for example, for bar elements we receive six internal forces and $N$, $M_x$, $Q_y$, $M_y$, $Q_x$, $M_z$ instead of three $N$, $M$, $Q$ and even if any element works "in plane" that nonzero values (probably small on size) can have all six internal forces. How small must be certain forces, so that they can be neglected, is not specified.

For example, the concept of a beam used in [6] and [5], obviously implies the ability to neglect the influence of longitudinal force in comparison with the influence of moments. But if in the first case for steel structures in 1.6.2.2 there is a record that for the value of the given relative eccentricity $m_{ef} > 20$ the calculation can be performed as for the bent element (i.e. to neglect the influence of longitudinal force), then for reinforced concrete structures such idealization is not defined. And for steel structures, the limit $m_{ef}$ = 20 is specified to test the stability, and whether this recommendation of standards allows a common interpretation is unknown.

Of course, a competent engineer can determine this limit in each case, but some rule is required for the software implementation, and its absence creates a situation for unnecessary controversy.

The above is a fairly typical situation when the normative document contains some information (for example, tabular values), but in the program it is more profitable to calculate them than to borrow it from the table. What degree of disagreement is permissible (or non-existent) is the subject of many meaningless discussions. But the requirements of design norms are not laws of nature, they only approximate these laws with one or another degree of accuracy. Unfortunately, information about the errors that are permissible according to regulatory documents can be found nowhere. The only exception that can be found is the use of 10.0 instead of the exact value of the acceleration of gravity 9.81 when converting the normative values of loads from kPa in kgf / $m^2$ in building regulations SNiP 2.01.07-85* of 1985 edition or 0.1 instead of $1/\pi^2$ in the formula (108) of building rules SP 16.13330.2017.

The problem of joining results at conditional borders is connected with the delimitation of basic concepts. Since some simplifying hypotheses were used in various variants of the stress-strain state, belonging to one or another category of normalization (compressed-bent bar, bent beam, etc.), it is often difficult to implement a smooth border crossing.

Especially many problems are connected with necessity (possibility, desirability?) of performance of the general static calculation taking into account geometrical and physical nonlinearity declared by standards.

For example, in State Building Codes of Ukraine DBN B.2.6-198: 2014 [6] it is formulated as "*5.3.6... Steel structures should, as a rule, be calculated as a entire spatial system taking into account the factors that determine the stress and strain state if necessary, taking into account the nonlinear properties of the design schemes*".

And at the same time there are such statements: "*When dividing the system into separate elements, the design forces (longitudinal and shear forces, bending moments and torques) in statically indeterminate structures may be calculated without taking into account geometry changes and with the assumption of steel elasticity ... Calculation of statically indeterminate structures as entire systems can be performed using deformed model within the limits of elastic work of steel*".

However, the following question remains unclear. Is it possible to apply the results of the calculation which was performed taking into account geometry changes if it concerns the coefficient of longitudinal bending φ? This coefficient is of great importance for the stability of compressed steel rods analysis and calculated using deformed model (but for the element, not entire system).

**Problems that cannot be solved when using nonlinear calculation**

For a number of computational cases that inevitably arise in the actual design, regulations establish rules necessarily requiring a linear approach to solving the problem. An example is dynamic analysis closely related to such concepts of linear dynamics of structures as the frequency and mode of the natural vibrations of the system. For a nonlinear system, the very concept of individual forms of natural oscillations disappears and all recommendations based on this (i.e. the procedure of decomposition of motion into a superposition of normal modes) lose their meaning.

An alternative approach suitable for accounting for nonlinear effects is sometimes (though rarely) present in standards, such as direct dynamic calculation by instrumental or synthesized accelerograms, but more often it is not only not mentioned, but simply not developed. Analysis of the response to the pulsating wind loadings can be typical here.

Another problem that is not solved in the nonlinear analysis is the problem of choosing unfavorable load combination. In practice, there are virtually no structures that work only on one load option. It is usually necessary to anticipate the possibility of the occurrence of many temporary loads and, therefore, it is necessary to somehow determine their estimated combination. This problem has a solution with a linear approach to the calculation, when you can use the principle of superposition. If you focus on nonlinear analysis, then at the same time you should specify for which combination of loads you should perform strength and stability analysis. This type of instructions in regulations is often missing.

**Stable equilibrium**

Examination of the equilibrium stability of the complex bar type structure in the general case requires the calculation accounting the geometric nonlinearity and inelastic operation of the structural elements. Calculation of this type, in addition to computational complexity, also requires overcoming a number of other difficulties associated with the great uncertainty of the design assumptions (patterns of load change, idealization of material properties, initial irregularities, residual stresses, etc.). In this regard, in engineering practice there is a tradition of performing an idealized elastic calculation of the stability of the system as a whole in combination with checking individual elements for which more detailed account of the inelastic behavior of the material, initial bends and eccentricities and other circumstances is performed.

Most often, the stability problem is replaced by a refined calculation of the deformed model with increasing bending moments in compressed bars or other similar way by multiplying by some buckling length coefficient φ or coefficient of bending moments increasing $\eta=1/(1-N/N_{cr})$. The critical value (in the sense of loss of stability) of the value of compressive force takes part in the choice of the value of these coefficients and this fact ties the calculation of the deformed model to the stability analysis of the idealized model.

A natural question arises about the relationship between these two approaches. To what extent and for what purposes can their results be used separately and what is the link between them? It is believed that the bridge that combines these two approaches will be the buckling lengths of the elements of the system. Therefore, the fundamental question is of the method of determining the buckling lengths.

Note also that the use of the concept of buckling length involves the division of bar type systems into separate elements, it is necessary to take into account the interaction of the element with the foundation and other elements (primarily adjacent to it in the nodes).

The buckling length of the bars of the same system is different for different combinations of loads, although in design practice usually use a simplified approach (it is allowed, for example, by paragraph 13.3.2 of [7]), according to which it is allowed to determine the buckling lengths only for such a loads combination, which gives the largest values of longitudinal forces, and the resulting value is used for other loads combinations. It is implicitly assumed that there is a combination in which the compressive forces in all elements take the maximum values. But it is easy to imagine an example of a design where this assumption is not fulfilled and, therefore, the problem of choosing a combination of loads to the stability analysis is still relevant.

The logic of most of the standards recommendations is focused on flat computational models or, at least, on a separate consideration of the spatial scheme in two orthogonal planes. If we turn to spatial systems, they may have difficulties of a completely different kind, associated with the use of the concept of flexibility in the two orthogonal planes of inertia of the bar element.

Following the classical approach of F.S. Yasinski, the buckling length of the bar is usually understood as the conditional length f a simple bar, the critical force of which when hinged its ends is the same as for a given rod. In terms of physical content, the buckling length of the bar with arbitrary fixings is the largest distance between two inflection points of the bent axis, which are determined from the stability analysis of this bar by the Euler method.

In the works of Yasinski himself and in numerous subsequent works, where the concept of the buckling length of the bar appears, the use of plane calculation models and, accordingly, plane deformation models is implicitly implied. Only for them it makes sense to consider the distance between the inflection points of the curved axis, taken as the calculated length.

Since even for plane models, the buckling length of compressed bars should be determined both in the plane and from the plane of the system, then here there is a mismatch with the definition of F.S. Yasinski. Indeed, imagine a spatial cantilever bar in which the cross section has moments of inertia $J_x$ and $J_y = 4J_x$. Under central compression, such a bar loses stability under load $P_{cr,x}=\pi^2 EJ_x./(2l)^2$ ($l_{ef,x}=2l$).

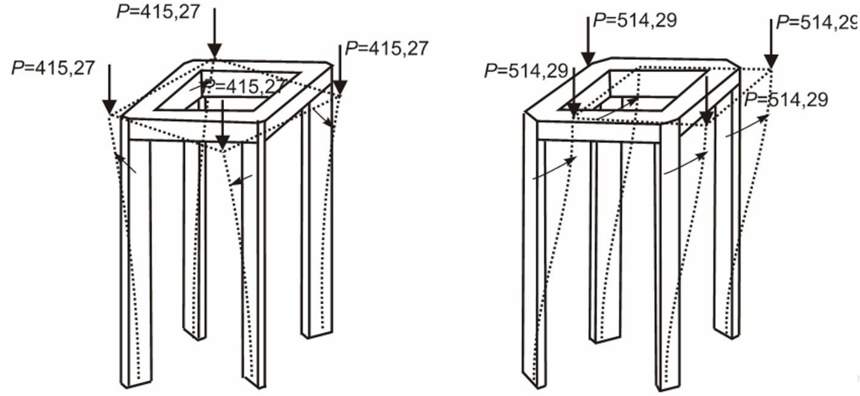
From the point of view of standards, apparently, it is possible to imagine a situation when two calculations on stability are performed during which deformation in one or in another main plane of inertia is alternately forbidden (for example, considering that $J_x = \infty$ and then $J_y = \infty$) , and after that the coefficients of the buckling length $\mu_x$ and $\mu_y$ are determined. But, as far as we know, for any complex systems, such even calculations in design practice are not performed.

Other problems arise when in the spatial system the main axes of inertia of the elements are not parallel to each other and the mode of stability loss, as well as the free lengths, is dependent on the orientation of these axes.

A fairly typical example is shown in Fig. 1, which shows the modes of stability loss and values of critical loads for two structures, which differ in that the cross-sections of the struts have different orientations of the main axes of inertia.

The model showed in Fig. 1 (a) has the coefficient of the buckling length in the plane of minimum rigidity $\mu_x$ = 0,597, while the model showed in Fig. 1 (б) has $\mu_x$ = 0,523. In the first case, the loss of stability mode is such that all the column are deformed in the plane of least rigidity. In the second case such deformation is observed only in two columns while the other two are deformed in the plane of greatest rigidity.

It should be noted that the solution of F.S. Yasinski refers to an elastic centrally compressed bar of constant cross-section, which when lost stability buckles in the form of a plane curve. Since the magnitude of the free length does not depend on the transverse load and is determined only by boundary conditions, this concept has been extended to elastic eccentrically compressed elements that bend in one of the main planes of inertia. Therefore, the in plane bending is implicitly assumed, because only in this case it makes sense to consider the distance between the inflection points of the bent axis, taken as the buckling length.




(a) (b)

Fig. 1

However, even a single bar can lose stability by having a spatial bending curve that occurs, for example, when the ends of the bar have cylindrical hinges whose axes are not parallel to each other [2]. Another example that limits the scope of the classical concept of the buckling length is the case of the torsional mode of stability loss. A number of other examples that indicate the difficulties arising here are given, for example, in [9].

However, the convenience of using the concept of the buckling length has made this method extremely popular, in almost all countries it is included in the regulations governing the verification of the bar structures equilibrium stability.

The buckling length of the elastic bar was used for normative calculation in the inelastic stage of the bar loading. It should be recognized that there is, in fact, no clear theoretical justification for this, and it should be considered a heuristic technique. And the widespread use of this technique is most likely due to the fact that engineers needed at least some practical method of calculating the bar structures for stability. Therefore clarity, associated with the solution of the simplest problems, replaced the reasoning of accuracy.

**Dynamic calculations**

Almost all regulations in the field of dynamics focus on the use of decomposition into modes of natural vibrations. Thus, the use of linear equations is implicitly assumed, and only in a few cases do software systems consider the linearized behavior of a nonlinearly deformable structure, i.e. analyze small oscillations around the deformed equilibrium position.

When focusing on the eigenmode decomposition, many regulatory documents indicate the number of eigenvalue forms to be taken into account, with no indication of the calculation model used. As a result, it has repeatedly happened that the first few natural frequencies (namely they are recommended to take into account by the standards) determine the local partial modes of motion, while the main mode of deformation is not the first.

The second problem of dynamic calculations, which is often mentioned indirectly by regulations, is the excessive simplification of dynamic models. This simplification due to tradition is often perceived as a characteristic of real behavior, which can lead to misunderstandings. Thus, the long-standing habit of using the cantilever calculation model in the seismic analysis has led to the fact that the detection of torsional vibrations as one of the lower is treated as a shortcoming, although no one could indicate what is the defect of this design.

It is necessary to mention one more aspect of dynamic calculations using eigenmode decomposition. It is associated with summation of modal contributions, which often follows the well-known "root-sum-squares" (RSS) rule. But this approach is based on the hypothesis that all modal reactions are normally distributed random variables with the same correlation coefficients, which is consistent with many observations, although not an established fact. Therefore, the absolutization of the RSS rule is rather doubtful. An example is the calculation using the accelerogram in those models where the equations of motion are solved by eigenmode decomposition, and summation fulfilled according to the RSS rule. But if the integration of equations of motion is performed, for example, by the Adams method, then we come to a completely different result. Nevertheless, since one and the same problem was solved, the result should not depend on the method of its solution.

The summation of internal forces, which are calculated by the usual rules for each of the eigenmodes, is also performed by the RSS method, but there may be another disappointment. The use of modules of moments, longitudinal and shear forces leads, for example, to disappearing of compressed-bent bars, that is all of them become stretched-bent. Similar effects of sign loss are possible in shell-type elements. To overcome this phenomenon in some software systems, the total values of internal forces are assigned signs, as in similar forces corresponding to the first eigenmode. It is difficult to substantiate such an approach, even if we assume that it is the first eigenmode that realizes the main contribution to the total value of each of the components of the response vector.

**Accuracy requirements**

Verification of compliance with structural design standards sometimes leads to uncertainties or errors due to the fact that the standards describe only one load or one stress-strain state. Detailed recommendations are given for this isolated situation, and in such a "ultimate" formulation (for example, as a calculation formula), which does not allow to understand what type of assumptions and simplifications were used. But in the real calculation it may be necessary to consider a less refined case and then there arise a number of difficulties.

As an example, we can point to the stability analysis of the plane bending of steel structures. The coefficient $\varphi_b$, the value of which is calculated in accordance with DBN B.2.6-198-2014 and depends, inter alia, on the location of the load within the beam height of (see table N4). But it may happen that the calculated combination of loads contains loads located both above and below the beam. In this case, the direct use of the rules becomes impossible.

If we take the opportunity to study the shell model of a thin-walled bar and with sufficiently detailed modeling to solve the problem of plane bending stability using the finite element method, it turns out that in the case of exact coincidence of loading options with the normative situation, we will get a solution. which does not coincide with the provisions of the design codes. This is because some approximations of exact expressions were laid down in the formulas of the appendix N [7], by means of which the coefficients $\varphi_b$ are calculated. The discrepancy may be small, but the rules by which they can be considered acceptable are unknown.

What degree of discrepancy is acceptable is the subject of much nonsensical debate. But the requirements of design standards are not laws of nature, they only approximate these laws with one or another degree of accuracy. Unfortunately, nowhere can be found information about the errors that allowed by the authors of the standards. The only exception that can be found is the use of the value of 10,0 instead of the exact value of the acceleration of gravity 9,81 when translating the normative values loads from kPa to kgf/m$^2$ in building regulations SNiP 2.01.07-85* of 1985 edition or 0,1 instead of $1/\pi^2$ in the formula (108) of building rules SP 16.13330.2017.

The problem of permissible discrepancy of results arises when the rules have some alternatives. The developers themselves were more likely to compare the results (if any) for a "typical case", but such a comparison does not follow a good correlation of the results in any case. An example is the analysis of methods for determining the width of cracks presented in [21], when the use of different alternative solutions, allowed by the standards showed more than 59% variance of the results.

There should be some measure which allow estimate the result of the comparison. After all, in engineering calculations there is no complete coincidence of results. The generally accepted norm of similarity in the form of a five percent discrepancy must also be specified and it is necessary to know to what results (displacement, effort, etc.) and to what values (extreme, average or other) it should refer. This problem would be greatly mitigated if the comparison was conducted only by the designer. However, submitted to the experts, such comparisons will be the subject of numerous and often pointless discussions.

**Programming as a means of controlling a regulatory document**

In the pre-computer period, the vague or ambiguous recommendations, although they were evil, but this evil was not as dangerous as it is today. Today, formal compliance with the rules in the software package is hidden from the eyes of the end user, and an unambiguous interpretation of the new paragraphs of the rules is primarily needed by software developers. And these points themselves should be set out in the wording, which should be in the nature of a clearly defined algorithm of action. It seems to us that this cannot be achieved without certain organizational changes.

Software implementation of the normative document is a good test procedure, which reveals discrepancies, logical inconsistencies, incompleteness and vagueness of the formulation and other shortcomings of the draft rules, in particular, compatibility with computer methods of analysis. As an example, we can refer to the construction of the bearing area of the element taking into account the full range of proposed requirements [11, 16] which revealed some inconsistencies that lead to the rupture of the boundary and non-convexity of the permissible loads area. The construction of this area is based on the analysis of calculations that contain several hundred variants of the internal forces values. Such mass verification was simply impossible in the era of manual arithmetic.

In addition, programming reveals those aspects of the normative document that are not formulated explicitly, as the developers of the norms focused on a qualified user who can independently decide on the use of a provision, based on the specifics of the calculation situation. This is not possible for a computer program, so it will definitely be installed during programming.

It is important that such verification work is performed without the participation of the developers of the regulatory document, which would ensure the purity of the experiment.

**Possible actions**

How can the contradiction between the desire to develop simple and understandable design rules (traditional approach to rationing) and the ability of modern computer systems to solve problems without the use of dubious simplifications (modernist approach) be eliminated?

It seems to us that two solutions are possible here:

• develop different versions of regulations for manual and computer calculation;

• create a special regulatory and methodological document on the rules for implementing the requirements of design standards in software.

The first option can be implemented in the traditional form, when formulating general requirements and necessary hypotheses, based on which one can create a software implementation. After that there appears a text such as "allowed ...", which presents a simplified version of the standardized provision.

And in the second option, the document should reflect:

• requirements for accuracy of calculations and permissible deviations from the literal implementation of regulatory guidelines;

• the procedure for verification and coordination with the authors of the standards concerning methods of numerical solution of design problems, which expand the possibilities of verifying regulatory requirements, but not available for manual calculation;

• requirements for software developers to inform users about the peculiarities of the implementation of regulatory requirements in case of deviation from their literal implementation.

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*Стаття надійшла 28.01.2020*

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**РОЗРАХУНКИ НА МІЦНІСТЬ В НОРМАТИВНИХ ДОКУМЕНТАХ І ПРОГРАМНИХ ЗАСОБАХ**

Сучасні норми будівельного проектування мають вже досить довгу історію. За цей час вони зазнали ряд змін, але деякі їх положення та рекомендації, будучи раз проголошеними, залишаються незмінними. І хоча вони не відповідають сучасним можливостями розрахункового аналізу, але продовжують своє існування в силу сформованої традиції. У цій роботі звертається увага лише на деякі із згаданих колізій, які пов'язані з програмною реалізацією нормативних вимог.

**Ключові слова**: несуча здатність, будівельні норми, комп’ютерний аналіз

*Perelmuter A.V.*

**STRENGTH ANALYSIS IN REGULATORY DESIGN DOCUMENTS AND COMPUTATIONAL SOFTWARE**

Modern building design standards have a long history. During this time, they have undergone a number of changes, but some of their provisions and recommendations, once proclaimed, remain unchanged. And although they do not meet the modern possibilities of computational analysis, but continue to exist due to the established tradition. In this paper, attention is paid to only some of the mentioned conflicts, which are related to the software implementation of regulatory requirements.

**Keywords:** load-bearing capacity, building codes, computer analysis.

*Перельмутер А.В.*

**РАСЧЕТЫ НА ПРОЧНОСТЬ В НОРМАТИВНЫХ ДОКУМЕНТАХ И ПРОГРАММНЫХ СРЕДСТВАХ**

Современные нормы строительного проектирования имеют уже довольно длинную историю. За это время они претерпели ряд изменений, но некоторые их положения и рекомендации, будучи раз провозглашенными, остаются неизменными. И хотя они не соответствуют современным возможностями расчетного анализа, но продолжают свое существование в силу сложившейся традиции. В этой работе обращается внимание лишь на некоторые из упомянутых коллизий, связанных с программной реализацией нормативных требований.

**Ключевые слова**: несущая способность, строительные нормы, компьютерный анализ

УДК 624.07+006.036
*Перельмутер А.В.* **Розрахунки на міцність в нормативних документах і програмних засобах** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 89-102.
*У статті звертається увага на невідповідність деяких положень і рекомендацій, наведених в нормах будівельного проектування, сучасним можливостями розрахункового аналізу.*
Іл. 1. Бібліогр. 16 назв.

UDC 624.07+006.036
*Perelmuter A.V.* **Strength analysis in regulatory design documents and computational software** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 89-102.
*The article draws attention to the discrepancy between some provisions given in the building design standards and modern possibilities of computational analysis.*
Fig. 1. Ref. 16.

УДК 624.07+006.036
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Ил. 1. Библиогр. 16 назв.

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UDC 539.3

# CREATION OF MATHEMATICAL MODEL OF PLATFORM-VIBRATOR WITH SHOCK, DESIGNED FOR CONCRETE PRODUCTS COMPACTION AND MOLDING

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DOI: 10.32347/2410-2547.2020.104.103-116

Platform-vibrators are the main molding equipment in the production of precast concrete elements. Shock-vibration technology for the precast concrete production on low-frequency resonant platform-vibrators significantly improves the quality of the products front surfaces and the degree of their factory readiness. This technology is used to produce large elements.

We describe the creation of a mathematical model for platform-vibrator that uses shock to produce asymmetric oscillations. The values of the upper and lower accelerations of the mold with concrete have different values with shock-vibration technology.

The created mathematical model corresponds to the two-body 2-DOF vibro-impact system. It is strongly nonlinear non-smooth discontinuous system. It has some peculiar properties, namely: the upper body with very large mass breaks away from the lower body during vibrational motion; both bodies move separately; the upper body falls down onto the soft constraint; the impact that occurs is soft one due to the softness and flexibility of the constraint. The soft impact simulation requires special discussion. In this paper, we simulate a soft impact by a nonlinear contact force in accordance with the Hertz quasistatic contact law.

The numerical parameters for this system were chosen in such a way that: firstly they provide the fulfillment of requirements for real machine, and secondly they allow analyzing its dynamic behavior by nonlinear dynamics tools. The created model is well enough to fulfill a number of requirements, namely: *T*-periodic steady-state movement after passing the transient process; the appropriate value of mold oscillations amplitude; the satisfactory value of the asymmetry coefficient that is the ratio of lower acceleration to the upper acceleration. We believe that the created model meets all the necessary requirements.

**Keywords:** platform-vibrator, shock, vibro-impact, mold with concrete, upper and lower accelerations.

## 1. Introduction

Molding processes are one of the most important in the manufacture of reinforced concrete structures. Now vibration and shock-vibration technologies for concrete mixtures compaction and concrete products molding have the greatest distribution in the construction industry. This priority is likely to continue in the future. Therefore, the issues of optimizing vibration modes, proper selection of vibration equipment do not lose their significance [1].

At the end of the last century 60s extensive technological researches began to optimize the molding modes. Gradually, it became clear that low-frequency compaction modes have undoubted advantages: they allow obtaining high-density concrete with a shorter compaction time. The asymmetric modes turned out to be very effective. The values of the upper and lower accelerations have different values under these regimes. Such modes are called shock-vibrational, and technology – shock-vibration. For their implementation, low-frequency resonant platform-vibrators were created. In recent years, shock-vibration modes have been widely used in various fields of technology.

Shock-vibration technology for the precast concrete production on low-frequency resonant vibratory platforms significantly improves the quality of the products front surfaces and their factory readiness degree. Platform-vibrators are the main molding equipment in the production of precast concrete elements [2]. Such equipment are produced in several factories in Russia.

Studies have confirmed that low-frequency resonant vibration platforms and shock-vibration technology allow to obtain higher quality products in comparison with other vibration platforms.

A complex process of concrete mixture particles interaction with each other occurs under vibration influence. Many aspects of this process are not well understood.

The proposed models and the corresponding equations of the concrete mixture state, the criteria for the compaction effectiveness and the front surfaces quality remain debatable [3].

Many experiments have been conducted to study various aspects of the concrete compaction process.

The magnitude of the working body acceleration $w$ was taken as one of the most important factors affecting the compaction process. It determines the values of dynamics strengths in machine elements to a large extent. So, it links technology process characteristics and strength machine characteristics.

If the working body makes asymmetric oscillations, it is advisable to take into account the upper acceleration $w_U$ and the lower one $w_L$. Upper acceleration is the acceleration of the mold with concrete at its highest position, and lower acceleration is the acceleration of the mold in its lowest position [4].

Studies were carried out at various values of the working body oscillations frequency.

Experiments with concrete mixtures of other compositions have confirmed that the concrete macrostructure formation is faster and better at a lower frequency and increased amplitude of the working body vibrations.

The goal of the paper is to create a mathematical model of platform-vibrator with shock and to select its numerical parameters so that:

a) the model maximally corresponded to the requirements for a real machine;

b) the model made it possible to analyze its dynamic behavior using nonlinear dynamics methods.

**2. Two body platform-vibrator with shock**

When developing resonant vibratory machines, designers have to take into account a large number of various requirements presented by modern production to new equipment. The realization of close to optimal operating mode is one of the most significant and fundamental. Its successful implementation is largely determined by the successful choice of the principle vibro-machine scheme and, as a consequence of this, its design scheme.

The specifics of many dynamic schemes was carefully analyzed in the work on resonant vibratory machines for compaction of concrete mixtures. It turned out that the conditions for optimal functioning can be satisfied by relatively simple two-mass systems. The creation of vibroforming machines with the number of main moving masses more than two is impractical, since this leads to an unjustified complication of the machines design.

The two-mass platform-vibrator with shock is one of the successful solutions for vibration equipment that implements shock-vibration technology for concrete mixtures compaction and reinforced products molding [5]. Its principled scheme is shown in Fig.1.

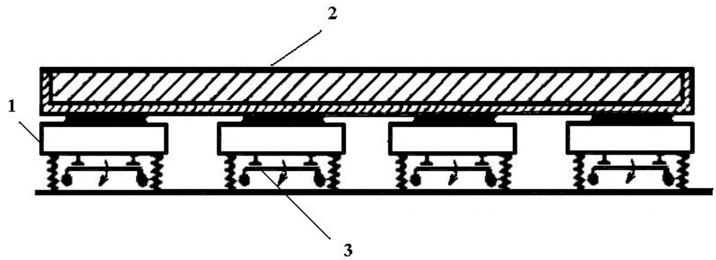


Fig.1 Principled scheme of platform-vibrator:
1 – working body; 2 – mold with concrete; 3 – vibration exciter.

The shock-vibration platform of block type consists of separate blocks on which rubber plates limiters are fixed. The mold with concrete mix is installed on the stops without fastening. With oscillations, the mold breaks away from the limiters and then falls on them. The mold collides with the limiters with oncoming movement. Two general vibration exciters are installed on each block.

One of the most important factors affecting the compaction process is the value of the working body acceleration $w$. More precisely, the ratio of the lower acceleration $w_L$ to the upper $w_U$ is important.

This can be explained as follows. With vertical oscillations, separation of the mixture from the pallet is possible only when the inertial forces applied to the particles of the concrete mixture act upward. The mixture is pressed to the mold pallet when inertial forces on the particles act downward [4].

The process of concrete mixture compaction is made more intensive due to such asymmetric vertical form vibrations. Accelerations $w_U$ that tear off mixture from the form pallet become smaller, and the pressing accelerations $w_L$ become larger with such oscillations. These asymmetric vibrations can be obtained precisely in shock-vibration compaction machines [4].

It is necessary to take into account the influence of concrete mix on the machine dynamics when calculating vibration compaction machines. So, it is necessary to consider the compacting machine and concrete mixture as a single dynamic system. But the concrete mixture is a complex viscoplastic medium. It has some elastic properties in the presence of air (especially in the initial period of compaction). All this makes it extremely difficult to solve a single dynamic system “compacting machine − concrete mixture”. The question about the nature of the machine interaction with the medium being processed remains the most complex and least investigated. However, the influence of the concrete mixture can be taken into account as an attached mass and some additional damping when practical calculating and, in particular, determining the amplitude-frequency responses of a vibratory machine [3].

The forces of resistance, despite their relative smallness, play a significant role in resonant and close to them oscillation modes. These forces are "used" by the system to compensate the energy coming from the external load. Therefore, it is necessary to take into account the resistance force in the resonance zone. They have not significant effect on the result outside this zone.

The attenuation coefficients are calculated from experimental data. One can assume that the dissipation energy is relatively small and significantly affects only on the resonating harmonic. Such an assumption may simplify the problem. However, accounting for energy dissipation in elastic elements is very important when studying the vibrations of an elastic system in the resonance region. Internal friction is determined by a number of factors. Their influence is difficult to take into account directly. There are many hypotheses to describe dissipative forces. The most widely used is the Kelvin-Voigt hypothesis. It is often called the viscous friction hypothesis. It suggests that dissipative forces are proportional to the strain rate of elastic bonds [6].

The parameters of the exciting force and of the system (the mass of the frame with the mold and concrete mixture, the stiffness coefficients of the vibration limiters, etc.) are selected so that the machine operating mode is close to resonant. That is, so that the natural frequency of the system is close to the frequency of the exciting force.

Resonant vibration platforms have wide possibilities for regulating the modes of working body oscillations. One can change both the amplitude and frequency, and the very law of working body oscillations in the machine tuning process.

The presence of two main moving masses allows us to solve two problems: to provide the necessary law of working body movement and to create an effective vibration isolation system. This can be achieved by appropriate selection of the elastic bonds characteristics.

One of the main disadvantages of resonance modes is associated with high system sensitivity to a change in its parameters, which is due to its strong nonlinearity. One can observe such sensitivity in real vibratory machines due to changes in the technological load mass, the characteristics of shock dampers and so on.

The calculation schemes for determining the amplitude-frequency responses of resonant vibration machines are based on assumptions usual to most applied problems of the oscillations theory. The main moving masses are assumed to be absolutely rigid. The masses of elastic bonds are not taken into account due to their relative smallness. Conditions guaranteeing single-axis motion are also fulfilled.

Such assumptions turn out to be quite acceptable and do not introduce significant errors in the final results of resonant vibratory machines calculating.

**3. The mathematical model of platform-vibrator with shock**

The calculation scheme of platform-vibrator is shown in Fig.2. Exciting force $F(t) = P\cos(\omega t + \varphi_0)$, its period is $T = 2\pi/\omega$.

Platform shock table with mass $m_1$ is attached to the base by linear vibration isolating spring of stiffness $k_1$ and a linear dashpot with damping factor $c_1$. Exciting external periodic force $F(t)$ is generated by electric motors mounted under the table. Elastic rubber gasket with thickness $h$ and stiffness $k_0$ is attached to the table. A linear dashpot with damping factor $c_0$ is placed between the table and the mold. Mold with concrete with mass $m_2$ is placed on the gasket but is not fixed both to the gasket and to the table. So it can tear herself away from the gasket and bounce. The machine starts their movement when the electric motors begin their work. First, the table and the mold move vertically together. Then the mold comes off from the gasket. The table and the mold are moving separately until the mold falls down onto the rubber gasket. Impact occurs. The bodies move together again until the mold comes off the gasket and so on.

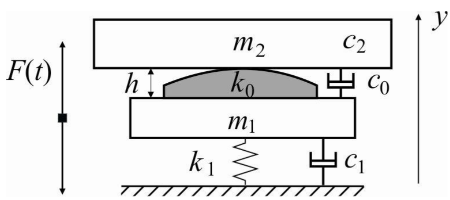


Fig.2

This shock-and-vibration machine is two-body 2-DOF vibro-impact system. The model has the following features: a large mass of the falling body – mold with concrete; softness, flexibility of one contacting surface – rubber gasket; separation of the one body (mold) from another (table with gasket) and their separate motion. One can consider the impact between mold and table with gasket as soft one because of the softness, flexibility of the rubber gasket.

Thus, we consider three states of the platform vibrator: the initial joint movement of both bodies, separate movement in case of loss of contact between them, and joint movement during the impact due to the form falling onto the gasket.

The forces acting on the bodies are as follows.

The elastic force in spring is

$$F_{k1} = k_1 \Delta l_1 = k_1 (y_1 - \lambda_{st})] . \quad (1)$$

The elastic force in rubber gasket is

$$F_{k0} = k_0 \Delta l_0 = k_0 [h - (y_2 - y_1)] . \quad (2)$$

The origin of coordinate $y$ is chosen in the table centre in the state of static equilibrium. The static deformation of spring is

$$\lambda_{st} = \frac{(m_1 + m_2) g}{k_1} . \quad (3)$$

The damping forces are taken to be proportional to the first degree of velocity:

$$F_{damp1} = c_1 \dot{y}_1 , \; F_{damp\,0} = c_0 \dot{y}_1 . \quad (4)$$

The influence of the concrete mixture can be taken into account as some additional damping $c_2 \dot{y}_2$ .

Then the primary joint movement of the table and the form until the first separation is described by the equations:

$$\begin{aligned} m_1 \ddot{y}_1 &= -m_1 g - F_{k1} - F_{damp1} - F_{k0} + F_{damp0} + F(t), \\ m_2 \ddot{y}_2 &= -m_2 g - c_2 \dot{y}_2 + F_{k0} - F_{damp0}. \end{aligned} \quad (5)$$

We introduce the standard notation:

$$\frac{k_1}{m_1} = \omega_1^2 , \; \frac{k_0}{m_2} = \omega_2^2 , \; \frac{c_0}{m_2} = 2\xi_0 \omega_2 , \; \frac{c_1}{m_1} = 2\xi_1 \omega_1 , \; \frac{c_2}{m_2} = 2\xi_2 \omega_2 , \; \frac{m_2}{m_1} = \chi . \quad (6)$$

The equations of primary joint movement will be written as follows;

$$\begin{aligned} \ddot{y}_1 &= g\chi - \omega_1^2 y_1 - \omega_2^2 \chi [h - (y_2 - y_1)] - 2\dot{y}_1 (\xi_1 \omega_1 - \xi_0 \omega_2 \chi) + \frac{1}{m_1} F(t), \\ \ddot{y}_2 &= -g + \omega_2^2 [h - (y_2 - y_1)] - 2\omega_2 (\xi_2 \dot{y}_2 + \xi_0 \dot{y}_1). \end{aligned} \quad (7)$$

The initial conditions are:

at $t = 0$ we have $\varphi_0 = 0 , \; y_1 = 0 , \; \dot{y}_1 = 0 , \; y_2 = h - \lambda_0 , \; \dot{y}_2 = 0$ . (8)

The static deformation of gasket is: $\lambda_0 = \dfrac{m_2 g}{k_0}$ .

The equations during the separate movement of bodies are:

$$\begin{aligned} \ddot{y}_1 &= \chi g - \omega_1^2 y_1 - 2\xi_1 \omega_1 \dot{y}_1 + \frac{1}{m_1} F(t), \\ \ddot{y}_2 &= -g - 2\xi_2 \omega_2 \dot{y}_2 . \end{aligned} \quad (9)$$

Impact occurs when the mold falls on the gasket. The mold with concrete and the table are moving jointly during impact. The equations of this movement are:

$$
\begin{aligned}
&\ddot{y}_1 = g\chi - \omega_1^2 y_1 - 2\xi_1\omega_1\dot{y}_1 + \frac{1}{m_1}F(t) + \\
&+H(z)\left\{2\xi_0\omega_2\chi\dot{y}_1 - \omega_2^2\chi[h-(y_2-y_1)] - \frac{1}{m_1}F_{con}(y_1-y_2)\right\}, \\
&\ddot{y}_2 = -g - 2\xi_2\omega_2\dot{y}_2 + \\
&+H(z)\left\{\omega_2^2[h-(y_2-y_1)] - 2\xi_0\omega_2\dot{y}_1 + \frac{1}{m_2}F_{con}(y_1-y_2)\right\}.
\end{aligned}
\qquad (10)
$$

Here $H(z)$ is Heaviside step function relatively bodies' rapprochement $z = h-(y_2 - y_1)$.

$$
H(z) = \begin{cases} 1, & z \ge 0 \\ 0, & z < 0 \end{cases}. \qquad (11)
$$

One can see that equations (10) include equations (9). When an impact occurs, $H(z) = 1$ and the terms in figure brackets begin to act.

$F_{con}(y_1 - y_2)$ is contact interactive force that simulates an impact and acts only during an impact. The type of this function requires the special consideration. It can be either a linear or non-linear function. Previously, we studied the impact simulation in vibro-impact systems with various impact types [7,8]. We'll describe the simulation of soft impact in detail in another paper. We'll compare the simulation by linear forces with different proportionality coefficients and the simulation by nonlinear Hertz' force. Now, when we'll choose the numerical system parameters in Section 4, we'll use the nonlinear Hertz's force [9,10] for calculations.

It is worth to point out that Hertz' contact theory requires that the strains in the contact region be sufficiently small to be within the scope of the linear theory of elasticity. "Metallic solids loaded within their elastic limit inevitably comply with this latter restriction. However, caution must be used in applying the results of the theory to low modulus materials like rubber where it is easy to produce deformations which exceed the restriction to small strains" [11]. For an example we have calculated the impact duration in three cases. We have compared the values obtained by known formula [11,12] and by numerical integration of the motion equations (10). The results are shown in Table 1.

Table 1

| **Moduli of elasticity of contacting bodies** | | **Impact duration by formula [11,12], s** | **Impact duration by integration of equations (10), s** |
|---|---|---|---|
| $\boldsymbol{E_1}$, N·m$^{-2}$ | $\boldsymbol{E_2}$, N·m$^{-2}$ | | |
| 3.5 E+06 | 2.0 E+11 | 0.152 | 0.0135 |
| 3.5 E+08 | 2.0 E+11 | 0.0256 | 0.0125 |
| 2.0 E+11 | 2.0 E+11 | 0.00257 | 0.00259 |

Nevertheless, Hertz's contact theory is widely used to model impact. It provides fairly good and reliable results.

The calculation scheme of shock-and-vibration machine corresponds to two-body 2-DOF vibro-impact system. It is strongly nonlinear non-smooth discontinuous system. It changes its structure during oscillatory motion. The right-hand sides of motion differential equations are discontinuous.

**4. Selection of numerical system parameters**

The basis for the choice of numerical parameters was "Recommendations for vibratory molding of reinforced concrete products" [5]. But, as we wrote in Section 2, this system is very sensitive to changes in its parameters due to strong nonlinearity. The parameters of the exciting force and of the system (the mass of the mold with concrete mixture, the stiffness coefficients of the vibration limiters, etc.) are selected in the machine tuning process. In particular, they are selected in such a way that the operation mode of the machine is close to resonant.

First of all, it is necessary to set damping ratios $\xi_1$ ,.$\xi_2$ , $\xi_0$. We can assume that the damping ratio should remain in the range $0\le \xi \le 1$ [13].

The damping effect plays a key role when the excitation of oscillations occurs at a frequency close to the natural system frequency. With precise resonance, the amplitude of the oscillations will tend to infinity until damping is taken into account. The actual amplitude in resonance is determined in fact by the magnitude of such damping.

We choose the damping ratios so as to obtain firstly, steady-state $T$-periodic oscillatory process after transient period, and secondly, the amplitude of mold oscillation, close to the required 0.8 – 1 mm.

Then we select the parameters of stiffness for spring $k_1$ and for gasket $k_0$. It is known that the parameters of rigidity are relevant ones. They strongly affect the oscillatory process. They also determine the natural system frequency. When selecting these parameters, we are guided by the same principles as when choosing damping coefficients.

**Note 1.** Elastic moduli of mold and gasket, Poisson's ratios, and gasket radius are included in the expression of contact Hertz's force. The gasket surface is flat. But we consider it as sphere of large radius in order to use the expression for contact Hertz' force.

It must be said that we cannot calculate the natural frequency of the oscillatory system. It changes its structure during movement because the mold comes off the table and then falls down on it. Its oscillatory motion is described by the equations (10). It is seen that Heaviside function $H(z)$ provides a change in structure. Separate motion of bodies and their joint motion during an impact are described by different equations. Therefore, the stiffness matrixes are different for these equations. We'll see a resonance after the formation of amplitude-frequency responses. Then we can clarify some system parameters

After many numerical experiments, we take the numerical parameters in vibro-impact system as shown in Table 2.

Table 2

| | |
|---|---|
| Mass of table $m_1$, kg | 7400 |
| Mass of mold with concrete $m_2$, kg | 15000 |
| Stiffness of rubber gasket $k_0$, N·m$^{-1}$ | $3.0 \cdot 10^8$ |
| Stiffness of spring $k_1$, N·m$^{-1}$ | $2.6 \cdot 10^7$ |
| Thickness of gasket $h$, m | 0.0275 |
| Damping ratio of dashpot 1 (spring) $\xi_1$ | 0.5 |
| Damping ratio of dashpot 0 (gasket) $\xi_0$ | 0.02 |
| Damping ratio in concrete mixture $\xi_2$ | 0.03 |
| Elastic modulus of mold $E_2$, N·m$^{-2}$ | $2 \cdot 10^{11}$ |
| Elastic modulus of rubber gasket $E_1$, N·m$^{-2}$ | $3 \cdot 10^7$ |
| Poisson's ratio of mold $\nu_2$ | 0.3 |
| Poisson's ratio of rubber gasket $\nu_1$ | 0.4 |
| Radius of gasket $R$, m | 10 |
| Amplitude of exciting force $P$, N | $2.44 \cdot 10^5$ |
| Frequency of exciting force $\omega$, rad·s$^{-1}$ | 157 |

**Note 2.** After integrating the equations of motion (7) and (10), we get a complete picture of the system motion, including the impact time. It should be noted that we were forced to significantly reduce the integration step during an impact. This decrease was much stronger than we did before when we considered a rigid (hard) impact between solids. We think this is due to the great softness of the rubber gasket. Its modulus of elasticity is small; therefore, its deformations during an impact may be not small.

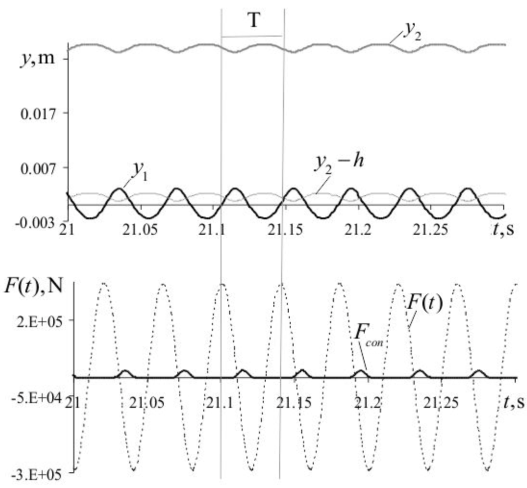


Fig. 3

We get a graph of contact forces and graphs of all other forces (Fig.5,6) acting in the system during movement. We give the graphs of both bodies' velocities and accelerations.

Time histories and contact force graph (Fig.3) show that the movement is *T*-periodical with one impact per cycle. We can clearly see the great penetration one body into another when we look at the curve ($y_2 - h$). The impact is very soft due the softness and suppleness of the rubber gasket and the penetration is quite large. Amplitude of mold oscillations is $A$=0.76 mm.

The phase trajectories (Fig.4) show that *T*-periodical movement is steady-state after passing the transient process.

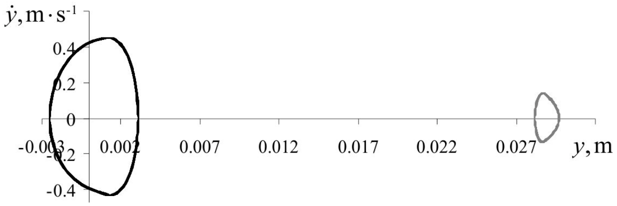


Fig. 4

The speed graph (Fig.5) shows the gradual, not instantaneous reverses in velocities during an impact because the impact is not instantaneous, its duration is long. Impact time $T_{impact}$ is marked by vertical lines in Fig.5.

And finally, the acceleration graph (Fig. 5) gives the picture of asymmetric accelerations. We have the lower acceleration $w_L = 39.4$ m·s$^{-2}$ ≈ 4 *g* and the upper acceleration $w_U = 11.2$ m·s$^{-2}$ ≈ 1.1 *g* (*g* is the gravitational acceleration).

Its ratio is $\frac{w_L}{w_U} = 3.6$.

**5. Conclusions**

The created mathematical model of platform-vibrator with shock corresponds to the two-body 2-DOF vibro-impact system. It is strongly nonlinear non-smooth discontinuous system. It has some peculiar properties, namely: the upper body with very large mass breaks away from the lower body during vibrational motion; both bodies move separately; the upper body falls down onto the constraint that is on the rubber gasket; the impact that occurs is soft one due to the softness and flexibility of the rubber gasket. The numerical parameters for this system were chosen in such a way that: firstly, they provide the fulfillment of requirements for real machine, and secondly, they allow analyzing its dynamic behavior by nonlinear dynamics tools. The created model provides: *T*-periodic steady-state movement after passing the transient process; the appropriate value of mold oscillations amplitude $A$= 0.76 mm; the satisfactory value of the asymmetry coefficient – the ratio of lower acceleration to the upper acceleration is $\frac{w_L}{w_U} = 3.6$.

Thus, we believe that the created model meets all the necessary requirements.

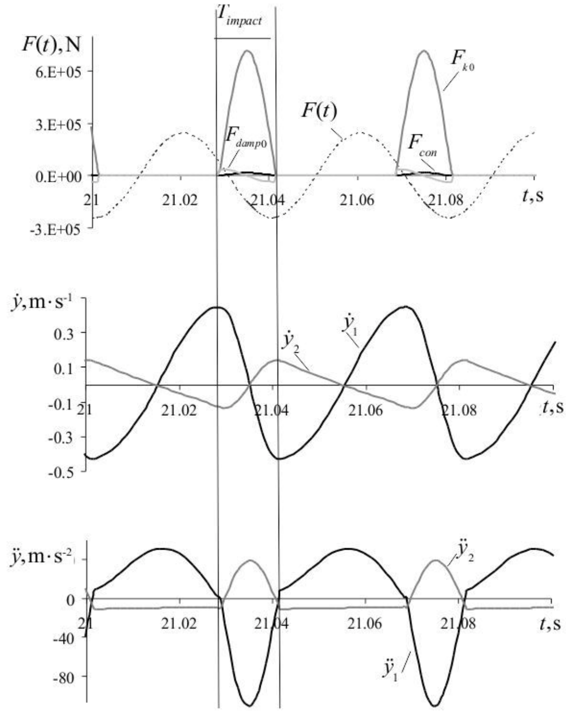


Fig. 5

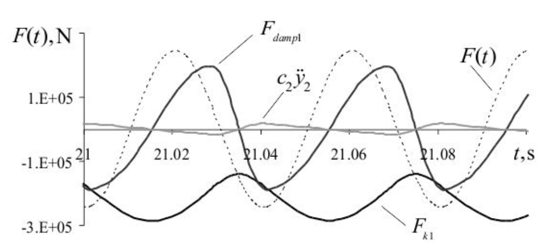


Fig. 6

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*Стаття надійшла 28.01.2020*

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**СТВОРЕННЯ МАТЕМАТИЧНОЇ МОДЕЛІ УДАРНО-ВІБРАЦІЙНОГО МАЙДАНЧИКА ДЛЯ УЩІЛЬНЕННЯ ТА ФОРМУВАННЯ БЕТОННИХ ВИРОБІВ**

Вібраційні майданчики є головним обладнанням при виробництві бетонних та залізобетонних виробів. Ударно-вібраційна технологія при виробництві збірного залізобетону на низькочастотних резонансних вібро-майданчиках значно поліпшує якість виробів та ступінь їхньої заводської готовності. Ця технологія використовується для виробництва великогабаритних виробів.

В статті описується створення математичної моделі ударно-вібраційного майданчика, де реалізується режим асиметричних коливань, у якому верхнє та нижнє прискорення форми з бетоном мають різні значення. Створена математична модель відповідає двох-масовій вібро-ударній системі з двома ступнями вільності. Це сильно нелінійна негладка розривна система, яка має такі особливості: верхнє тіло дуже великої маси відривається під час коливального руху від нижнього тіла, і тоді тіла рухаються окремо; потім верхнє тіло падає на м'який обмежник; відбувається м'який удар. Моделювання м'якого удару потребує окремого обговорення. У цій статті удар моделюється нелінійною контактною силою відповідно до квазістатичного контактного закону Герца.

Числові параметри системи вибиралися таким чином, щоб по-перше, вони забезпечували виконання вимог до реальної машини, та по-друге, дозволили виконати аналіз її динамічної поведінки засобами нелінійної динаміки. Створена модель достатньо добре забезпечує виконання низки вимог, а саме: *T*–періодичний усталений рух після перехідного процесу; придатне значення амплітуди коливань форми; задовільну величину коефіцієнту асиметрії, а саме відношення нижнього прискорення до верхнього.

**Ключові слова**: ударно-вібраційний майданчик, вібро-ударна система, форма з бетоном, верхнє та нижнє прискорення.

UDC 539.3

*Bazhenov V.A., Pogorelova O.S., Postnikova T.G.* **Creation of mathematical model of platform-vibrator with shock, designed for concrete products compaction and molding *L*yapunov exponents estimation for strongly nonlinear nonsmooth discontinuous vibroimpact system** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 103-116.

*Platform-vibrators are the main molding equipment in the production of precast concrete elements. Shock-vibration technology for the precast concrete production on low-frequency resonant platform-vibrators significantly improves the quality of the products front surfaces and the degree of their factory readiness. This technology is used to produce large elements.*

*We describe the creation of a mathematical model for platform-vibrator that uses shock to produce asymmetric oscillations. The values of the upper and lower accelerations of the mold with concrete have different values with shock-vibration technology.*

*The created mathematical model corresponds to the two-body 2-DOF vibro-impact system. It is strongly nonlinear non-smooth discontinuous system. It has some peculiar properties, namely: the upper body with very large mass breaks away from the lower body during vibrational motion; both bodies move separately; the upper body falls down onto the soft constraint; the impact that occurs is soft one due to the softness and flexibility of the constraint. The soft impact simulation requires special discussion. In this paper, we simulate a soft impact by a nonlinear contact force in accordance with the Hertz quasistatic contact law.*

*The numerical parameters for this system were chosen in such a way that: firstly they provide the fulfillment of requirements for real machine, and secondly they allow analyzing its dynamic behavior by nonlinear dynamics tools. The created model is well enough to fulfill a number of requirements, namely: T-periodic steady-state movement after passing the transient process; the appropriate value of mold oscillations amplitude; the satisfactory value of the asymmetry coefficient that is the ratio of lower acceleration to the upper acceleration. We believe that the created model meets all the necessary requirements.*

Table 2. Fig. 6. Ref. 13

УДК 539.3

*Баженов В.А., Погорелова О.С., Постнікова Т.Г.* **Створення математичної моделі ударно-вібраційного майданчика для ущільнення та формування бетонних виробів** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 103-116. – Англ.

*Вібраційні майданчики є головним обладнанням при виробництві бетонних та залізобетонних виробів. Ударно-вібраційна технологія при виробництві збірного залізобетону на низькочастотних резонансних вібро-майданчиках значно поліпшує якість виробів та ступінь їхньої заводської готовності. Ця технологія використовується для виробництва великогабаритних виробів.*

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Табл 2. Рис. 6. Бібліогр. 13.

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UDC 355.41

# EXPERIMENTAL INVESTIGATION OF IMPACT OF INJURY MEASURES ON THE PROTECTION SCREENS OF COMBAT ARMOURED VEHICLES

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DOI: 10.32347/2410-2547.2020.104.117-135

**Abstract.** In order to evaluate the relative properties of the protective screen to the striking energy of the means of defeat, an experimental study was conducted, which allowed to test the hypothesis regarding the protection of the combat armoured vehicles against the means of defeat which, under the action of kinetic energy, destroy the armoured obstacle. Based on the data obtained during the experimental study, a mathematical model was constructed that describes the punching momentum of the protective screen. The use of this model makes it possible to calculate the energy losses caused by deformation and destruction of the obstacle. Built as a result of the multifactor experiment, the regularity of the impact of the means of defeat on the protective screen takes into account the speed of the means of defeat, the angle of encounter of the means of defeat with the protective screen, the thickness of the front and back layer and the hardness of the means of defeat.

**Keywords**: Combat armoured vehicles, protective screens, experimental study, mathematical model.

**1. Introduction**. Carrying out experimental studies concerning the security of combat armored vehicles (CAV) against firearms is crucial to substantiate the feasibility of using additional CAV protective screens. Tests are conducted to evaluate the effectiveness of the protection of the finished specimens, during which firearms or the means of imitation of firearms are used. Most often, such tests are performed as experiments to further validate the sample to ensure that the CAV sample withstands certain effects or to verify the claimed sample characteristics.

Therefore, the purpose of laboratory testing is to obtain useful information for assessing the relativity of persistent or weak properties to the effects of firearms on the elements of the sample (system). Laboratory tests are the basis for evaluating changes in the properties of elements that are not sensitive to the effects of the firearms lesion, as well as for getting empirical data about the behaviour of many elements critical of the firearms lesion. In addition, they are linked to test methods that can serve for more effectively align of requirements to the stability of remedies with other requirements which are presented to the providers of protection means.

**2. Problem Formulation**

Given the fact that experimental studies require a considerable investment of time and money, so in the study of these phenomena most often use a stochastic approach, which abstracts from a number of factors. In this case, experimental and statistical methods of research are applied, in which real processes are considered as processes of probability, and the object of study is represented as a cybernetic system (black box), which is investigated by means of mathematical modelling [1]. Herewith about the functioning of the system is judged by its reaction $Y$ (baseline , response) at the output of the system when it causes certain influences $X$ (factors) at its input (fig. 1).

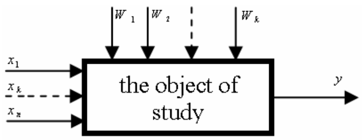



Fig. 1. The system of "Black box", the model of the object of study

For this purpose, we obtain a mathematical model of the process under study that adequately describes the relationship of process results (baseline indicators of $y$) to external influences (input factors of $x$).

It does not deny the possibility of further investigation of internal phenomena, but also generalizes information for much deeper disclosure of the cause-and-effect links in the processes that take place. Inasmuch as each of the responses is related to the input factors of the objectively existing dependency. Equation of state of the system, which, of course, is unknown. But, based on observations of the system's appeal , namely, the correspondence of the output indicators to the input at each point in time, the equation of state can be approximated by another function of the form [2].

$$Y = B_0 + \sum_{1 \le i \le k} B_i X_i + \sum_{1 \le i < l \le k} B_{il} X_i X_l + \sum_{1 \le i \le k} B_{ii} X_i^2 + \ldots$$

where $Y$ an indicator of the process under study; $X_i$ – factors affecting the process under study; $B$ – regression equation coefficients; $B_0, B_i B_{il} B_{ii}$ – polynomial coefficients; $k$ – number of independent variables.

This regression equation is a polynomial model in the form of a Taylor series segment that describes well the function response of the local plane of the factor space and is convenient to use due to the versatility and comparative simplicity of their methods of construction based on experimental data [3].

The analysis of scientific studies showed that in the work [4], the results of the analysis of armoured machine body are given.The problems of providing an adequate level of ballistic and mine resistance are identified. The combined nature of the causes is due to the welding of steels used in the production and structural features of a number of housings, but it is not specified exactly how to increase the ballistic stability of the CAV housings in this work. In the dedicated work [5] on numerical modeling of process of penetration of protective ceramic elements with different design, the effectiveness of the developed protective ceramic elements for the protection of CAV was confirmed. But in this work is not specified as a means of defeat loses energy characteristics when breaking through an armour obstacle. The paper [6] presents a finite element model based on data obtained as a result of dynamic and static testing of composite materials to predict the response and behavior of failure of hybrid plates at low shock load. In this study, the impact of the velocity of the impactor, the angle of its incidence, and the thickness of the impactor were taken into account, but the hardness of the impactor was not taken into account and damaged obstacles were not investigated. In the work [7] that is devoted to increasing the level of protection of multi-purpose design vehicles, many variants of technical solutions for increasing the level of ballistic protection were proposed. The results of observations on vulnerable sections of vehicles from small firearms are presented, but it is not determined how to increase the level of protection against the means of damage.

That is, in these works [4–7] are not defined how the means of lesion will lose kinetic energy depending on the velocity, the angle of encounter with the protective screen, the thickness of the face and the rear layer of the protective screen, and the hardness of the means of lesion.

Therefore, the purpose of this article is to elucidate the results of an experimental study of the effect of the means of lesions on the protective screens of the CAV and to build a mathematical model based on experimental studies of energy loss by the means of lesion during the break of the protective screens of the CAV.

## 3. Experimental study

### 3.1 Experimental equipment

To evaluate the stability of the protective screen, the technique of investigation of the parameters of the breakdown with the registration of the shock pulse, which allows to carry out of rapid assessment of the resistance of materials to deformation and fracture during cross-cutting. [8–9]. However, it is possible to obtain a quantitative assessment of the stability of the material of the armor obstacle design to the breakdown, taking into account the conditions of interaction, physical and mechanical properties and geometric parameters of the impactor and obstacle. The essence of the technique lies in the fact that the obstacle samples are broken through by the impactor, so that the pendulum is given a shock impulse, which causes the deviation of the latter from the equilibrium position by the value $L$, which can calculate the energy losses caused by the deformation and destruction of the obstacle fig. 2.

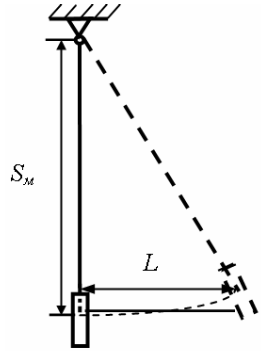


Fig. 2. Schema of ballistic pendulum

Bulletproof resistance studies were conducted in the ballistic track of the Weapons Scientific Testing Laboratory and special protective materials in accordance with the requirements [10] of the experimental setup (fig. 3) [11].

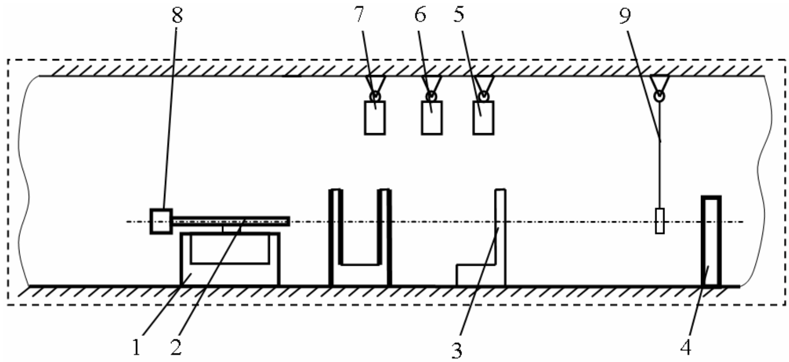



Fig. 3. Schema of ballistic installation:
1 - ballistic barrel mounting stand; 2 - barrel, 3 - velocity determination device, 4 - box, 5 - illumination level determination device, 6 - noise level recording device, 7 - temperature recording sensor, 8 - duct pressure sensor in the barrel channel, 9 - ballistic pendulum

In doing so, a ballistic pendulum weighing $M_б$ having the length of the pendulum from the point of hit of the ball in the sample to the axis of the swivel was used $S_M$ (fig.1). The pendulum was suspended from the ceiling in the ball holder of the ballistic track . The firing was carried out from a ballistic weapon by single-shots, mounted in a special device fastening of the product. The impactor was accelerated with the powder gases through the ballistic barrel channel, which at speeds of 300 – 900 m/s. interacted with an obstacle that was fixed on the ballistic pendulum. After the breaking through , he fell into the "unobstructed catcher". Shots were made with the help of an electro trigger. The velocity of the bullet was measured by the optoelectronic measuring complex IBX – 731.3, located at a distance of 2.5 m from the cut of the barrel. The distance from the section of the ballistic barrel to the point of defeat of the sample, which is rigidly fixed on the ballistic pendulum, was 10 m.

To ensure the free passage of the impactors to the ballistic pendulum with simultaneous cutting off of the powder gases, special cut-offs are used to prevent them from influencing the sample fixed to the pendulum.

The study was conducted under the following conditions:

ambient temperature 0C 20 ± 5

relative humidity,% not more than 80

atmospheric pressure, kPa 87 – 107

The object of the test is selected, proposed in the paper [12] protective screen fig. 4, the front and back layer of which is made of BT70SH steel and 10 mm thick porous AlSi7 cast aluminum is selected as the porous material.

As an impactor was used a cylindrical device with rounded ends with a diameter of 15 mm and a length of 40 mm made of У8А steel.

Before the research the ballistic pendulum was calibrated. For this purpose, a non-penetrating indenter shot was used, which is used in the study. In this case, all the energy of the ball is spent on the deflection of the pendulum from an equilibrium state. Indenters and sample BT70SH by weight $M_n$ were used for calibration. All shots were carried out normal to the plane of the specimen. The thickness of the specimen was chosen to ensure that it was not punched. The following ratio was used to determine the calibration factor for each shot

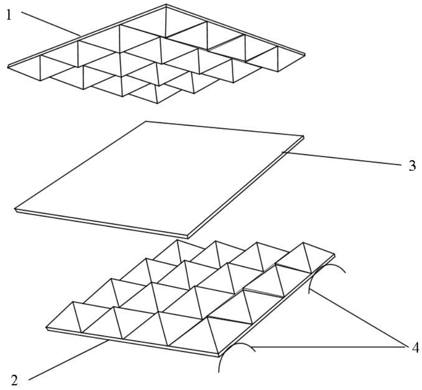


Fig. 4. Protective screen CAV:
1 – the front layer, 2 – the back layer, 3 – the porous material, 4 – elastic elements

$$k_{ni} = (mV_{2,5})(S_p)^{-1},$$

where $k_{ni}$ the calibration factor of the ballistic pendulum for each shot; $m$ mass of the intender, kg; $V_{2,5}$ speed of the intender at a distance of 2.5 m from the cut of the barrel, m/s; $S_p$ the magnitude of the deviation of the pendulum from its equilibrium state after the indenter is hit in the sample, m.

Three series of experiments were carried out to determine the calibration factor, taking into account different speeds of the intender. The values of the calibration coefficients for each shot were calculated by the average value of the calibration coefficient equal to

$$k_n = (\sum_{i=1}^{N} k_{ni})(N)^{-1} .$$

The positioning of the ballistic barrel, the speedometer and the ballistic pendulum was constant, changing only the angle of inclination of the test specimen in accordance with the plan of the experimental study [13].

**3.2 Construction of mathematical model**

In order to evaluate the relative properties of the protective screen to the striking energy of the means of lesion, in the first stage of the study experimental tests were conducted which became the basis for obtaining empirical data on the behavior of many elements critical to the kinetic energy of the means of lesions.

The data obtained in the experimental study allowed us to test the hypothesis regarding the protection of CAV from the defeat agents, which under the action of kinetic energy destroys the armor obstruction, thereby causing premature failure of the CAV [14].

Previous studies have shown that the kinetic energy parameters of the means of defeat in one way or another affect the stability of the armoured obstacle, which leads to its destruction. Eventually leads to failure of the CAV. The basis of this nature of influence is a complex of physical phenomena that accompany the process of destruction of the armoured obstacle.

In order to calculate the model, information about the value of the response, which is investigated in the selected area of the factor space, is accumulated during the experiment. The most effective way of doing this is through an active experiment on the basis of multi-factor planning [15].

Planning a multifactorial experiment involves choosing the type of mathematical model. Insomuch as the real nature of the processes that occur under the influence of the lesion on the CAV in general is largely unknown then it is quite difficult to build a model adequate to the real process in advance. In this case, it would be most rational to use a priori information on similar studies.

First of all, it is about choosing a model class [8], namely about choosing a function

$$M(y) = f(x_1, x_2, ..., x_k), \quad (1)$$

where $y$ indicator of the process under study (response); $x_1, x_2, ..., x_k$ variable factors.

As stated in the works [1–3, 8, 13–15], in the study of a large group of technical processes, it is better to use as a specified function step series, or rather segments of step series - algebraic polynomials. On the one hand, these are fairly simple equations, in terms of mathematical processing, and on the other hand, there is a high probability of obtaining an adequate model.

The next step is to choose the degree of the polynomial. In situations where there is no a priori information on the order of the polynomial, the mathematical model of the process under study is selected, starting with the simplest linear equation, and consequently increasing the degree of the polynomial to obtain an adequate model. The process of obtaining a mathematical model in these situations is as follows. Initially, a full factorial experiment $2^k$ or an experiment represented by a fractional replica is implemented, $2^{k-p}$, where $p$ is the number of interaction effects replaced by the new variables.

According to the results of experiments performed in accordance with these plans, the coefficients of the linear regression equation are found. If this equation proves to be inadequate, then the regression coefficients are found for the interaction factors. If the regression equation for the interactions of the factors is also inadequate, then the previously performed experiments supplement the experiments at the "star" points with the shoulder $\alpha$ and experiments at the center of the plan, the number of which is equal $n_0$. The number of experiments at "star" points is equal $2k$. according to the results of experiments performed according to the plan $2^k$ or $2^{k-p}$ and with additional experiments at the “star” points and at the center of the plan, the second order polynomial coefficients are estimated. It should be noted that the process under study can often be described as a second-order polynomial. If the second-order polynomial is inadequate, proceed to the third-order planning and describe the process under consideration by the third-degree polynomial [1–3].

For five factors, they consider a valuable, second-rate, central composite rotatable plan. In this plan, each variable varies in only three levels: +1, 0, -1. The use of this plan, which involves only three levels of variation of factors, simplifies and reduces the cost of the experiment [16].

Based on the results of the experiments presented according to the considered plan, the coefficients of the regression equation can be determined

$$y=b_0+b_1x_1+b_2x_2+...+b_kx_k+b_{12}x_1x_2+...+b_{k-1,k}x_{k-1}x_k+b_{11}x_1^2+...+b_{kk}x_k^2.. \quad (2)$$

Equation coefficients (2) can be determined using the least squares method, which is one of the basic methods of regression analysis for estimating unknown parameters of regression models by sample data [17].

Thus, the technique is based on the experimental-statistical method of mathematical modeling of the process of the impact of the energy of means of defeat on the armoured obstacle of CAV, in which the experiment is considered as the main source of information about the process, and methods of probability theory and mathematical statistics is considered the main means of processing the results of the experiment.

Experimental studies include a fairly large set of interdependent sequential operations that can be divided into several stages. The logical sequence of the experimental study is shown in fig. 5. It should be noted that the planned experiment can only be successful under a number of conditions.

Firstly, the object of the study should be manageable that is to say it should be possible to unambiguously identify these factors in the selected area and unambiguously determine the relevant responses. In addition, the baseline (responses) should be quantitative and should be measured with any possible combination of selected factor levels. The factors must be independent, unambiguous and compatible. The process under study must be carried out in the entire area of the chosen factor space, that is, in the whole range of change of the selected factors. Furthermore, the researchable object must satisfy the reproducibility requirement of repeatedly repetition of the same experiment, and its responses should have a scatter not exceeding some specified value [14].

Thus, the task is to determine how the impactor will affect the destruction of the armoured obstacle.

These problems can be solved by staging an extreme study. In planning, the following factors were adopted as variables: $v_f$ - the speed of the means of defeat, m/s; $\gamma$ - the angle of the indenter meeting with the protective screen, deg.; $h_f$ – thickness of the front layer, mm; $h_r$ – thickness of the back layer, mm; $H$ – hardness of the impactor, HB (300-600 steel U8A);

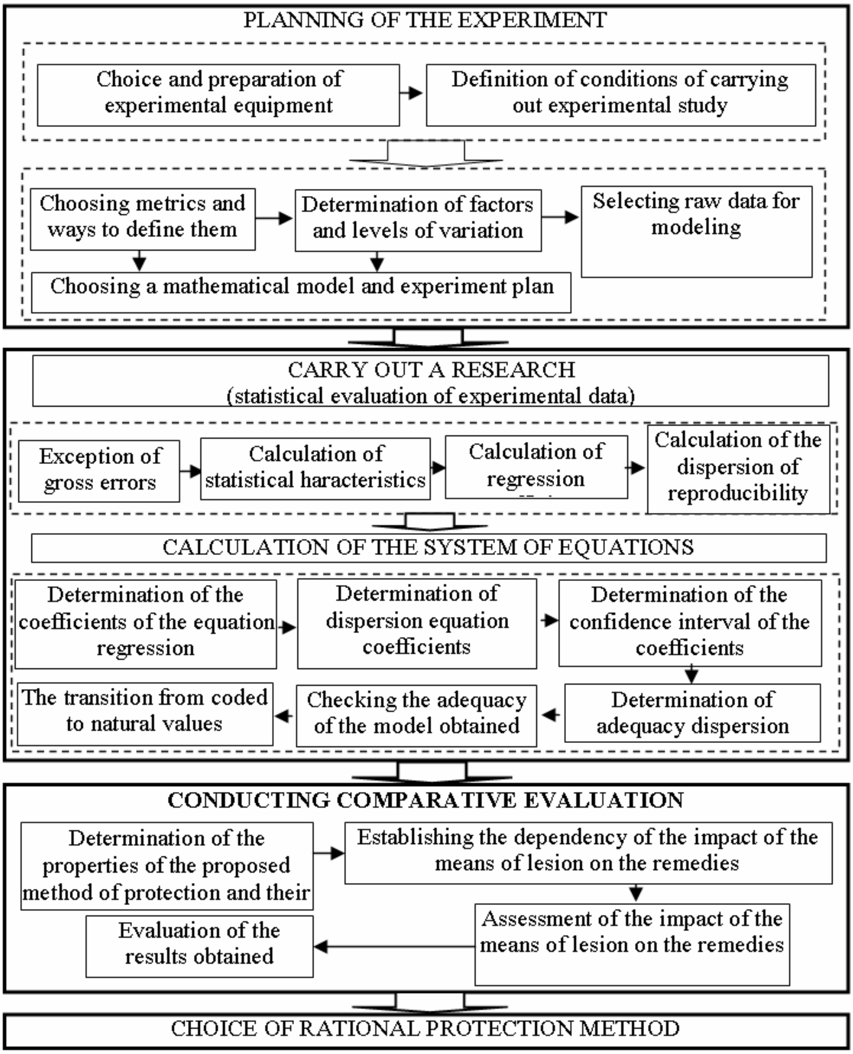



Fig. 5. Structural and logical scheme of the experimental study of the effect of means of lesion on the protective screen of CAV

The impulse deflection of the ballistic pendulum is taken as the optimization parameters. The main levels and intervals of variation of factors [18] are selected by the results of previous experiments, the intervals of variation and levels of factors are given in table 1.

To obtain the model of the process in the form of a second-degree polynomial [19], a central composite second-order rotatable plan is presented, which is presented in table 2.

Table 1

Levels and intervals of variation of factors

| Factors | Variation intervals | Levels of factors | | |
|---|---|---|---|---|
| | | Main 0 | Upper +1 | Lower –1 |
| $x_1$ – the speed of the means of defeat $v_f$, m/s | 100 | 800 | 900 | 700 |
| $x_2$ – angle of the indenter meeting with the protective screen, $\gamma$, deg | 15 | 45 | 60 | 30 |
| $x_3$ – thickness of the front layer, $h_f$, mm | 5 | 10 | 15 | 5 |
| $x_4$ – the thickness of the back layer, $h_r$, mm | 5 | 10 | 15 | 5 |
| $x_5$ – hardness of the impactor, $H$, HB | 150 | 450 | 600 | 300 |

As we can see, the chosen planning matrix satisfies the general properties of the planning matrix, which allows us to quickly calculate the objective function:

symmetry with respect to the zero level, it means that the algebraic sum of the elements of the column of each factor, is equal to zero;

the sum of the squares of the column elements of each of the factors is equal to the number of experiments (property of normalization).

The product of any two different vector columns of factors is zero.

Table 2

Planning matrix

| Experiment number | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $y_z$ |
|---|---|---|---|---|---|---|---|
| 1 | + | – | – | – | – | + | 54,6 |
| 2 | + | + | – | – | – | – | 60,2 |
| 3 | + | – | + | – | – | – | 53,3 |
| 4 | + | + | + | – | – | + | 55,4 |
| 5 | + | – | – | + | – | – | 56,7 |
| 6 | + | + | – | + | – | + | 58,9 |
| 7 | + | – | + | + | – | + | 59,4 |
| 8 | + | + | + | + | – | – | 50,3 |
| 9 | + | – | – | – | + | – | 51,4 |
| 10 | + | + | – | – | + | + | 52,7 |
| 11 | + | – | + | – | + | + | 58,9 |
| 12 | + | + | + | – | + | – | 57,2 |
| 13 | + | – | – | + | + | + | 56,4 |
| 14 | + | + | – | + | + | – | 56,8 |
| 15 | + | – | + | + | + | – | 55,7 |
| 16 | + | + | + | + | + | + | 57,4 |
| 17 | + | +2 | 0 | 0 | 0 | 0 | 58,5 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 18 | + | -2 | 0 | 0 | 0 | 0 | 54,1 |
| 19 | + | 0 | +2 | 0 | 0 | 0 | 58,3 |
| 20 | + | 0 | -2 | 0 | 0 | 0 | 53,1 |
| 21 | + | 0 | 0 | +2 | 0 | 0 | 58,8 |
| 22 | + | 0 | 0 | -2 | 0 | 0 | 53,9 |
| 23 | + | 0 | 0 | 0 | +2 | 0 | 59,7 |
| 24 | + | 0 | 0 | 0 | -2 | 0 | 53,2 |
| 25 | + | 0 | 0 | 0 | 0 | +2 | 56,2 |
| 26 | + | 0 | 0 | 0 | 0 | -2 | 53,6 |
| 27 | + | 0 | 0 | 0 | 0 | 0 | 54,3 |
| 28 | + | 0 | 0 | 0 | 0 | 0 | 55,8 |
| 29 | + | 0 | 0 | 0 | 0 | 0 | 53,6 |
| 30 | + | 0 | 0 | 0 | 0 | 0 | 55,2 |
| 31 | + | 0 | 0 | 0 | 0 | 0 | 56,1 |
| 32 | + | 0 | 0 | 0 | 0 | 0 | 54,7 |

The variances of the predicted values of the optimization parameter are the same at equal distances from the zero level (the rotatability property of the planning matrix).

The coefficients of the given equation (2), with the number of factors $k = 5$ and the kernels of the plan represented by $2^{5-1}$ $(1 = x_1x_2x_3x_4x_5)$, are determined using formulas of the form:

$$b_0 = \frac{A}{N}\left[2\lambda^2(k+2)\sum_{j=1`}^{N} y_j - 2\lambda c \sum_{i=1}^{k}\sum_{j=1}^{N} x_{ij}^2 y_j\right], \quad (3)$$

$$b_i = \frac{c}{N}\sum_{j=1}^{N} x_{ij} y_j, \quad (4)$$

$$b_{il} = \frac{c^2}{N\lambda}\sum_{j=1}^{N} x_{ij} x_{lj} y_j, \quad (5)$$

$$b_{ii} = \frac{A}{N}\left\{c^2\left[(k+2)\lambda - k\right]\sum_{j=1}^{N} x_{ij}^2 y_j + c^2(1-\lambda)\sum_{i=1}^{k}\sum_{j=1}^{N} x_{ij}^2 y_j - 2\lambda c\sum_{j=1}^{N} y_j\right\}. \quad (6)$$

To find the regression equation coefficients in the first stage of experimental data processing, we find the sums of the equation (3 – 6) [20-21]

$$\sum_{j=1}^{32} y_i = 1784; \quad \sum_{j=1}^{32} x_{1j} y_j = 11{,}3; \quad \sum_{j=1}^{32} x_{2j} y_j = 10{,}3; \quad \sum_{j=1}^{32} x_{3j} y_j = 17{,}7;$$

$$\sum_{j=1}^{32} x_{4j} y_j = 10{,}7; \quad \sum_{j=1}^{32} x_{5j} y_j = 17{,}3; \quad \sum_{j=1}^{32} x_{1j} x_{2j} y_j = -16{,}5; \quad \sum_{j=1}^{32} x_{1j} x_{3j} y_j = -12{,}1;$$

$$\sum_{j=1}^{32} x_{1j} x_{4j} y_j = 0{,}9; \quad \sum_{j=1}^{32} x_{1j} x_{5j} y_j = -12{,}3; \quad \sum_{j=1}^{32} x_{2j} x_{3j} y_j = -11{,}9;$$

$$\sum_{j=1}^{32} x_{2j}x_{4j}y_j = 23{,}9\,;\; \sum_{j=1}^{32} x_{2j}x_{5j}y_j = 17{,}1;\; \sum_{j=1}^{32} x_{3j}x_{4j}y_j = 4{,}3\,;$$

$$\sum_{j=1}^{32} x_{3j}x_{5j}y_j = 13{,}1\,;\; \sum_{j=1}^{32} x_{4j}x_{5j}y_j = \text{-}3{,}5\,;\; \sum_{j=1}^{32} x_{1j}^2 y_i = 1346\,;\; \sum_{j=1}^{32} x_{2j}^2 y_j = 1341\,;$$

$$\sum_{j=1}^{32} x_{3j}^2 y_j = 1346\,;\; \sum_{j=1}^{32} x_{4j}^2 y_j = 1347\;\; \sum_{j=1}^{32} x_{5j}^2 y_j = 1335\,;\; \sum_{i=1}^{5}\sum_{j=1}^{32} x_{ij}^2 y_j = 6714\,.$$

The next step is to determine the value

$$A = \frac{1}{2\lambda\left[(k+2)\lambda - k\right]} = 0{,}4929,$$

$$c = \frac{N}{\sum\limits_{j=1}^{N} x_{ij}^2} = 1{,}333,$$

$$\lambda = \frac{k(n_c + n_0)}{(k+2)n_c} = 0{,}879120879.$$

After some calculations, equations (3 – 6) will take the form

$$b_0 = 10{,}82\sum_{j=1}^{32} y_j - 2{,}344\sum_{i-1}^{5}\sum_{j=1}^{32} x_{ij}^2 y_i, \tag{7}$$

$$b_i = 0{,}042\sum_{j=1}^{32} x_{ij}y_j, \tag{8}$$

$$b_{il} = 0{,}063194444\sum_{j=1}^{32} x_{ij}x_{lj}y_j, \tag{9}$$

$$b_{ii} = 0{,}0316\sum_{j=1}^{32} x_{ij}^2 y_j + 0{,}0033\sum_{i=1}^{5}\sum_{j=1}^{32} x_{ij}^2 y_j - 0{,}0361\sum_{\text{j}=1}^{32} y_j. \tag{10}$$

Substituting the sums obtained into formulas (7–10), we find the values of the regression equation coefficients for the pendulum deflection impulse.

Based on the coefficients obtained, the equation (2) will take the form

$$y_z = 54{,}946389 + 0{,}4708x_1 + 0{,}4292x_2 + 0{,}7375x_3 + 0{,}4458x_4 + 0{,}7208x_5 -$$
$$-1{,}04271x_1x_2 - 0{,}76465x_1x_3 + 0{,}05688x_1x_4 - 0{,}77729x_1x_5 - 0{,}75201x_2x_3 +$$
$$+1{,}51035x_2x_4 + 1{,}08063x_2x_5 + 0{,}27174x_3x_4 + 0{,}82785x_3x_5 - 0{,}22118x_4x_5 +$$
$$+0{,}30863\text{x}_1^2 + 0{,}156963\text{x}_2^2 + 0{,}321269\text{x}_3^2 + 0{,}346546\text{x}_4^2 - 0{,}04526x_5^2.$$

The dispersion of the coefficients of the regression equation can be found using the formula

$$s^2\{b_0\} = \frac{2A\lambda^2(k+2)}{N}s_y^2, \tag{11}$$

$$s^2\{b_i\}=\frac{c}{N}s_y^2, \tag{12}$$

$$s^2\{b_{il}\}=\frac{c^2}{\lambda N}s_y^2, \tag{13}$$

$$s^2\{b_{ii}\}=\frac{Ac^2\left[(k+1)\lambda-(k-1)\right]}{N}s_y^2. \tag{14}$$

Accordingly, the dispersion of the coefficients will be equal

$$s^2\{b_0\}=0,1667s_y^2, \tag{15}$$

$$s^2\{b_i\}=0,0417s_y^2, \tag{16}$$

$$s^2\{b_{il}\}=0,0632s_y^2, \tag{17}$$

$$s^2\{b_{ii}\}=0,0349s_y^2. \tag{18}$$

The dispersion $s^2\{y_z\}$ of the optimization parameter is determined by the results of experiments in the center of the plan (table 3):

Table 3

Auxiliary table for calculating the dispersion $s^2\{y_z\}$

| The number of the study | $y_{zu}$ | $\overline{y}_z$ | $y_{zu}-\overline{y}_z$ | $(y_{zu}-\overline{y}_z)^2$ |
|---|---|---|---|---|
| 27 | 54,3 | 54,95 | -0,65 | 0,4225 |
| 28 | 55,8 | | 0,85 | 0,7225 |
| 29 | 53,6 | | -1,35 | 1,8225 |
| 30 | 55,2 | | 0,25 | 0,0625 |
| 31 | 56,1 | | 1,15 | 1,3225 |
| 32 | 54,7 | | -0,25 | 0,0625 |
| $\sum_{u=1}^{6} y_{zu}=329,7$ | | | | $S_E=\sum_{z=1}^{6}(y_{zu}-\overline{y}_z)^2=4,415$ |

$$s^2\{y_z\}=\frac{\sum_{u=1}^{6}(y_{zu}-\overline{y}_z)^2}{n_0-1}=0,883,$$

where $n_0=6$ the number of the studies in the center of plan.

The dispersion of the coefficients of the regression equation $y_z$ are determined using the formulas (15–18):

$$s^2\{b_0\}=0,00096278;\ s^2\{b_i\}=0,00024069;\ s^2\{b_{il}\}=0,00036505;$$

$$s^2\{b_{ii}\}=0,00020165.$$

Mean square errors in the determination of the regression coefficients for $y_z$ respectively equal

$$s\{b_0\}=0{,}3836231;\ s\{b_i\}=0{,}1918115;\ s\{b_{il}\}=0{,}2362217;$$

$$s\{b_{ii}\}=0{,}1755655.$$

We define the confidence intervals for the coefficients:

$$\Delta b_0=\pm ts\{b_0\}=\pm 2{,}57\times 0{,}3836231=\pm 0{,}985911313;$$

$$\Delta b_i=\pm ts\{b_i\}=\pm 0{,}492955656;$$

$$\Delta b_{i1}=\pm ts\{b_{il}\}=\pm 0{,}607089785;\ \Delta b_{ii}=\pm ts\{b_{ii}\}=\pm 0{,}451203305.$$

Coefficients that, according to the absolute value of less than the corresponding confidence intervals, can be considered statistically insignificant and excluded from the regression equation [22]. The result of rotatable planning of the regression equation is written in the form

$$y_z=54{,}946389+0{,}4708x_1+0{,}4292x_2+0{,}7375x_3+0{,}4458x_4+0{,}7208x_5-$$
$$-1{,}04271x_1x_2-0{,}76465x_1x_3+0{,}05688x_1x_4-0{,}77729x_1x_5-0{,}75201x_2x_3+$$
$$+1{,}51035x_2x_4+1{,}08063x_2x_5+0{,}27174x_3x_4+0{,}82785x_3x_5-0{,}22118x_4x_5+$$
$$+0{,}30863\mathrm{x}_1^2+0{,}156963\mathrm{x}_2^2+0{,}321269\mathrm{x}_3^2+0{,}346546\mathrm{x}_4^2-0{,}04526x_5^2.$$

To determine $s_{\text{ad}}^2$ we should calculate the sum $s_R$ f squares of deviations of the calculated $y_{zj}$ values of the response function from experimental ones $y_{zj}$ at all points in the plan (table 4).

The number of degrees of freedom is determined by the formula

$$f=N-k'-(n_0-1)=9,$$

where $k'$ the number of statistically significant coefficients of the model; $N$ – the total number of experiments; $n_0$ – the total number of experiments at the center of the plan.

The dispersion of adequacy is determined by equation

$$s_{\text{ad}}^2=\frac{s_R-s_E}{f}=3{,}745723$$

The adequacy of the obtained model is verified by $F$ – criterion:

$$F_p=\frac{s_{\text{ad}}^2}{s_y^2}=4{,}242041899,$$

where $s_{\text{ad}}^2$ – the dispersion of adequacy; $s_y^2$ – the dispersion of the optimization parameter.

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104

Table 4

Auxiliary table for calculating $s_R$

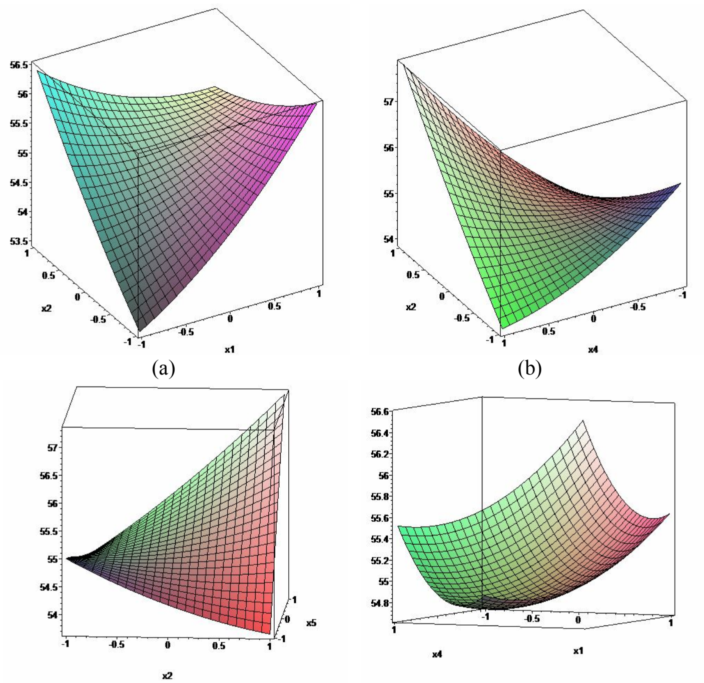
| Experiment number | $y_{zj}$ | $y_{zj}$ | $y_{zj} - y_{zj}$ | $(y_{zj} - y_{zj})^2$ |
|---|---|---|---|---|
| 1 | 0,33 | 0,28 | 0,05 | 0,0025 |
| 2 | 0,47 | 0,45 | 0,02 | 0,0004 |
| 3 | 0,52 | 0,55 | -0,03 | 0,0009 |
| 4 | 0,34 | 0,33 | 0,01 | 0,0001 |
| 5 | 0,71 | 0,65 | 0,06 | 0,0036 |
| 6 | 0,35 | 0,35 | 0,00 | 0,0000 |
| 7 | 0,77 | 0,63 | 0,14 | 0,0196 |
| 8 | 1,64 | 1,79 | -0,15 | 0,0225 |
| 9 | 0,50 | 0,42 | 0,08 | 0,0064 |
| 10 | 0,29 | 0,28 | 0,01 | 0,0001 |
| 11 | 0,51 | 0,36 | 0,17 | 0,0289 |
| 12 | 0,20 | 0,35 | -0,15 | 0,0225 |
| 13 | 0,31 | 0,26 | 0,05 | 0,0025 |
| 14 | 0,74 | 0,72 | 0,02 | 0,0004 |
| 15 | 0,81 | 0,83 | -0,02 | 0,0004 |
| 16 | 0,42 | 0,41 | 0,01 | 0,0001 |
| 17 | 0,54 | 0,44 | 0,10 | 0,0100 |
| 18 | 0,54 | 0,69 | -0,15 | 0,0225 |
| 19 | 0,65 | 0,60 | 0,05 | 0,0025 |
| 20 | 0,28 | 0,38 | -0,10 | 0,0100 |
| 21 | 0,95 | 0,96 | -0,01 | 0,0001 |
| 22 | 0,31 | 0,35 | -0,04 | 0,0016 |
| 23 | 0,23 | 0,27 | -0,04 | 0,0016 |
| 24 | 0,57 | 0,58 | -0,01 | 0,0001 |
| 25 | 0,15 | 0,33 | -0,18 | 0,0324 |
| 26 | 0,72 | 0,59 | 0,13 | 0,0169 |
| 27 | 0,25 | 0,19 | 0,06 | 0,0036 |
| 28 | 0,25 | 0,18 | 0,07 | 0,0049 |
| 29 | 0,25 | 0,33 | -0,08 | 0,0064 |
| 30 | 0,25 | 0,27 | -0,02 | 0,0004 |
| 31 | 0,25 | 0,19 | 0,06 | 0,0036 |
| 32 | 0,25 | 0,35 | -0,10 | 0,0100 |
| | | | | $s_R = \sum_{j=1}^{32} (y_{zj} - y_{zj})^2 = 0,2342$ |

Thus, the mathematical model in which $s_y^2$ is accepted as $s^2\{y_z\}$=0.883, the value of the criterion $F$ = 4,242. Tabular value of $F_T$ – criterion at 5% significance level, in particular degrees of freedom for the numerator 9 and for the denominator 5 is equal to 4,85. The value $F_p < F_T$, therefore, the resulting model can be considered adequate.

The transition from coded ($x_1, x_2, x_3, x_4, x_5$) to natural ($v_f, \gamma, h_f, h_r, H$) values of the factors is carried out in accordance with the experimental conditions (table 1) by the formulas

$$x_1 = \frac{v_f\text{-}800}{100};\ x_2 = \frac{\gamma\text{-}45}{15};\ x_3 = \frac{h_f\text{-}10}{5};\ x_4 = \frac{h_r\text{-}10}{5};\ x_5 = \frac{H\text{-}450}{150}.$$

Thus, using the mathematical model obtained from the experimental data, we can construct the following dependencies.




(a) (b)

(c) (d)

Fig. 5. Dependence of the pendulum deflection pulse (a) from the speed of the impactor and the angle of the protective screen (b) from the tilt angle of the protective screen and the thickness of the back layer (c) from the tilt angle of the protective screen and the hardness of the impactor (d) from the thickness of the back layer and the speed of the impactor

## Conclusions

Analyzing the pattern of impact of the means of lesion on the protective screen revealed by the multivariate experiment, we can conclude that the parameters of the means of the lesion in one way or another affect on the protective screen. That is, the means of lesion can break through this protective screen, but during its penetration by the impactor is lost the amount of energy that is not enough in the future to break through the main armoured obstacle.

Thus, using of a protective screen will protect the CAV from breaking through the main armoured obstacle. It should also be noted that as the thickness of the protective screen increases, the weight of the CAV will increase, which will result in loss of buoyancy and decrease in other CAV characteristics..

In the future, it is necessary to carry out an experimental study taking into account the sixth factor - the rigidity of the elastic element. Based on theoretical and experimental data on the impact of the means of lesion on the armoured obstacle, it is necessary to develop recommendations on the choice of a rational method of protection of CAV.

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*Стаття надійшла 21.01.2020*

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**ЕКСПЕРИМЕНТАЛЬНЕ ДОСЛІДЖЕННЯ ВПЛИВУ ЗАСОБІВ УРАЖЕННЯ НА ЗАХИСНІ ЕКРАНИ БОЙОВИХ БРОНЬОВАНИХ МАШИН**

Для оцінки відносних властивостей захисного екрана до уражаючої енергії засобів ураження було проведено експериментальне дослідження, яке дозволило перевірити гіпотезу щодо захисту бойових броньованих машин від засобів ураження, на які діє кінетична енергія, засобів ураження. На підставі даних, отриманих під час експериментального дослідження, була побудована математична модель, яка описує імпульс удару ударника у захисний екран. Використання цієї моделі дає можливість розрахувати втрати енергії викликаної деформацією та руйнуванням перешкоди. Побудований в результаті багатофакторного експерименту математична модель описує вплив засобів ураження на захисний екран із врахуванням швидкості засобів ураження, кута нахилу захисного екрану, товщини переднього і заднього шару і твердості засобів ураження.

**Ключові слова:** бойові броньовані машини, захисні екрани, експериментальне дослідження, математична модель.

УДК 355.41
*Дачковський В.О., Даценко І.П., Коцюруба В.І., Яльницький О.Д., Голда О.Л., Неділько О.М., Сиротенко А.М.* **Експериментальне дослідження впливу засобів ураження на захисні екрани бойових броньованих машин** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 117-135.

*Для оцінки відносних властивостей захисного екрана до уражаючої енергії засобів ураження було проведено експериментальне дослідження, яке дозволило перевірити гіпотезу щодо захисту бойових броньованих машин від засобів ураження, на які діє кінетична енергія, засобів ураження. На підставі даних, отриманих під час експериментального дослідження, була побудована математична модель, яка описує імпульс удару ударника у захисний екран. Використання цієї моделі дає можливість розрахувати втрати енергії викликаної деформацією та руйнуванням перешкоди. Побудований в результаті багатофакторного експерименту математична модель описує вплив засобів ураження на захисний екран із врахуванням швидкості засобів ураження, кута нахилу захисного екрану, товщини переднього і заднього шару і твердості засобів ураження.*

Іл. 5. Бібліог. 22 назв.

UDC *355.41*
*Dachkovskyi V.O., Datsenko I.P., Kotsiuruba V.I., Yalnytskyi O.D., Holda O.L., Nedilko O.M., Syrotenko A.M.* **Experimental investigation of impact of injury measures on the protection screens of combat armoured vehicles** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2020. – Issue 104. – P. 117-135.

*In order to evaluate the relative properties of the protective screen to the striking energy of the means of defeat , an experimental study was conducted, which allowed to test the hypothesis regarding the protection of the combat armoured vehicles against the means of defeat which, under the action of kinetic energy, destroy the armoured obstacle.. Based on the data obtained during the experimental study, a mathematical model was constructed that describes the punching momentum of the protective screen.. The use of this model makes it possible to calculate the energy losses caused by deformation and destruction of the obstacle. Built as a result of the multifactor experiment, the regularity of the impact of the means of defeat on the protective screen takes into account the speed of the means of defeat, the angle of encounter of the means of defeat with the protective screen, the thickness of the front and back layer and the hardness of the means of defeat.*
Fig. 5. Ref. 22

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UDC 539.3

# BUCKLING AND VIBRATIONS OF THE SHELL WITH THE HOLE UNDER THE ACTION OF THERMOMECHANICAL LOADS

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DOI: 10.32347/2410-2547.2020.104.136-146

The paper outlines the fundamentals of the method of solving static problems of geometrically nonlinear deformation, buckling, and vibrations of thin thermoelastic inhomogeneous shells with complex-shaped midsurface, geometrical features throughout the thickness, under complex thermomechanical loading. The technique is based on the geometrically nonlinear equations of three-dimensional thermoelasticity, the finite element formulation of the problem in increments, and the use of the moment finite-element scheme. A thin shell is considered by this method as a three-dimensional body. We approximate a shell by one spatial universal finite element (FE) throughout the thickness. The universal FE is based on an isoparametric spatial FE with polylinear shape functions for coordinates and displacements. The universal element has additional variable parameters introduced to expand its capabilities. The method of modal analysis of the shell is based on an approach that at each current stage of thermomechanical loading takes into account the stresses accumulated at the previous stages. The developed algorithm allows one to study geometric nonlinear deformation and buckling of elastic shells of an inhomogeneous structure with a thin and medium thickness, as well as to study small vibrations of the shells relative to the reference deformed state caused by static loading, taking into account large displacements and a prestressed state. An analysis of the stability and vibration of the spherical panel with the hole is carried out. The effect on the frequencies and mode shapes of the shell of the sequential action of thermal and mechanical loads is investigated.
**Keywords:** elastic shell, hole, buckling, natural frequency, mode shapes, thermo-mechanical load, universal finite element.

## Introduction

Shells as elements of thin-walled structures are widely used in various engineering applications such as construction, engineering, shipbuilding, aviation and space technology, transport and other branches.

The shells can be weakened by holes, channels, cavities, and dents in accordance with technological necessity. During operation such structures can be subjected to loads of various nature including mechanical and thermal. At the same time static loads significantly affect both the stress-strain state of the structure and the dynamic characteristics which include the frequencies and modes of natural vibrations.

Obtaining information about the natural frequencies and modes of the shell is one of the important aspects of the complex analysis of the thin-walled structure. This modal information plays a key role in the design of these structures and can provide the strength of the elastic system even at the design stage.

There are a large number of theoretical, numerical and experimental studies of shells of various shapes. Background and bibliography can be found in Ref. [1–3, 9-15]. Although the basic equations and relations of the theory of shells were obtained long ago, until now analytical solutions to problems have been obtained only for some relatively simple classes of shells with predominantly canonical form. Therefore methods of numerical analysis are widely used to solve the problems of shell theory. Currently, there is a fairly large arsenal of these methods. On their basis effective approaches have been developed to solve a wide class of problems on the stress-strain state, stability and vibration of thin plates and shells. A large number of monographs are devoted to the presentation of these approaches [1-3, 7, 10, 12, 14-21]. In general a broad bibliographic description is devoted to various aspects of shell researches [22].This description has been compiled by David Bushnell since 2011 and is currently being updated. On this website page there are people who have made a significant contribution directly to the field of stability loss, as well as people who have laid the foundations of the theory and methods of researching various aspects of analysis for shell structures. The authors as researchers involved in the study of geometrically nonlinear deformation, stability, buckling, and oscillations of thin elastic shells [1-3] are also included in Shell_Buckling_People.

In recent decades the number of articles on the analysis of elastic shells has expanded significantly. Among them much attention is paid to analyzing elastic thin shells reinforced by ribs [1-3, 5,6,7,11, 23-25]. Much less research has been devoted to investigating shells with various weakening [1-8].

The article is a continuation of studies of deformation, buckling, and vibrations of shell structures. Research is devoted to modal analysis of a thin shell with a hole.

**1 Problem statement and research method**

The methodology for studying the natural vibrations of thin-walled shell structures, taking into account the effects of static thermo-mechanical loading, is based on an integrated approach. The finite element method [1-2] for investigating static problems of the stress–strain state, buckling, and postbuckling behavior of thin inhomogeneous shells, and the method [3, 26] for modal analysis of shells taking into account the pre-stressed state at each step of the thermo-mechanical load are used. Thus, the problem of determining the natural frequencies and vibration modes of the shell is solved by the incremental method in two stages.

At the first stage, the static problem of nonlinear deformation of inhomogeneous shells is solved by the method given in Ref. [1-2]. At this stage for the corresponding increments of the static load the parameters of the stress-strain state for the finite-element shell model (FESM) are determined. These parameters include: deformed shape (new coordinates of the nodes and increment of displacements for them), the stresses in the finite elements (FE), and others. This problem is solved for each increment of thermo-mechanical load.

The method is based on the geometrically nonlinear equations of three-dimensional thermoelasticity, the finite element formulation of the problem in increments, and the use of the moment finite-element scheme (MFES).

To develop the FESM, we approximate a thin shell by one spatial FE throughout the thickness. The universal FE is based on an isoparametric spatial FE with polylinear shape functions for coordinates and displacements. Additional variable parameters have been introduced to enhance the capabilities of this FE [1-2]. The nonlinear deformation of shells is analyzed using the incremental method based on the general Lagrangian formulation. The problem of nonlinear deformation, buckling, and post-buckling behavior of inhomogeneous shells is solved by a combined algorithm. The algorithm employs the parameter continuation method, and a modified Newton–Kantorovich method at the step of the load's increment [1-2].

At the second stage of the current step, the thermo-mechanical load is assumed to be zero (i.e., "deleted") and the parameters of natural vibrations are determined [3, 26]. At this stage we use the new shell shape and the pre-stressed state which has been determined at the first stage. For each load increment the natural frequencies and mode shapes are computed until a negative value of the fundamental tone (lowest frequency) appears. This is because of according to the dynamic criterion, the moment of the loading at which a negative value of the frequency appears may be taken as the moment of loss of stability of the shell and this load is adopted as critical [3, 26, 27].

The determination of the natural frequencies of the shell is not performed at the next steps of the thermo-mechanical load increment. Next, only the post-buckling behavior of the shell is investigated. The accuracy of the calculation for the natural vibrations of the shell taking into account the pre-stressed state is confirmed by the coincidence of the value of the upper critical load with that obtained in another way.

This approach allows us to analyze the joint effect of thermo-mechanical load parameters and the geometric characteristics of the shells on the buckling and natural vibrations of shell structures.

## 2 Buckling and natural vibrations of a panel with a hole

A shallow spherical panel of square planform hinged at the edges and having a central square hole is considered (Fig. 1). The shell is under the action of thermo-mechanical loading.

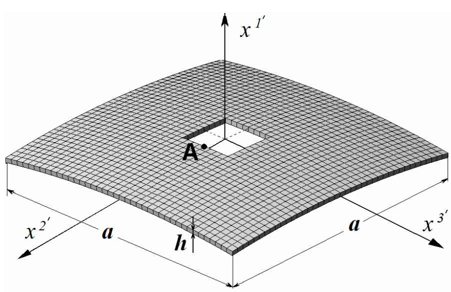


Fig. 1. A shallow spherical panel with a central hole

Curvature of the panel is defined by the parameter $K=2a^2/(Rh)=32$, where: $h=1$ cm is the thickness, $a=60h$ is a size of the panel in the plan, $R=225h$ is the radius of mid-surface. The input data: width of the hole $b_o = 12h$, elastic modulus $E = 2.1 \cdot 10^6$ kg/cm$^2$, Poisson's ratio $\nu = 0.3$, linear expansion coefficient $\alpha = 0.12 \cdot 10^{-4}$ deg$^{-1}$, $\rho = 7.85 \cdot 10^{-3}$ kg/cm$^3$. The data are taken from Ref. [7, part. 2], where the problem of panel stability under the action of pressure alone is considered.

The effect of the thermo-mechanical load on the panel consists of two stages:

(i) the shell is gradually heated by the temperature field whose parameter increases from $0°$C to a set value $T°$C. So, at the first stage the stress–strain state of the shell is perturbed by the temperature field;

(ii) the panel is subjected to uniform normal pressure of intensity $q$ in addition. So, at the second stage the temperature field is remaining constant.

We consider three options for preheating at $T = -20°, 0°, 20°$C. Results are presented in terms of dimensionless parameters: $\overline{q} = a^4 q/(Eh^4)$, $\overline{u}^{1'} = u^{1'}/h$, where $u^{1'}$ is the deflection of the panel along the axis $x^{1'}$.

The results of investigations of the processes of geometrically nonlinear deformation and buckling of a smooth panel and a panel with a hole are details presented in in Ref. [1-2, 28].

Examination of the dependence of the frequencies and modes of natural vibrations of a smooth panel on the mechanical load is given in [4, 26]. It is shown that neglecting the prestressed state (only the new deformed state of the shell was taken into account) leads to an incorrect determination of the upper critical load and frequencies.

The calculating results of a smooth panel are basic for analyzing the effect of geometric features such as holes on the natural vibrations of a shallow shell. There is the dashed line with the mark "▬" for the solution of the smooth panel on the "load – deflection" ("$\overline{q} - \overline{u}$") and "load – frequency" ("$\overline{q} - \omega$") curves. The calculation results for a panel with a hole are marked "■∷■". For the panel without hole the deflection have been considered at its center, and for the panel with a hole the deflection have been considered at the point $A$ (Fig. 1). The design model is the panel with mesh $40 \times 40$ FEs.

The accuracy of calculations in the problems of buckling of the indicated panels had been determined by a comparative analysis of the solutions obtained using the MFES and calculations performed using the software LIRA [29] (Fig. 2, Fig. 3).

A comparison of the "$\overline{q} - \overline{u}$" curves obtained by the MSFE and software LIRA for shells without hole (▬) and with hole (■∷■) when their loading only pressure ($T = 0°$C) reveals agreement between the "$\overline{q} - \overline{u}$" curves in the prebuckling domain and when loss of stability. The difference between the values of $\overline{q}_{\text{cr}}^{\text{up}}$ is respectively -1.9% and 2.9% (Fig. 2).

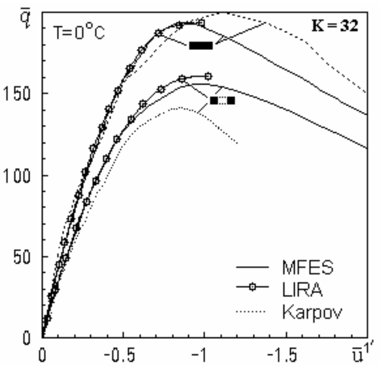



Fig. 2

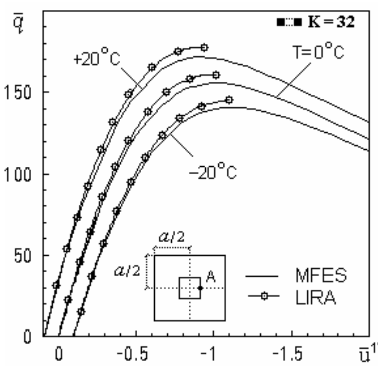
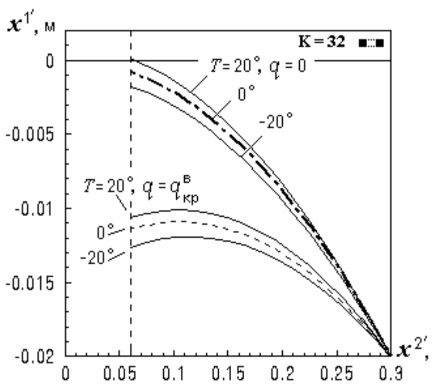
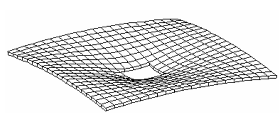



Fig. 3

For all heating cases, both solutions are in good agreement with each other throughout the "$\overline{q}-\overline{u}$" curve (Fig. 3). The disagreement between the values of $\overline{q}_{\text{cr}}^{\text{up}}$ is an area $3.0-3.5\,\%$. Configurations for the deformed shell after pre-cooling to $T=-20^{\circ}\text{C}$ and preheating to $T=+20^{\circ}\text{C}$ obtained by both methods, are in complete agreement with each other and have little difference from the original form ($T=0^{\circ}\text{C}$, $\overline{q}=0$). Forms of buckling are in good agreement too (Fig. 4 (a)). Buckling of the panel occurs with click of its central part (Fig. 4 (b)).




(a)



(b)

Fig. 4

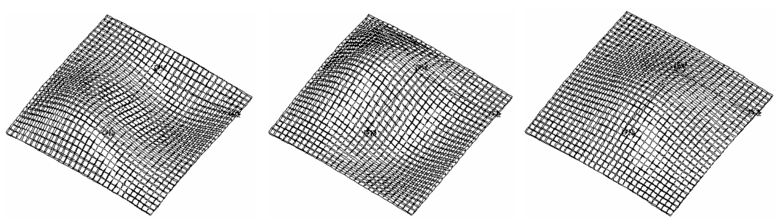
We have found that that weakening of the smooth panel ("▬") by the central hole ("■::■") reduces the critical load $\overline{q}_{\text{cr}}^{\text{up}}$ by 19.2% (Fig. 2). For the shell with the hole ("■::■"), pre-cooling and preheating leads to a change in the critical load $\overline{q}_{\text{cr}}^{\text{up}}$ by 9.78 and -9.97% compared to the corresponding unheated panel ($T=0^{\circ}\text{C}$) (Fig. 3).

An analysis of the natural vibrations of the smooth ("▬") and weakened ("■::■") panel shows that for unloaded shells ($T=0^{\circ}\text{C}$, $\overline{q}=0$) the presence of a hole reduces the frequency $\omega_1$ by 3.3% (Table 1). At the same time, frequencies $\omega_1$ and $\omega_2$ are double for a shell without a hole, and frequencies $\omega_2$ and $\omega_3$ are double for a panel with a hole. Therefore, the mode shapes differ for the respective shells. For the smooth panel, the mode shapes that correspond to double frequencies $\omega_1$ and $\omega_2$ are conjugate, and the mode that corresponds to the frequency $\omega_3$ is characterized by the oscillation of the central part of the shell (Fig. 5). The opposite nature of the mode shapes is observed for a panel with a hole (Fig. 6). Modes transform in accordance with the change in the number of double frequencies during loading (Table 1). At buckling domain, the vibration modes have the same shape for the shell without a hole (Fig. 5 (c)) and with it (Fig. 6 (a)).

Table 1

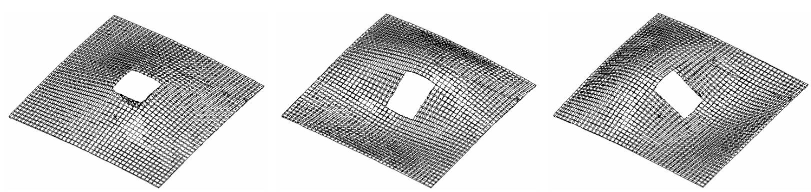
Panel natural frequencies $\omega_i$ at various load values $\overline{q}^i$ ($T = 0°\text{C}$)

| № $\overline{q}^i$ | $\omega_1$, $Hz$ | $\omega_2$, $Hz$ | $\omega_3$, $Hz$ | $\omega_4$, $Hz$ | $\omega_5$, $Hz$ | $\omega_6$, $Hz$ |
|---|---|---|---|---|---|---|
| 0 | .53378e+3 | .53378e+3 | .54740e+3 | .69124e+3 | .79609e+3 | .81664e+3 |
| 0 | .51604e+3 | .51938e+3 | .51938e+3 | .60926e+3 | .71434e+3 | .81960e+3 |
| 1 | .51213e+3 | .51251e+3 | .51251e+3 | .59917e+3 | .70329e+3 | .81049e+3 |
| 2 | .49805e+3 | .49805e+3 | .50057e+3 | .58103e+3 | .68530e+3 | .79348e+3 |
| 3 | .47453e+3 | .47453e+3 | .48176e+3 | .55137e+3 | .65603e+3 | .76620e+3 |
| 4 | .43368e+3 | .43368e+3 | .44905e+3 | .49943e+3 | .60537e+3 | .72019e+3 |
| 5 | .34814e+3 | .34814e+3 | .37947e+3 | .38773e+3 | .49978e+3 | .63141e+3 |
| 6 | .27518e+3 | .27518e+3 | .29254e+3 | .33249e+3 | .45578e+3 | .58481e+3 |
| 7 | .24566e+3 | .24566e+3 | .25091e+3 | .31914e+3 | .44342e+3 | .57459e+3 |
| 8 | .18104e+3 | .19133e+3 | .19133e+3 | .28532e+3 | .41071e+3 | .55581e+3 |
| 9 | .14446e+3 | .16503e+3 | .16503e+3 | .27148e+3 | .39684e+3 | .54946e+3 |
| 10 | .85034e+2 | .13249e+3 | .13251e+3 | .26427e+3 | .38724e+3 | .54844e+3 |
| 11 | -.31833e+5 | .93008e+2 | .93082e+2 | .26060e+3 | .37989e+3 | .55059e+3 |



(a) (b) (c)

Fig. 5



(a) (b) (c)

Fig. 6

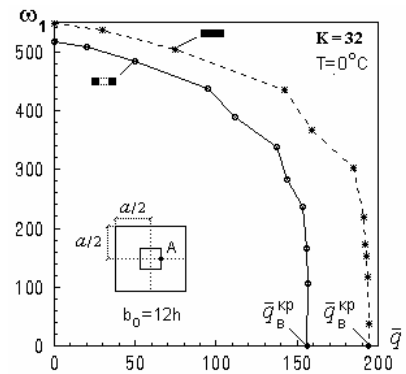



Fig. 7

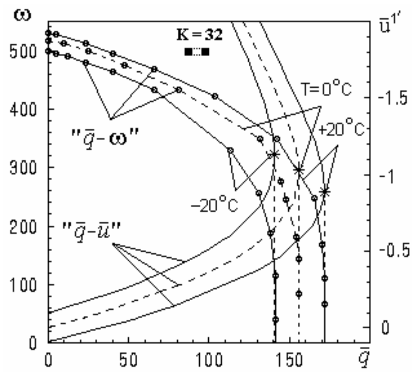



Fig. 8

The "$\overline{q}-\omega_1$" curves have the same fashion for the investigated shells when only pressure is acting ($T = 0°$ C) (Fig. 7). The load moments at which the natural vibrations have been calculated are shown in the figure. These moments correspond to the load $\overline{q}^i$ ($i = \overline{1, 11}$) (Table 1). The applied pressure causes a restructuring of both frequency multiplicity and vibration modes. The frequencies $\omega_2$ and $\omega_3$ become double when $i \geq 8$. Accordingly, the mode shape for the frequency $\omega_1$ becomes the simplest (Fig. 6 (a)).

Preheating and pre-cooling of the shell leads to small changes in the frequencies (Table 2). In the case of pre-cooling by $T = -20°$ C, the mode shapes are similar to ones shown in Fig. 6. In the case of preheating by $T = +20°$ C, the mode shapes are similar to the (b), (c), (a) shapes of Fig. 6.

The "$\overline{q}-\omega_1$" curves have the same fashion for the investigated shell in all cases of heating (Fig. 8). At buckling domain, the mode shape is similar to the shape from Fig. 6 (a) for all cases of heating.

Table 2

Natural frequencies for the heated panel ("■::■", $\overline{q}$ =0)

| $T$°C | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ |
|---|---|---|---|---|---|
| 0 | .51604e+3 | .51938e+3 | .51938e+3 | .60926e+3 | .71434e+3 |
| +20 | .53017e+3 | .53028e+3 | .53151e+3 | .61665e+3 | .71662e+3 |
| $\Delta_0^{+20}$ | +2.37 | +2.10 | +2.33 | +1.21 | +0.32 |
| -20 | .49977e+3 | .50829e+3 | .50838e+3 | .60211e+3 | .71205e+3 |
| $\Delta_0^{-20}$ | -3.15 | -2.13 | -2.12 | -1.17 | -0.32 |

**Conclusions**

The finite element approach for determining the natural vibrations of shells of an inhomogeneous structure is developed on the basis of a modified isoparametric spatial finite element with polylinear shape functions. The algorithm for modal analysis of the shells is based on the finite element method for studying shells with geometric features throughout thickness. The prestressed state of the deformed shell is taken into account at each stage of thermo-mechanical loading.

An analysis of the stability and vibration of the spherical panel with the hole is carried out. The effect on the frequencies and mode shapes of the shell of the sequential action of temperature and pressure is investigated. We have shown that the developed method is an effective tool for a comprehensive study of the stability and vibrations of inhomogeneous shells of thin and medium thickness under the action of thermo-mechanical loads.

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*Стаття надійшла 21.01.2020*

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**ВТРАТА СТІЙКОСТІ ТА КОЛИВАННЯ ОБОЛОНКИ З ОТВОРОМ ПІД ДІЄЮ ТЕРМОСИЛОВОГО НАВАНТАЖЕННЯ**

У статті викладені основи методу розв’язання статичних задач геометрично нелінійного деформування, втрати стійкості та коливань тонких термопружних неоднорідних оболонок зі складною формою середньої поверхні, з геометричними особливостями за товщиною, в умовах дії складного термомеханічного навантаження. Метод заснований на геометрично нелінійних співвідношеннях тривимірної термопружності, скінченно-елементному формулюванні задачі в приростах і використанні моментної схеми скінченних елементів. За цим методом тонка оболонка розглядається як тривимірне тіло, яке моделюється по товщині одним універсальним просторовим скінченним елементом (СЕ). Універсальний СЕ розроблений на основі ізопараметричного просторового СЕ з полілінійними функціями форми для координат і переміщень. Можливості модифікованого елемента розширені за рахунок введення додаткових змінних параметрів. Методика модального аналізу неоднорідних оболонок базується на підході, за яким на кожному кроці термосилового навантаження враховуються накопичені на попередніх кроках напруження. Розроблена методика дозволяє комплексно досліджувати геометрично нелінійне деформування та стійкість тонких і середньої товщини пружних оболонок неоднорідної структури та вивчати малі коливання оболонки відносно відлікового деформованого стану, що викликаний довільним статичним навантаженням, з урахуванням великих переміщень і переднапруженого стану. Виконано аналіз стійкості та коливань сферичної панелі з отвором. Досліджено вплив послідовної дії теплових і силових навантажень на частоти і форми коливань оболонки.

**Ключові слова:** пружна оболонка, отвір, втрата стійкості, власна частота, форма коливань, термосилове навантаження, універсальний скінченний елемент.

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**ПОТЕРЯ УСТОЙЧИВОСТИ И КОЛЕБАНИЯ ОБОЛОЧКИ С ОТВЕРСТИЕМ ПОД ДЕЙСТВИЕМ ТЕРМОСИЛОВОЙ НАГРУЗКИ**

В статье изложены основы метода решения статических задач геометрически нелинейного деформирования, потери устойчивости и колебаний тонких термоупругих неоднородных оболочек со сложной формой срединной поверхности, с геометрическими особенностями по толщине, в условиях действия сложной термомеханической нагрузки. Метод основан на геометрически нелинейных соотношениях трехмерной термоупругости, конечно-элементной формулировке задачи в приращениях и использовании моментной схемы конечных элементов. Тонкая переменной толщины оболочка сложной геометрической формы рассматривается согласно методу как трехмерное тело, которое моделируется по толщине одним универсальным пространственным конечным элементом (КЭ). Универсальный КЭ разработан на основе изопараметрического пространственного КЭ с полилинейными функциями формы для координат и перемещений. Возможности модифицированного элемента расширены за счет введения дополнительных переменных параметров. Методика модального анализа оболочки базируется на подходе, когда на каждом шаге термосилового нагружения учитываются напряжения, накопленные на предыдущих шагах. Разработанная методика позволяет комплексно исследовать геометрически нелинейное деформирование и устойчивость тонких и средней толщины упругих оболочек неоднородной структуры, а также изучать малые колебания оболочек относительно отсчетного деформированного состояния, вызванного произвольной статической нагрузкой, с учетом больших перемещений и преднапряженного состояния. Выполнен анализ устойчивости и колебаний сферической панели с отверстием. Исследовано влияние последовательного воздействия тепловых и силовых нагрузок на частоты и формы колебаний оболочки.

**Ключевые слова:** упругая оболочка, отверстие, потеря устойчивости, собственная частота, форма колебаний, термосиловая нагрузка, универсальный конечный элемент.

УДК 539.3
*Баженов В.А., Кривенко О.П.* **Втрата стійкості та коливання оболонки з отвором під дією термосилового навантаження** // Опір матеріалів і теорія споруд: наук.-тех. збірник – К.: КНУБА, 2020. – Вип. 104. – С. 136-146. – Англ.

*Досліджено вплив послідовної дії нагріву та тиску на стійкість і коливань сферичної панелі з отвором.*
Табл. 2. Ил. 8. Библиогр. 29 назв.

UDC 539.3
*Bazhenov V.A., Krivenko O.P.* **Buckling and vibrations of the shell with the hole under the action of thermomechanical loads** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 136-146.

*Effect of the sequential action of heating and pressure on the stability and vibration of a spherical panel with a hole is investigated.*
Tabl. 2. Fig. 8. Bibliograf. 29 ref.

*Баженов В.А., Кривенко О.П.* **Потеря устойчивости и колебания оболочки с отверстием под действием термосиловой нагрузки** // Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУБА, 2020. – Вып. 104. – С. 136-146. – Англ.

*Исследовано влияние последовательного воздействия нагрева и давления на устойчивость и колебаний сферической панели с отверстием.*
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# NONLINEAR OSCILLATIONS OF A PRESTRESSED CONCRETE BRIDGE BEAM SUBJECTED TO HARMONIC PERTURBATION IN THE CONDITIONS OF INDETERMINACY OF PARAMETERS

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DOI: 10.32347/2410-2547.2020.104.147-163

**Abstract.** This paper deals with the nonlinear oscillations of a prestressed reinforced concrete beam firmly attached to two supports. The beam is subjected to a harmonic force. The calculations of such beams are associated with a number of uncertainties in the initial data. This publication is devoted to questions of their correct accounting.

For a long period of time in mechanics, to tack into account some uncertainties, they have been using the probability theory for modeling and such theory dominates. It have been proven that the probability theory can solve a lot of problems but nevertheless it has some weaknesses. In particular, the lack of statistical information or incomplete information does not adequately reflect the real object of study in a mathematical model. Recently, many researchers have noted that the uncertainty in construction is not only stochastic in nature, and this provides an impetus for the introduction of new developing methods and theories of soft computing. Among them, theories of fuzzy and rough sets, the reliability of which has already been proven in solving control problems, etc. They are the most popular and effective theories now.

For the beam under consideration, the amplitude of beam oscillations is determined, provided that its parameters are indeterminate (fuzzy) and vary within certain limits. An example of determining the amplitude of the oscillation of the 33-meter-long prestressed beam designed by Soyuzdorproekt is studied. The membership function for the amplitude of the beam transverse oscillations using the theory of fuzzy numbers is constructed. The influence analysis of the fuzziness of the disturbance frequency value on the amplitude of oscillations is performed.
It has been revealed that even a small indeterminacy in the frequency setting can cause the beam damage, although there will not yet be any damage when setting the accurate frequency. Thus for the value $\omega_3^{(0)} = 18.2$ , the corresponding value $A_3^{(0)}$ of the right endpoint of the amplitude interval exceeds the maximum acceptable value of 0.076 m, although the modal value of the amplitude does not exceed the acceptable value. Therefore, when calculating the amplitude of structural oscillations, the interval endpoints of the frequency variation should be taken into account, and not its modal value. Analysis of the table shows that further increase in the oscillations frequency leads to resonance, because it moves beyond the acceptable limits both the endpoints of the interval of undetermined amplitude, and the modal value.

**Keywords:** forced oscillations of prestressed concrete beam, membership function, perturbation frequency, the theory of fuzzy numbers.

## 1. INTRODUCTION

The project designing is connected with the parameters of materials needed for its creation such as the elasticity modulus of concrete and steel. They are not determined as well as the dimensions of units of the unbuilt construction Therefore, at the design stage one should take into account the indeterminacy of parameters and foresee its further consequences. We will show how to take into account the indeterminacy of the parameters for defining the amplitude of the oscillations of the prestressed concrete T-shaped cross section beam objected to harmonic perturbation. Prestressed concrete beams are widely applied in bridge construction due to the use of high-strength reinforcement. It is known that concrete is well-compressed but it does not work well in tension. Therefore, the reinforcing frame includes high-strength rebar. To fully use the carrying capacity, the high-strength rebar is stretched between stops before its concreting. Without pre-tension of the reinforcement the concrete layer inside it is not able to withstand stretching and may crack. This cannot be allowed, because the moisture that penetrates into the cracks from outside will cause corrosion of the reinforcement. In addition, cyclic freezing and thawing destroys the beam. Therefore, pre-tension of the reinforcement is applied. Such 33-meter-long beams have been designed by "Soyuzdorproekt" and applied in bridges for over 50 years. The precast beams are manufactured with the help of the rolling stands. First, the reinforcement frame including 10 bunches of 5 mm high-strength wire is mounted on a metal rolling stand. Each bunch consists of 24 wires. There are anchors at the ends of the bunch. The main task of an anchor is to pass on the tensile force of the bunch to the concrete after his release from the catch. After stretching the bunches to the designed size the stand with the frame and tensioned bunches is rolled into the casting workshop. After casting with concrete, the beam doesn't reach the designed strength. Thus the beam is rolled into the steaming chamber where it is kept for 24 hours at a temperature of $90^0 C$. It reaches the designed strength in this chamber (under normal conditions it takes 28 days). After steaming the finished beam is rolled to the warehouse (Fig. 1). The bunches are released from the catches there. As concrete compresses the beam flexes upwards. Calculation of the tensile force of the bunches provides the absence of cracks in the top layer of the beam.

Fig. 1. A prestressed beam on a stand

Consider the forced transverse vibrations $y(z,t)$ of the beam with the constant moment of inertia of section $I$, the modulus of elasticity of concrete $E$, the cross-sectional area $S$, the length $l$, and the linear mass $m$. Here $z$ is the abscissa of the point of the beam axis, $t$ is time. Let us consider the case where both supports on which the beam rests, for some reason are stationary (Fig. 2). In this case, a horizontal reaction $H$ arises under the transverse displacement, and it is determined by the following formula

$$H = \frac{ES}{l} \cdot \int_0^l y_z^2 dz \ . \qquad (1.1)$$

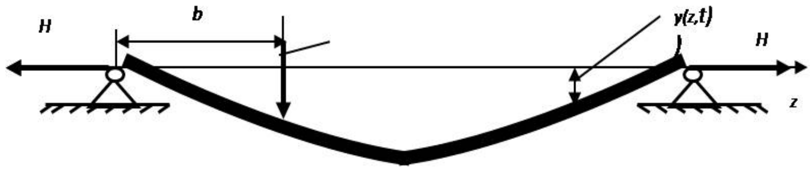


Fig. 2. Sketch of the beam fixed on two supports

The transverse variable force affects the beam $F_0(t)$. Taking into account the formula (1.1) and according to N.G. Bondar [1], we obtain the following equation of oscillations

$$L(z,t) = \frac{EI}{m} \cdot \left( y_{zzzz} - a \cdot y_{zz} \cdot \int_0^l y_z^2 dz \right) + y_{tt} - \frac{F_0(t) \cdot \delta(z-b)}{m} = 0 \, . \qquad (1.2)$$

Here, the subscript for the function $y$ denotes the partial derivative with respect to the corresponding variable, $\delta(z) - \delta$ is the Dirac function, and $b$ is a point of application of a force. We seek a solution in the following form

$$y(z,t) = x \cdot \sin \frac{\pi z}{l} \, , \qquad (1.3)$$

where $x = x(t)$. This solution satisfies zero geometric and force conditions. In accordance with the Bubnov-Galerkin method, we substitute function (1.3) for equation (1.2) and minimize the functional

$$\int_0^l L(z,t) \cdot \sin \frac{\pi z}{l} dz \, .$$

We result at the Duffing's equation

$$\ddot{x} + a \cdot x + \beta \cdot x^3 = F_1(t) \ , \qquad (1.4)$$

Where

$$a = \frac{\pi^4 EI}{ml^4}, \quad \beta = a \cdot \frac{S}{4I}, \quad F_1(t) = \frac{2F_0(t)}{l \cdot m} \sin \frac{\pi b}{l} \, . \qquad (1.5)$$

Let $F_0(t) = f \cdot \sin \omega \cdot t$, where $f$ is the amplitude of the perturbing force, $t$ is time, $\omega$ is the frequency of harmonic perturbation. Let the perturbing force be applied in the middle of the beam ($\sin \frac{\pi b}{l}$ =1). After the replacement of $\frac{2 \cdot f}{l \cdot m} = F$, equation (1.4) is expressed as

$$\ddot{x} + a \cdot x + \beta \cdot x^3 = F \sin \omega t \, . \qquad (1.6)$$

Thus, the problem of oscillations of a beam with the geometric nonlinearity leads to the solution of the Duffing's equation with a strict characteristic of the

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104

restoring force $(a>0,\beta>0)$ The problem of oscillations of a beam with the physical nonlinearity also leads to the Duffing's equation (1.4) , when the tension is connected with the relative elongation by the relationship

$$\sigma = E\cdot\varepsilon + \beta_0\cdot\varepsilon^3 .$$

In this case, the coefficient $\beta$ can be either positive or negative.

In articles [2, 3] a fuzzy double crisis is observed in the forced Duffing's oscillator with multiplicative fuzzy noise. The Duffing's equation contains only one fuzzy parameter with triangular membership function. In this paper, we consider the forced Duffing's oscillator having several triangular fuzzy parameters. Let us consider the stationary mode of oscillations of a system, according to which the principal component of the solution has the form of the right-hand part. Naturally, this regime occurs under certain initial conditions.

## 2. PROBLEM DEFINITION

Let us construct an approximate solution by the Duffing's method. In order to reduce the quantity of equation parameters, we proceed to dimensionless variables. Let $x_0$ be the static deviation of the corresponding linear system

$$x_0 = \frac{F}{a}. \tag{2.1}$$

A new dimensionless variable $y$ can be defined by the equality

$$y = \frac{x}{x_0} . \tag{2.2}$$

This is a relative displacement. Taking into account the equality (2.1), we obtain from the equality (2.2) the following

$$y = \frac{x\cdot a}{F}. \tag{2.3}$$

We proceed to the dimensionless argument $\tau$ connected with the variable $t$ by the equality

$$\sqrt{a}\cdot t = \tau . \tag{2.4}$$

Considering the equations (2.3) and (2.4), we result at the equation in dimensionless variables, which has already got one parameter $\gamma$ instead of three

$$\frac{d^2 y}{d\tau^2} + y + \gamma\cdot y^3 = \sin\nu\cdot\tau . \tag{2.5}$$

Here

$$\gamma = \frac{\beta\cdot F^2}{a^3}, \tag{2.6}$$

$$\nu = \frac{\omega}{\sqrt{a}}. \tag{2.7}$$

Let the null approximation have the form of the right-hand part and is a harmonic

$$y = A \sin \nu \cdot \tau \tag{2.8}$$

with not yet defined amplitude $A$. Depending on the initial conditions, the value of the amplitude $A$ can be either positive, which corresponds to the in-phase oscillations with the active force, or negative, which corresponds to the oscillations in the antiphase, respectively. The null approximation satisfies the initial conditions

$$\tau = \frac{\pi}{2 \cdot \omega}, \; y = A, \; \frac{dy}{d\tau} = 0 . \tag{2.9}$$

In accordance with Dufing's idea, we add to both parts of equality (1.6) the expression $\nu^2 \cdot y$. We get the following

$$\frac{d^2 y}{d\tau^2} + \nu^2 \cdot y + y + \gamma \cdot y^3 = \sin \nu \cdot \tau + \nu^2 \cdot y .$$

Substituting the expression (2.8) for the variable $y$ for the right-hand part of the equation as well as the third and the fourth members from the left-hand part of it, we result at the equality

$$\frac{d^2 y}{d\tau^2} + \nu^2 \cdot y = \sin \nu \cdot \tau \cdot (1 - A + \nu^2 A - \frac{3}{4} \cdot \gamma \cdot A^3) + \frac{1}{4} \cdot \gamma \cdot A^3 \sin 3 \cdot \nu \cdot \tau . \tag{2.10}$$

The eigenfrequency of the linear system, artificially created as a result of the adding the summand $\nu^2 \cdot y$ to both parts of the equation, coincides with the frequency of the first right-hand part summand. To exclude resonance, the expression in parentheses from the right-hand part should be equated to zero. This is the sense of the Duffing's idea. We reach the equation for determining the amplitude

$$1 - A + \nu^2 A - \frac{3}{4} \cdot \gamma \cdot A^3 = 0 , \tag{2.11}$$

The last equality is the amplitude-frequency characteristic equation. Now the equation (2.10) is expressed as

$$\frac{d^2 y}{d\tau^2} + \nu^2 \cdot y = \frac{1}{4} \cdot \gamma \cdot A^3 \sin 3 \cdot \nu \cdot \tau ,$$

A particular solution of this equation which satisfies the initial conditions (2.9) is expressed by the equality

$$y = A \sin \nu \cdot \tau + \frac{\gamma \cdot A^3}{32 \cdot \nu^2} \cdot (\sin \nu \cdot \tau - \sin 3 \cdot \nu \cdot \tau) .$$

This equality describes the first approximation of the equation solution (2.5). The equation (2.11) of the amplitude-frequency characteristic (AFC) of the null approximation contains only one parameter $\gamma$. The diagram of the function $\nu$ under $\gamma = 1$ is shown in Fig. 3.

By replacing the variables in this equation, you can get rid of this parameter as well. First, we find the minimum point of the diagram of the perturbation frequency $\nu$ and the oscillations amplitude $A$ relationship. This point has the following coordinates

$$A^* = -\sqrt[3]{\frac{2}{3\cdot\gamma}}, \quad \nu^* = \sqrt{1+3\cdot\sqrt[3]{\frac{3\cdot\gamma}{16}}}\ .$$

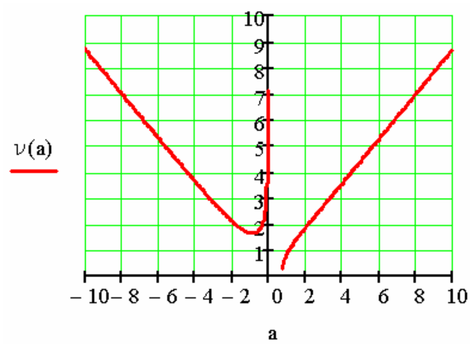


Fig.3. Diagram of the amplitude-frequency relationship

We enter new dimensionless variables $c$ and $d$ and express through them the amplitude $A$ and oscillations frequency $\nu$, by the formulas

$$A = -d\cdot\sqrt[3]{\frac{2}{3\cdot\gamma}}, \qquad (2.12)$$

$$\nu = \sqrt{1+3\cdot c\cdot\sqrt[3]{\frac{3\cdot\gamma}{16}}}\ . \qquad (2.13)$$

The substitution units in these equalities instead of variables $c$ and $d$ gives us the coordinates of the minimum point of the function $\nu$. Substituting the right-hand parts of equations (2.12) and (2.13) into equation (2.11), we result at the equation in new dimensionless variables $c$ and $d$:

$$d^3 - 3\cdot c\cdot d + 2 = 0, \qquad (2.14)$$

which no longer contains any parameter. The diagram of the relationship between the variables $c$ and $d$ is shown in Fig. 4. From equality (2.13) we express the variable $c$ through the frequency $\nu$:

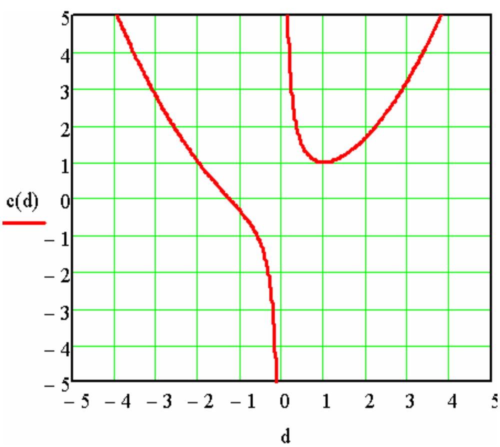


Fig. 4. Diagram of relationship between $c$ and $d$

$$c = \frac{4\cdot(\nu^2-1)}{3\cdot\sqrt[3]{12\cdot\gamma}}\ .$$

Taking into account the equalities (2.6) and (2.7), we find

$$c = \frac{4\cdot(\omega^2-\alpha)}{3\cdot\sqrt[3]{12\cdot\beta\cdot F^2}}\ . \qquad (2.15)$$

The equality (2.3) shows that the amplitude $A_0$ of the oscillations of the variable $x$ is related to the amplitude $A$ of the dimensionless variable $y$ by the equality

$$A_0 = \frac{A \cdot F}{a}.$$

Taking into account the equalities (2.6) and (2.12), after simplification we obtain

$$A_0 = -d \cdot \sqrt[3]{\frac{2 \cdot F}{3 \cdot \beta}}. \tag{2.16}$$

According to the Cardano formulas, from the equation (2.14) we find the value $d$ as a function $c$:

$$d = d(c) = W(c) + V(c) \text{ for any value of } c; \tag{2.17}$$

$$d = da(c) = -0.5 \cdot \left(W(c) + V(c)\right) + 0.5 \cdot \left(W(c) - V(c)\right) \cdot \sqrt{-3} \text{ , if } c > 1; \tag{2.18}$$

$$d = db(c) = -0.5 \cdot \left(W(c) + V(c)\right) - 0.5 \cdot \left(W(c) - V(c)\right) \cdot \sqrt{-3} \text{ , if } c > 1. \tag{2.19}$$

Here the following is expressed

$$W(c) = \sqrt[3]{-1 + \sqrt{1 - c^3}}, \qquad V(c) = \sqrt[3]{-1 - \sqrt{1 - c^3}}.$$

The diagram of the function $d$ is shown in Fig. 5. The equation (2.14) has a single real root if $c < 1$, and it is defined by the formula (2.17). If $c > 1$, the equation (2.14) has three real roots, and they are defined by formulas (2.17), (2.18) and (2.19). In this case, the branch of the diagram that corresponds to the formula (2.18) for $c > 1$, and formula (2.17) for $c < 1$, determines the negative values of the root $d$ and corresponds to the large (in-phase) oscillations of the beam.

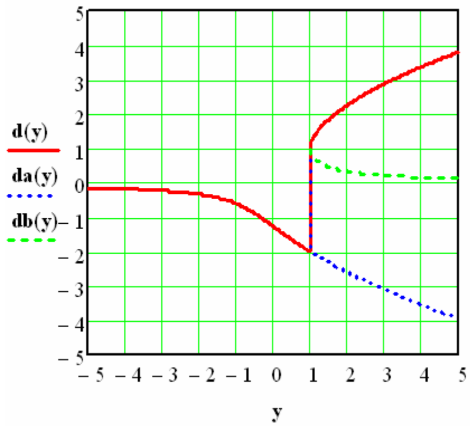

Fig. 5. Diagram of the function *d*

The branch of the diagram, which corresponds to the formula (2.17) for $c > 1$, determines the large positive values of the root. It is proved that they correspond to unstable points of the amplitude-frequency characteristic, so they should not be taken into account. The branch which is defined by the formula (2.19) for $c > 1$ determines the smaller positive values.

It corresponds to the small (antiphase) oscillations of the beam. The relationship between the function $d$ and parameter $c$ for large oscillations takes the following form

$$d = dm(c) = \begin{cases} da(c), if & c > 1, \\ d(c), if & c \le 1 \end{cases}.$$

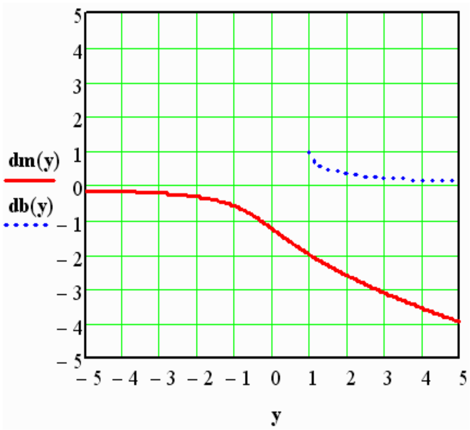

Fig.6. Diagram of the function *d* for large and small oscillations

The diagram of the relationship between the function $d$ and parameter $c$ denoted by a continuous line for large (in-phase) oscillations, and by a dashed line for small (antiphase) oscillations is shown in Fig. 6.

The realization of large or small oscillations depends on the initial conditions. Taking into account equalities (2.15) and (2.16), we obtain the amplitude of oscillations $A_0$.

Let us calculate the amplitude of oscillations under undetermined values of parameters of the T-shaped prestressed beam. We will consider the beam parameters as undetermined triangular numbers, because they have valuable properties such as the simplicity of the description and the clarity of the interpretation, the keeping of the form when adding and subtracting, and the convenience of decomposition on a $\alpha$ - level system. Besides, there is no statistics for such a problem. A cross section of the beam is shown in Fig. 7. All sizes are given in millimeters. Here $h = 1730\,\text{mm}$, $a = 200\,\text{mm}$, $b = 580\,\text{mm}$, $e = 1400\,\text{mm}$, $c = 20\,\text{mm}$. We calculate the moment of inertia of the beam cross section. First, we determine the position of the neutral axis $y_0$ with respect to the lower face of the cross section. The standard stress $P_{ar}$ in one bunch is equal to $499300\,\text{N}$ and corresponds to the stretching of the reinforcement by 198 mm. After cutting the bunches, the concrete shrinks and the stress in the beam decreases. We determine the total stress in ten bunches after concrete compression. The tension stress of the reinforcement after compression of concrete decreases and is expressed as $P_{sn} = 10 \cdot P_{ar} \cdot \dfrac{0.198 - x}{0.198}$. Equating it to the compression stress of concrete which is equal to $S \cdot \dfrac{x}{l} \cdot E$ we find the value $x$ of the

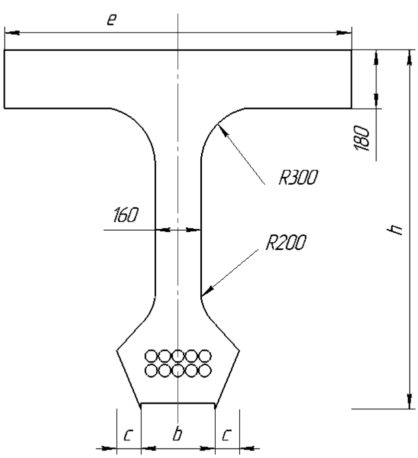

Fig. 7. Cross-section of the beam

contraction of the bunches: $x = 0.0086$ m. Here $S = 0.704\,\mathrm{m}^2$ is the area of the beam cross section, $E = 26\cdot 10^9$ Pa is the modulus of elasticity of concrete of a B 35 rate, $l = 33$m is the length of the beam.

The reduced tension force of the bunches $P_{sn}$ is $4.776\cdot 10^6$ N. The position of the neutral axis depends on the stress in the tensioned reinforcement. The manufactured beam lies on the rolling stand and is under the influence of its own weight and compressive force passed from the prestressed reinforcement. The beam lying on the stand, in accordance with the design, has a short-term bend $\Delta$ caused by the prestressing force and its own weight and it is equal to 32.5 mm.

We result at the equation with respect to the value $y_0$:

$$\frac{M(y_0)\cdot l^2}{8\cdot E\cdot I(y_0,\delta)}+\frac{5}{384}\cdot\frac{q\cdot l^4}{E\cdot I(y_0,\delta)}-\Delta=0\,. \quad (2.20)$$

Here the first summand is the inflection from the beam compression by stretched beams, the second summand is the deflection from the beam's own weight, $q$ is the load from the beam's own weight $q = 17218\,\mathrm{N/m}$, $M(y_0)$ is the moment of the compression of the concrete by prestressed reinforcement

$$M(y_0)=P_{sn}\cdot\left(y_a-y_0\right).$$

Here $y_a$ is the distance from the lower face of the cross section to the center of the bunches. The moment of inertia is a function of the position of the neutral axis $y_0$ and the deviation $\delta$ of the cross-sectional dimensions from the designed values. It is defined by the following formula

$$I(y_0,\delta)=2\Bigg[\int_0^{0.08+\delta}\int_0^{1.73+\delta}(y-y_0)^2\,dydx+$$

$$+\int_{0.08+\delta}^{0.138+\delta}\int_0^{1.73-\sqrt{0.04-(x-0.28-\delta)^2}}(y-y_0)^2\,dydx+\int_{0.138+\delta}^{0.31+\delta}\int_0^{0.51-x}\left(y-y_0\right)^2 dydx-$$

$$-\int_{0.29+\delta}^{0.31+\delta}\int_0^{10\cdot(x-0.29-\delta)}\left(y-y_0\right)^2 dydx+\int_{0.08+\delta}^{0.38+\delta}\int_{1.25+\sqrt{0.09-(x-0.38-\delta)^2}}^{1.73+x}\left(y-y_0\right)^2 dydx+$$

$$+\int_{0.38+\delta}^{0.7+\delta}\int_{1.55}^{1.73+\delta}\left(y-y_0\right)^2 dydx\Bigg]. \quad (2.21)$$

Similarly, we calculate the cross-sectional area as a function of deviations of the cross-sectional dimensions. Let the dimensions of the section have a deviation within the tolerance $\pm 0.003$ m. Solving the equation (2.20) and taking into account the equality (2.21), we can calculate the moment of inertia. Depending on the deviations of the cross-sectional dimensions, the cross-sectional area $S$ and the moment of inertia $I$ have the following values and intervals of variation

$$S = 0.704\,\mathrm{m}^2,\quad 0.691\,\mathrm{m}^2 < S < 0.718\,\mathrm{m}^2;$$

$$I = 0.285\,\mathrm{m}^4,\quad 0.281\,\mathrm{m}^4 < I < 0.29\,\mathrm{m}^4.$$

Let the undetermined length of the beam $l$, the linear mass $m$, the modulus of elasticity of the concrete $E$, the amplitude of the perturbation force $f$ and the perturbation frequency $\omega$, as well as their intervals of variation have the following values

$$l = 33\text{ m},\quad 32.99\text{ m} < l < 33.01\text{ m};$$
$$m = 1756\text{ kg/m},\quad 1724\text{ kg/m} < m < 1791\text{ kg/m};$$
$$E = 26\cdot 10^9\text{ Pa},\quad 25\cdot 10^9\text{ Pa} < E < 27\cdot 10^9\text{ Pa};$$
$$f = 50\text{ N},\quad 49.9\text{ N} < f < 50.1\text{ N};$$
$$\omega = 17.8\text{ Hz},\quad 17.7\text{ Hz} < \omega < 17.9\text{ Hz}.$$

## 3. DEFINITION OF AN UNDETERMINED TRIANGULAR NUMBER

An undetermined triangular number is a number with a carrier $Supp(A) = [a_1, a_3]$ with a single modal value for which $\mu A(x) = 1$ and the membership function [4]:

$$\mu A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2; \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3; \\ 0, & x < a_1, x > a_3. \end{cases} \tag{3.1}$$

The undetermined number function can be interpreted as a measure of the designer's confidence that all the points of a certain segment differ little from the determined value that belongs to it, and we probably do not know the determined value. It's natural that the longer the segment, the less confidence that all its points are close to the determined value. The membership function is a subjective evaluation. The values that the membership function takes are called the $\alpha$ -level of the undetermined number. For example, if according to the results of all the studies the modulus of elasticity of concrete is expressed by the interval $[a_1, a_3]$, then its $\alpha$ -level is equal to zero, and $\alpha$ -level of the determined number is equal to one, because the determined number is the interval the ends of which are equal to it. The undetermined number is unimodal. If the condition $\mu_A(x) = 1$ is true only for one value, this singular number is called a mode. It is obvious that the mode of the triangular number is $a_2$. Let all the parameters of the problem be unimodal undetermined numbers. We will operate with the undetermined parameters based on the interval method. The undetermined triangular number $A$ is completely defined by three determined numbers. Therefore it is expressed by $A = (a_1, a_2, a_3)$, and its $\alpha$ -level interval is written as $A_\alpha = \left[a_1^{(\alpha)}, a_3^{(\alpha)}\right]$. It's obvious that $a_1 = a_1^{(0)}$, $a_3 = a_3^{(0)}$, $a_2 = a_1^{(1)} = a_3^{(1)}$. Taking into account the expression (3.1), the ends of the interval $A_\alpha$ can be written as functions $\alpha$ :

$$A_\alpha = [(a_2 - a_1)\cdot\alpha + a_1, -(a_3 - a_2)\cdot\alpha + a_3].$$

## 4. OPERATIONS ON UNDETERMINED NUMBERS BASED ON THE INTERVAL METHOD

Let $A$ and $B$ be two undetermined, not necessarily triangular, but unimodal numbers with the $\alpha$-level intervals and $A_\alpha=\left[a_1^{(\alpha)},a_3^{(\alpha)}\right]$ and $B_\alpha=\left[b_1^{(\alpha)},b_3^{(\alpha)}\right]$, $\forall\alpha\in(0,1]$ . The operations on the $\alpha$-level intervals of undetermined numbers $A$ and $B$ are performed according to the following rules

$$A_\alpha+B_\alpha=\left[a_1^{(\alpha)},a_3^{(\alpha)}\right]+\left[b_1^{(\alpha)},b_3^{(\alpha)}\right]=\left[a_1^{(\alpha)}+b_1^{(\alpha)},a_3^{(\alpha)}+b_3^{(\alpha)}\right],$$

$$A_\alpha-B_\alpha=\left[a_1^{(\alpha)},a_3^{(\alpha)}\right]-\left[b_1^{(\alpha)},b_3^{(\alpha)}\right]=\left[a_1^{(\alpha)}-b_1^{(\alpha)},a_3^{(\alpha)}-b_3^{(\alpha)}\right],$$

$$A_\alpha\cdot B_\alpha=\left[a_1^{(\alpha)},a_3^{(\alpha)}\right]\cdot\left[b_1^{(\alpha)},b_3^{(\alpha)}\right]=$$

$$=\begin{bmatrix}\min\left\{a_1^{(\alpha)}\cdot b_1^{(\alpha)},a_1^{(\alpha)}\cdot b_3^{(\alpha)},a_3^{(\alpha)}\cdot b_1^{(\alpha)},a_3^{(\alpha)}\cdot b_3^{(\alpha)}\right\},\\ \max\left\{a_1^{(\alpha)}\cdot b_1^{(\alpha)},a_1^{(\alpha)}\cdot b_3^{(\alpha)},a_3^{(\alpha)}\cdot b_1^{(\alpha)},a_3^{(\alpha)}\cdot b_3^{(\alpha)}\right\}\end{bmatrix}$$

The multiplication of the $\alpha$-level interval of the undetermined number by a determined number $k$ is defined by the following rule

$$k\cdot A_\alpha=k\cdot\left[a_1^{(\alpha)},a_3^{(\alpha)}\right]=\left[\min\left\{k\cdot a_1^{(\alpha)},k\cdot b_3^{(\alpha)}\right\},\max\left\{k\cdot a_1^{(\alpha)},k\cdot b_3^{(\alpha)}\right\}\right].$$

The inverse $\alpha$-level interval of the undetermined number is the undetermined number

$$\left(A_\alpha\right)^{-1}=\frac{1}{A_\alpha}=\left[a_1^{(\alpha)},a_3^{(\alpha)}\right]^{-1}=\left[\min\left\{\frac{1}{a_1^{(\alpha)}},\frac{1}{a_3^{(\alpha)}}\right\},\max\left\{\frac{1}{a_1^{(\alpha)}},\frac{1}{a_3^{(\alpha)}}\right\}\right].$$

There is no need in division operation, because it can be reduced to multiplication by the inverse number.

Let us define the membership function of the amplitude of oscillations. First, let us calculate the $\alpha$-level of undetermined parameters $I,S,E,l,m,F,\omega$:

$$I_\alpha=\left[I_1^{(\alpha)},I_3^{(\alpha)}\right],S_\alpha=\left[S_1^{(\alpha)},S_3^{(\alpha)}\right],\ E_\alpha=\left[E_1^{(\alpha)},E_3^{(\alpha)}\right],\ l_\alpha=\left[l_1^{(\alpha)},l_3^{(\alpha)}\right],$$

$$m_\alpha=\left[m_1^{(\alpha)},m_3^{(\alpha)}\right],\ f_\alpha=\left[f_1^{(\alpha)},f_3^{(\alpha)}\right],\ \omega_\alpha=\left[\omega_1^{(\alpha)},\omega_3^{(\alpha)}\right].$$

Here the endpoints of the intervals are defined by the formulas:

$$I_1^{(\alpha)}=(I_2-I_1)\cdot\alpha+I_1,\ I_3^{(\alpha)}=-(I_3-I_2)\cdot\alpha+I_3;\ I_1=0.281,\ I_2=0.285,$$
$$I_3=0.29,$$

$$S_1^{(\alpha)}=(S_2-S_1)\cdot\alpha+S_1,\ S_3^{(\alpha)}=-(S_3-S_2)\cdot\alpha+S_3;\ S_1=0.691,\ S_2=0.704,$$
$$S_3=0.718,$$

$$E_1^{(\alpha)}=(E_2-E_1)\cdot\alpha+E_1,\ E_3^{(\alpha)}=-(E_3-E_2)\cdot\alpha+E_3;\ E_1=25\cdot10^9,\ E_2=26\cdot10^9,$$
$$E_3=27\cdot10^9,$$

$$l_1^{(\alpha)}=(l_2-l_1)\cdot\alpha+l_1,\ l_3^{(\alpha)}=-(l_3-l_2)\cdot\alpha+l_3;\ l_1=32.99,\ l_2=33,\ l_3=33.01,$$

$$m_1^{(\alpha)} = (m_2 - m_1)\cdot\alpha + m_1,\ m_3^{(\alpha)} = -(m_3 - m_2)\cdot\alpha + m_3;\ m_1 = 1724,\ m_2 = 1756,\ m_3 = 1791,$$

$$f_1^{(\alpha)} = (f_2 - f_1)\cdot\alpha + f_1,\ f_3^{(\alpha)} = -(f_3 - f_2)\cdot\alpha + f_3;\ f_1 = 49.9,\ f_2 = 50,\ f_3 = 50.1,$$

$$\omega_1^{(\alpha)} = (\omega_2 - \omega_1)\cdot\alpha + \omega_1,\ \omega_3^{(\alpha)} = -(\omega_3 - \omega_2)\cdot\alpha + \omega_3;\ \omega_1 = 17.7,\ \omega_2 = 17.8,\ \omega_3 = 17.9.$$

Here and below, the moment of inertia is expressed by $\mathrm{m}^4$, the cross-sectional area is expressed by $\mathrm{m}^2$, the modulus of elasticity is expressed by $\mathrm{Pa}$, the length is expressed by $\mathrm{m}$, the linear mass of the beam is expressed by $\mathrm{kg/m}$, the amplitude of the perturbing force is expressed by N, and the frequency of the perturbation is expressed by Hz.

We calculate the membership functions of the parameters (1.5) of the Duffing's equation. From the first and second equalities of the expression (3.1), according to the above mentioned rules of operations on undetermined numbers, we calculate the endpoints of the intervals

$$a_\alpha = \left[a_1^{(\alpha)}, a_3^{(\alpha)}\right] \text{ and } \beta_\alpha = \left[\beta_1^{(\alpha)}, \beta_3^{(\alpha)}\right]:$$

$$a_1^{(\alpha)} = \frac{\pi^4\cdot E_1^{(\alpha)}\cdot I_1^{(\alpha)}}{m_3^{(\alpha)}\cdot(l_3^{(\alpha)})^4},\ a_3^{(\alpha)} = \frac{\pi^4\cdot E_3^{(\alpha)}\cdot I_3^{(\alpha)}}{m_1^{(\alpha)}\cdot(l_1^{(\alpha)})^4},\ \beta_1^{(\alpha)} = \frac{a_1^{(\alpha)}\cdot S_1^{(\alpha)}}{4\cdot I_3^{(\alpha)}},$$

$$\beta_3^{(\alpha)} = \frac{a_3^{(\alpha)}\cdot S_3^{(\alpha)}}{4\cdot I_1^{(\alpha)}}.$$

Let us calculate the $\alpha$ -level of the undetermined number $c_\alpha$ guided by the equality (2.15) by transforming the latter to the following form

$$c_\alpha = R_\alpha W_\alpha,$$

where the following is denoted

$$R_\alpha = k\cdot\left(\frac{\omega_\alpha^2}{a_\alpha} - 1\right),\ W_\alpha = \frac{a_\alpha}{\sqrt[3]{\beta_\alpha\cdot F_\alpha^{\ 2}}}\cdot,\ k = \sqrt[3]{\frac{16}{81}},$$

$$R_\alpha = \left[R_1^{(\alpha)}, R_3^{(\alpha)}\right],\ W_\alpha = \left[W_1^{(\alpha)}, W_3^{(\alpha)}\right].$$

The endpoints of intervals are defined by the formulas:

$$R_1^{(\alpha)} = k\cdot\left(\frac{(\omega_1^{(\alpha)})^2}{a_3^{(\alpha)}} - 1\right),\ R_3^{(\alpha)} = k\cdot\left(\frac{(\omega_3^{(\alpha)})^2}{a_1^{(\alpha)}} - 1\right),$$

$$W_1^{(\alpha)} = \frac{a_1^{(\alpha)}}{\sqrt[3]{\beta_3^{(\alpha)}\cdot\left(F_3^{(\alpha)}\right)^2}},\ W_3^{(\alpha)} = \frac{a_3^{(\alpha)}}{\sqrt[3]{\beta_1^{(\alpha)}\cdot\left(F_1^{(\alpha)}\right)^2}}.$$

According to the rule of the triangular numbers multiplication, we get the $\alpha$ - level interval of the parameter $c$:

$$c_\alpha = R_\alpha\cdot W_\alpha = \left[c_1^{(\alpha)}, c_3^{(\alpha)}\right],$$

where the endpoints of the interval are defined by the formulas:

$$c_1^{(\alpha)} = \min\left\{R_1^{(\alpha)} \cdot W_1^{(\alpha)}, R_1^{(\alpha)} \cdot W_3^{(\alpha)}, R_3^{(\alpha)} \cdot W_1^{(\alpha)}, R_3^{(\alpha)} \cdot W_3^{(\alpha)}\right\},$$

$$c_3^{(\alpha)} = \max\left\{R_1^{(\alpha)} \cdot W_1^{(\alpha)}, R_1^{(\alpha)} \cdot W_3^{(\alpha)}, R_3^{(\alpha)} \cdot W_1^{(\alpha)}, R_3^{(\alpha)} \cdot W_3^{(\alpha)}\right\}.$$

After the calculation we have a non-triangular unimodal number

$$c_1^{(0)} = -434.23,\ c_1^{(1)} = -201.437,\ c_3^{(0)} = -8.758.$$

Let the initial conditions be such that the beam carries out large oscillations.

## 5. CALCULATION OF UNDETERMINED AMPLITUDE OF OSCILLATIONS

The diagram of the function $d$ (Fig. 6) decreases monotonely which simplifies the calculation of the undetermined $\alpha$-level number intervals $d_\alpha = \left[d_1^{(\alpha)}, d_3^{(\alpha)}\right]$. The endpoints of the interval are defined by the equalities:

$$d_1^{(\alpha)} = d(c_3^{(\alpha)}), \quad d_3^{(\alpha)} = d(c_1^{(\alpha)}).$$

Taking into account the equality (22), we calculate the endpoints of the $\alpha$-level intervals of the undetermined amplitude $A_{0\alpha} = \left[A_1^{(\alpha)}, A_3^{(\alpha)}\right]$:

$$A_1^{(\alpha)} = -\sqrt[3]{\frac{2}{3}} \cdot d\left(c_1^{(\alpha)}\right) \cdot \frac{(F_1^{(\alpha)})^{1/3}}{(\beta_3^{(\alpha)})^{1/3}},\quad A_3^{(\alpha)} = -\sqrt[3]{\frac{2}{3}} \cdot d\left(c_3^{(\alpha)}\right) \cdot \frac{(F_3^{(\alpha)})^{1/3}}{(\beta_1^{(\alpha)})^{1/3}}.$$

The membership function for the oscillation amplitude is convex but not triangular. The diagram of the undetermined amplitude membership function is shown in Fig. 8 which is calculated by the given undetermined parameters of the problem. The carrier of the undetermined amplitude of the nonlinear oscillations of the beam is the following interval

$$\left[A_1^{(0)}, A_3^{(0)}\right] = \left[2.559 \cdot 10^{-5}\,\text{m};\ 1.401 \cdot 10^{-3}\,\text{m}\right].$$

The mode of undetermined amplitude $A_2$ is $5.797 \cdot 10^{-5}\,\text{m}$. The average value of the undetermined amplitude is calculated by the formula

$$A_{sr} = \int_0^1 \frac{A_1(\alpha) + A_3(\alpha)}{2} d\alpha$$

and is equal to $5.797 \cdot 10^{-5}\,\text{m}$. In some cases, the middle of the interval for the $\alpha = 0.5$ level membership function can be taken as the expected value of the undetermined number. We have

$$A_{sr} = \frac{A_1(0.5) + A_3(0.5)}{2} = 7.744 \cdot 10^{-5}\ \text{m}.$$

Let us determine the largest amplitude of oscillations at which the yield of high-strength wire begins. We find the largest amplitude of oscillations provided that the deflection of the beam from the moment of the compression force of concrete by high-strength reinforcement, stretched up to the yield strength, is equal to the sum of the largest value of the oscillations amplitude and the deflection of the beam from its own weight. The largest compression-caused deflection of the beam $y_{tek}$ in the middle of the span is determined by the equality

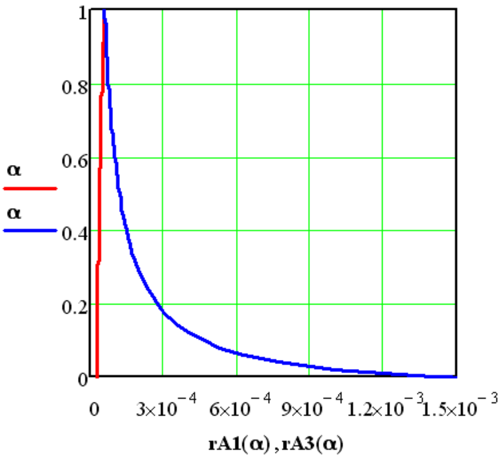


Fig. 8. Diagram of the undetermined amplitude of the oscillation function

$$y_{tek}=\frac{P_{tek}\cdot(y_0-y_a)\cdot l^2}{8\cdot E\cdot I}.$$

Here $P_{tek}$ is the total stress from the stretched bunches at which the high-strength reinforcement yield begins.

According to the laboratory tests, the yield strength force for a single 5 mm wire is 32,340 N, so we have $P_{tek}=7.762\cdot10^6$ N. Taking into account the equality $y_0-y_a=0.792$ m, we obtain the largest deflection of the beam which is equal to $y_{tek}=0.111$ m. The deflection from the own weight of the beam in the middle of the span is 0.035 m. Therefore the acceptable value of the oscillations amplitude is 0.076 m. Table 1 shows the values of the endpoints of the intervals of the large oscillations amplitude $A_1^{(0)}$, $A_3^{(0)}$, and the modal value $A_1^{(1)}$, expressed in meters, as well as the values of the endpoints of the oscillation frequency intervals $\omega_1^{(0)}$, $\omega_3^{(0)}$, and the modal value $\omega_1^{(1)}$ expressed in hertz, respectively.

Table 1

Values of the endpoints of the intervals of the large oscillations amplitude $A_1^{(0)}$, $A_3^{(0)}$, and the modal value $A_1^{(1)}$

| $\omega_1^{(0)}$ | $\omega_1^{(1)}$ | $\omega_3^{(0)}$ | $A_1^{(0)}$ | $A_1^{(1)}$ | $A_3^{(0)}$ |
|---|---|---|---|---|---|
| 9.9 | 10 | 10.1 | $5.594\cdot10^{-6}$ | $6.998\cdot10^{-6}$ | $8.778\cdot10^{-6}$ |
| 17.7 | 17.8 | 17.9 | $2.559\cdot10^{-5}$ | $5.797\cdot10^{-5}$ | $1.401\cdot10^{-3}$ |
| 17.75 | 18 | 18.2 | $2.637\cdot10^{-5}$ | $7.633\cdot10^{-5}$ | 0.284 |
| 18.1 | 18.15 | 18.2 | $3.358\cdot10^{-5}$ | $1.0\cdot10^{-4}$ | 0.284 |
| 19 | 19.1 | 19.2 | $1.232\cdot10^{-4}$ | 0.337 | 0.631 |
| 19 | 19.5 | 20 | $1.232\cdot10^{-4}$ | 0.458 | 0.816 |
| 26.84 | 26.85 | 26.86 | 1.259 | 1.527 | 1.844 |

## 6. CONCLUSIONS

Analysis of the results given in the table shows that even a small indeterminacy in the frequency setting can cause the beam damage, although there will not yet be any damage when setting the accurate frequency. Thus for the value $\omega_3^{(0)} = 18.2$, the corresponding value $A_3^{(0)}$ of the right endpoint of the amplitude interval exceeds the maximum acceptable value of 0.076 m, although the modal value of the amplitude does not exceed the acceptable value. Therefore, when calculating the amplitude of structural oscillations, the interval endpoints of the frequency variation should be taken into account, and not its modal value. Analysis of the table shows that further increase in the oscillations frequency leads to resonance, because it moves beyond the acceptable limits both the endpoints of the interval of undetermined amplitude, and the modal value.

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*Стаття надійшла 21.02.2020*

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**НЕЛІНІЙНІ КОЛИВАННЯ ПОПЕРЕДНЬО НАПРУЖЕНОЇ ЗАЛІЗОБЕТОННОЇ МОСТОВОЇ БАЛКИ ПРИ ГАРМОНІЙНОМУ ОБУРЕНІ В УМОВАХ НЕЧІТКИХ ПАРАМЕТРІВ**

**Анотація**. У даній роботі розглядаються нелінійні коливання попередньо напруженої залізобетонної балки, нерухомо закріпленої на двох опорах. Балка знаходиться під дією гармонійної сили. Розрахунки таких балок пов'язані з цілою низкою невизначеностей у вихідних даних. Питанням коректного їх врахування присвячується дана публікація.

Довгий час в механіці, для врахування невизначеностей, домінує використання теорії ймовірностей в моделюванні. Ця теорія довела свою ефективність у розв'язанні багатьох задач, але має і деякі слабкі сторони. Зокрема, недостатня статистична інформація або неповна інформація не дозволяє адекватно відображати реальний об'єкт дослідження в математичній моделі. Останнім часом багато дослідників відзначають, що невизначеність в будівництві носити не тільки стохастичний характер. Це дає поштовх для впровадження нових методів і теорій м'яких обчислень. Серед них найбільшу популярність і ефективність в даний час мають теорії нечітких і неточних множин, достовірність яких уже доведена при вирішенні задач управління і т.д.

Для розглянутої балки визначена амплітуда її коливань за умови, що її параметри є нечіткими і змінюються в певних межах. Розглянуто приклад визначення амплітуди коливань попередньо напруженої балки довжиною 33 м, запроектованої Союздорпроект. Побудована функція належності амплітуди поперечних коливань балки з використанням теорії нечітких множин. Виконано аналіз впливу нечіткості завдання частоти обурення на амплітуду коливань. Виявлено, що навіть мала нечіткість в завданні частоти може викликати руйнування балки, хоч при чіткому завданні частоти руйнування ще не буде. Так для значення $\omega_3^{(0)} = 18.2$ відповідне значення $A_3^{(0)}$ правого кінця інтервалу амплітуди перевищує граничне допустиме значення 0.076 м, хоча модальне значення амплітуди не перевищує допустиме значення. Отже, при обчисленні амплітуди коливань конструкцій в розрахунок слід брати кінці інтервалу зміни частоти, а не її модальне значення. Аналіз показує, що подальше збільшення частоти коливань веде до резонансу, тому що виводить за допустимі межі і кінці інтервалу нечіткої амплітуди, і модальне значення.

**Ключові слова:** попередньо напружена залізобетонна балка, теорія нечітких множин, функція приналежності, частота збурень, амплітуда коливань.

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## НЕЛИНЕЙНЫЕ КОЛЕБАНИЯ ПРЕДВАРИТЕЛЬНО НАПРЯЖЕННОЙ ЖЕЛЕЗОБЕТОННОЙ МОСТОВОЙ БАЛКИ ПРИ ГАРМОНИЧЕСКОМ ВОЗМУЩЕНИИ В УСЛОВИЯХ НЕЧЕТКОСТИ ПАРАМЕТРОВ

**Аннотация**. В данной работе рассматриваются нелинейные колебания предварительно напряженной железобетонной балки, неподвижно закрепленной на двух опорах. Балка находится под действием гармонической силы. Расчёты таких балок сопряжены с целым рядом неопределённостей в исходных данных. Вопросам корректного их учёта посвящается данная публикация.

Долгое время в механике, для учёта неопределённостей, доминирует использование в моделировании теории вероятности. Она доказала свою эффективность в решении многих задач, но имеет и некоторые слабые стороны. В частности, недостаток статистической информации или неполная информация не позволяет адекватно отображать реальный объект исследования в математической модели. В последнее время многие исследователи отмечают, что неопределённость в строительстве носить не только стохастический характер, и это даёт толчок для внедрения новых развивающихся методов и теорий мягких вычислений. Среди них наибольшую популярность и эффективность в настоящее время имеют теории нечётких и неточных множеств, достоверность которых уже доказана при решении задач управления и т.д.

Для рассмотренной балки определена амплитуда ее колебаний при условии, что её параметры являются нечеткими и изменяются в известных пределах. Рассмотрен пример определения амплитуды колебаний преднапряжённой балки длиной 33 м, запроектированной Союздорпроектом. Построена функция принадлежности амплитуды поперечных колебаний балки с использованием теории нечётких множеств. Выполнен анализ влияния нечёткости задания частоты возмущения на амплитуду колебаний. Выявлено, что даже малая нечеткость в задании частоты может вызвать разрушение балки, хоть при четком задании частоты разрушения ещё не будет. Так для значения $\omega_3^{(0)} = 18.2$ соответствующее значение $A_3^{(0)}$ правого конца интервала амплитуды превышает предельное допустимое значение 0.076 м, хотя модальное значение амплитуды не превосходит допустимое значение. Следовательно, при вычислении амплитуды колебаний конструкций в расчет следует брать концы интервала изменения частоты, а не ее модальное значение. Анализ показывает, что дальнейшее увеличение частоты колебаний ведет к резонансу, потому что выводит за допустимые пределы и концы интервала нечеткой амплитуды, и модальное значение.

**Ключевые слова**: предварительно напряженная железобетонная балка, теория нечётких множеств, функция принадлежности, частота возмущений, амплитуда колебаний.

УДК 517.11+519.92+539.3

*Баєв С.В., Волчок Д.Л.* **Нелінійні коливання попередньо напруженої залізобетонної мостової балки при гармонійному обурених в умовах нечітких параметрів** // Опір матеріалів і теорія споруд: наук.-тех. збірник – К.: КНУБА, 2020. – Вип. 104. – С. 147-163. – Англ.

*Для розглянутої балки визначена амплітуда її коливань за умови, що її параметри є нечіткими і змінюються в певних межах.*

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*For the beam under consideration, the amplitude of its vibrations is determined, provided that its parameters are fuzzy and vary within known limits.*

*Баев С.В., Волчок Д.Л.* **Нелинейные колебания предварительно напряженной железобетонной мостовой балки при гармоническом возмущении в условиях нечеткости параметров** // Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУБА, 2020. – Вып. 104. – С. 147-163. – Англ.

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UDC 681.518.5

# SITUATION FORECASTING AND DECISION-MAKING OPTIMIZATION BASED ON USING MARKOV FINITE CHAINS FOR AREAS WITH INDUSTRIAL POLLUTIONS

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DOI: 10.32347/2410-2547.2020.104.164-174

The paper proposes a methodology for modeling engineering-within-nature complex systems (further, "systems"), which will be helpful for researchers and operators of complex technical systems in predicting the emergencies using environmental monitoring systems.

**Keywords:** complex technical systems, engineering-within-nature complex systems, nature & engineering complex systems, environmental monitoring, forecasting emergencies, statistical research, decision support.

**Introduction.** The complexity of modern technical systems and the increase of man-made risks arising from environmental pollution requires not only the increased reliability of such systems, which can be achieved, in particular, by the use of advanced monitoring and diagnostics, but also by the availability of effective means to predict probable accidents that can occur in the operation of these systems as well as the availability of timely and optimal measures of response to the emergencies. The purpose of establishing the permanent system to monitor the faults occurrence lies in reducing the possibility of sudden unexpected events, undermining the economy of the industrial enterprise, dangerous production stops, damage to equipment and accidents to personnel, as well as to facilitate technical maintenance of equipment. Critical levels of environmental pollution should also be minimized. Although the pursuit of greater reliability and less cost may seem incompatible at first, a closer examination of this issue shows that this is not the case [1].

In Ukraine, a national methodology has been approved to predict the effects of chemical pollution on the environment [2]. In addition, in the areas where environmentally hazardous enterprises are located, specific engineering within nature complex systems are formed, which are characterized by certain trends of environmental changes, which sometimes lead to negative ecological and economic consequences. The material costs of restoring the natural equilibrium within such territories are usually extremely high. Therefore, the problem of creating adequate modeling techniques and forecasting the functioning of industrial enterprises to prevent accidents on them is a matter of first and foremost importance.

The review of existing approaches to forecasting the man-made (anthropogenic) risks. Predicting the state (condition) of the environment under the influence of dangerous man-made objects is becoming increasingly important when solving environmental problems associated with finding optimal forms of environmental safety management [3]. The most characteristic of the following tasks are the following:

• environmental monitoring;
• nature exploitation rationing;
• industrial sites environmental impact assessment.

One of the traditional approaches to predicting anthropogenic impact on the environment is the use of mathematical models that describe the processes and phenomena, which characteristic for the studied natural object [4]. The following methods are most often used in predicting the state of the environment:

• dynamic systems;
• time series (regressions);
• Markov models.

In mathematical modeling, let a natural object (water object, soil, stand, atmospheric air, etc.) be considered as a dynamic system containing $n$ components. In this case, the mathematical model of a natural object usually takes the form of a system of differential equations [5].

$$dY_i / dt = g_i(Y_1,...,Y_n,V_1,...,V_m,t),$$

where $Y = (Y_1,...,Y_n)$ is the vector that characterizes the state of the natural object; $V = (V_1,...,V_m)$ - vector of external factors that affect the state of a natural object; $t$ - time. The solution of the differential equation system is the functional dependencies $Y_i = Y_i(t)$ that allows predicting the state of a natural object.

In the absence of an adequate deterministic dynamic model, statistical forecasting methods are used. Among the statistical methods, the most common method for solving the environmental forecasting problem is regression analysis [6].

Suppose that an observation $y_i$ is the sum of a regular deterministic component and random interferences:

$$y_i = f_i(x_i, \alpha) + \varepsilon_i, \tag{1}$$

where $i = 1..N$ is the observation number; $x_i = (x_{i1}, x_{i2}, ..., x_{ik})$ vector of input factors; $f_i$ - regression function; $\alpha$ - unknown, generally multidimensional parameter, $\alpha \in \mathbb{R}^m$, $m \le N$; $\varepsilon_i$ – interferences (random variables) that have zero mathematical expectation, finite variance, and do not correlate with one another. Equation (1) is called the nonlinear regression model. The task is to evaluate an unknown parameter $\alpha$. As a method of estimation, the least squares method (LSM) is usually used, which leads to the optimization task $\sum_{i=1}^{N} (y_i - f_i(x_i, \alpha))^2 \to \min$. In this case, the value $\alpha$ that is the solution to this task is called an estimate under LSM.

The selected model is compared with the original data to check how accurately it describes the time series. A model is considered acceptable if the residuals are small and have a normal distribution.

Another statistical approach for modeling the behavior of these systems is based on stochastic modeling. Stochastic modeling does not use rigorous ratios, but expert and empirical evaluations and a universal mathematical apparatus. Stochastic modeling based on Markov finite-chain theory [7] has been successfully applied in various industries [8].

Suppose that the evolution of an ecosystem is described by the Markov chain. The transitions of the system from one state to another mean the moving a point that depicts the current state of the system from one set of phase space to another, and the corresponding system of phase space sets $A_j, j = 1..m$ is built on the basis of environmental standards. The transition probabilities matrix $P_{ij} = P\left\{\xi_{k+1} \in A_j / \xi_k \in A_i\right\}$, constructed on statistical information, where $\xi_k$ is the vector of system state on the time $t_k$, $\xi_{k+1}$ is the same vector at time $t_{k+1}$. Such a description allows you to solve at least three of the following tasks:

1. To determine system transition probabilities $P_{ij}^{(n)}$ from the state $A_i$ to the state $A_j$ in $n$ steps.

2. To find the vector of probabilities $P^{(n)}(B)$ of the system being in all possible states of the set $B$ in $n$ steps, if the state of the system is known at the initial moment.

3. For the specific states of the system, to determine the probabilities of getting into them in no more than $n$ steps and stationary probabilities, which allow to determine the measure (portion) of time that the system is in these states.

The main disadvantage of existing forecasting methods is the inability to estimate the average residence time of a system in one or another set of states and the lack of attention to economic effect of its evolution.

Formulation of the task to model the behavior of engineering-within-nature complex systems. The models based on Markov finite chains have the following characteristics:

• simplicity of the content;

• actual environmental standards and regulations are naturally taken into account, since the phase space is built on the basis of current environmental legislation;

• the possibility of reducing the set of estimated parameters to the elements of the transition matrix.

It should also be noted that the use of such an interpretation of the system evolutions eliminates the need to determine the distributions of random variables and processes that determine the state of the system.

The practical application of such models, due to the existing advanced theory of Markov chains allows us to use the following criteria of optimality:

• to minimize the probabilities of system states, which are extraordinary situations, in the steady state distribution of the respective chain;

• to maximize the average time of reaching the respective state;

• to minimize the damage caused by the system being in "ecologically disadvantaged" states;

• to maximize the economic impact of the system's operation, taking into account both profits from industrial sites, positive social shifts as well as losses related to environmental damages.

The purpose of this work is to develop a methodology for predicting the occurrence of ecological threats, to study the distribution of time spent by the engineering-within-nature complex system in safe and in unfavorable (unsafe) states, ecological and economic analysis of the consequences of its evolution.

Methods of forecasting and optimizing the economic effect on a discrete set of strategies. Let the system is responsible to control the territories, which represent an amalgamation of zones (regions) that will in future be considered as non-intersecting. The ecosystem phase space $\Omega$ is a direct product of $\Omega_l$, where $\Omega_l$ is the set of all possible ordered sets of concentrations of harmful substances in the air and water environments of the $l$-th region of controlled area. The model of each of these zones is a corresponding Markov chain [9]. The natural modification (version) of the Markov property for the situation under consideration is the following:

$$P(T_j \in A_k / T_1 \in A^{(1)},...,T_m \in A^{(m)}) = P(T_j \in A_k / T_{j1}^{(1)} \in A^{(j1)},...,T_{jr}^{(r)} \in A^{(jr)})$$

that is, the probability of finding the $j$-th zone in the $k$-th state $A_k$ is determined by the states $A^{(j1)},...,A^{(jr)}$, in which the adjacent zones $T_{j1}^{(1)},...,T_{jr}^{(r)}$ locate. Accordingly, statistical studies of the transition probabilities [10] for each zone should include a study of their dependencies on the "configuration" of the environmental situation in the adjacent zones. Note that in some cases there is even a deterministic dependence between the states of adjacent zones with a certain time lag. This is the case, for example, for the condition of air basins in the case of steady air currents, for water basins of zones settled sequentially downstream. Thus, the happening of an emergency even in one region (or vice versa, the normalization of the ecological situation in it) requires a consistent recalculation of transition probability matrices for the entire controlled area. It should be noted that such calculations often lead to controversial results (we obtain different transition matrices for the same region). To eliminate these contradictions, it is suggested that:

• to create, on the basis of statistical studies, a bank of scenarios that may occur in each zone;

• calculate time lags for interregional effects for each scenario;

• operational management shall be carried out by using standard scenarios with simultaneous control of their adequacy to the real situation.

It should be noted, that the forms of interdependence between transition matrices for adjacent zones can be calculated in two ways:

• based on available statistics;

• on the basis of a correspondence model [11], based on the available information about air currents, dynamics of water reservoirs, etc.

This requires:

• the previous choice of the most likely way of forming such a dependency related to the accumulated information;

• statistical control of incoming information in terms of selecting the most likely hypothesis according to its available dynamics.

The first question to be solved when using this model is the question of discreteness or continuity of time (in the evolution of a chain). The choice of Markov chains with discrete time is explained by not only available practical experience of using them [9], but also to the objectively available periodicity of information inflow into the control system from its primary links.

For discrete-time chains, the problem of homogeneity arises. Homogeneous chains are much easier to investigate, but applying them requires a solid statistical justification, which is not always possible. At the same time, it is natural to use several homogeneous models adapted to the operating modes of the enterprises and the seasons. Thus, one task of considering inhomogeneous chains can be replaced by several tasks of analyzing the corresponding homogeneous chains and developing an algorithm of transition from one of them to another depending on the current state.

The next question is the choice of the phase space structure (set of states) of the chain (Fig. 1). Its solution is based on a legislative base (framework) that determines the levels of environmental pollution and the need to consider such levels for each of the regions that form the controlled system. At the same time, phase space, constructed as a direct product of "local" phase spaces, which in turn duplicate the levels of pollution in the respective zones, is, on the one hand, too cumbersome (in the presence of $m$ regulated levels of pollution and $n$ zones it contains $m^n$ states) and, at the same time, is not always adequate from the point of view of the system description, since it does not contain clearly identified states of "threatening". From this point of view, it is advisable to isolate "transient" states in some zones, which would mean the approaching to the critical levels of pollution, or vice versa - the tendency to decrease them. At the same time, based on economic (potential loss) and other considerations, the primary phase space containing $(m^*)^n$ ($m^*$ - the number of levels of contamination, taking into account the abovementioned additional) phase space should be enlarged (i.e. to combine several states into one).

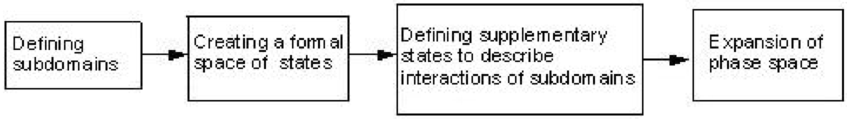


Fig. 1. Building a graph of the system states

After the final formation of the phase space (which may be different under the solving the problems of economic, ecological, political, social, technological and other directions) and determine the corresponding transition matrix, one can classify its states (Fig. 2).

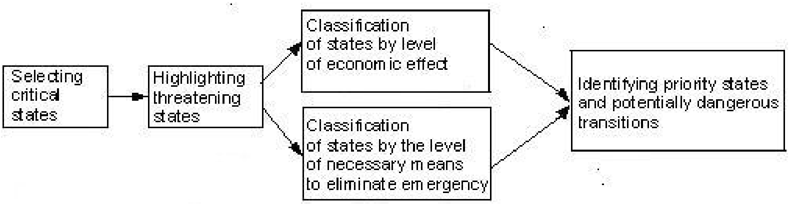


Fig. 2. Defining the basic characteristics of the system states

The opportunity of decomposing the phase space into classes of achievability, that is constructing the states graph of the chain (Fig. 3), requires in-depth analysis. By providing a high probability of defining the "starting" state of the system, it is possible to limit yourself to a chain that is non-decomposable.

Formal (mathematical) distribution of absorbing states of the system should be consistent with their character as states of ecological catastrophe resulting from the limited resources of natural environmental purification. For such states, they determine the average time to achieve them and the probability of achieving them no earlier than a fixed time. If in this case for a random time $\tau$ of achieving of one of such states we have $M[\tau]<\tau_0$, or for the probability $p^*$ of getting to such a state not earlier than a certain time, an inequality $p^* < p_0$ is true, where $\tau_0$, $p_0$ pre-set values, the system is considered as highly-insecure and needs structural changes (Fig. 4).

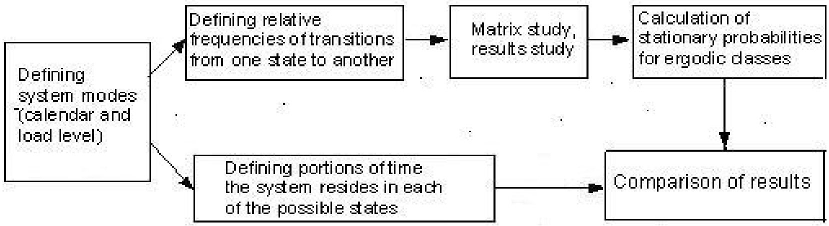


Fig. 3. Finding the transient probabilities of the Markov chain for non-critical states

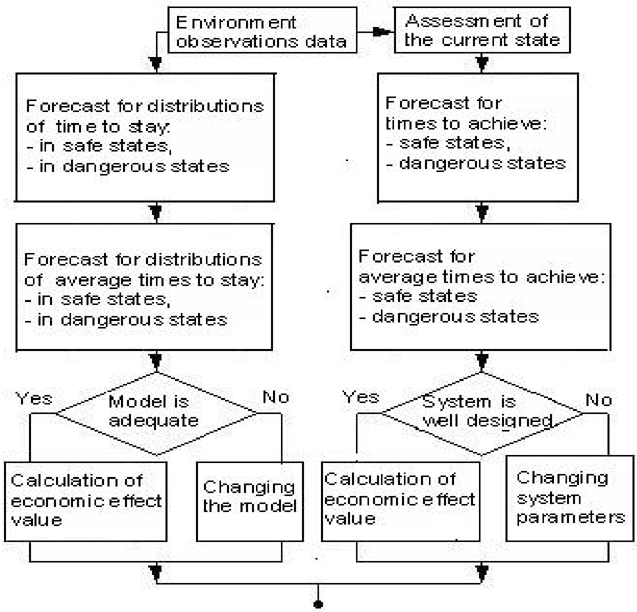


Fig. 4. Pattern cycle of checking the model adequacy and system quality

For each of the other states of the chain, along with its probabilistic characteristics, environmental and economic parameters are also considered: achievable economic effect, permissibility of the corresponding environmental pollution, possibility of exploitation of natural resources and technological objects. This allows not only to set the "rating" of the states of the system from the most desirable to the extremely undesirable, but also to set optimization tasks for it, based on the possibilities of choosing the initial state and influencing the matrix of transient probabilities [10]. Given the real characteristics of the system under consideration, it is proposed to use Markov periodic chains to model its behavior [12], which gives an additional opportunity to characterize and detect emergencies as a "deviation from periodicity" in the behavior of the system.

Another feature of the proposed methodology is the optimization of a discrete set of strategies based on a set of scenarios, which allows to avoid purely mathematical problems in solving extreme (and in some cases variational) tasks due to the really existing system of industrial installations (machinery) operating modes.

The paper proposes a methodology that allows to combine economic estimates with the ability to predict the situation and optimize decision making to improve the environmental situation in areas potentially exposed to chemical pollution.

The stages of implementation of the methodology are as follows:

1. By using available transition matrices for the chains that describe the environmental situation in the zones, they identify such modes of operation of technological systems that, during a predetermined period (no more than $N$ transitions), can lead to situations that are classified as extremely dangerous, in at least one of the regions.
2. For situations that are considered unfavorable, a mode of operation is selected in which the weighted sum of the probabilities of their achieving (attaining) is minimized by no more than $M$ transitions or stationary probabilities of these states for a particular chain. The coefficients in such a weighted sum shall be chosen on the basis of the need to ensure that the expected average level of contamination for each of its components is properly restrained:

$$\begin{cases} \alpha_{11} \cdot p_1 + ... + \alpha_{1l} \cdot p_l \le s_1 \\ ... \\ \alpha_{k1} \cdot p_1 + ... + \alpha_{kl} \cdot p_l \le s_k \end{cases}.$$

3. After performing the previous steps for each of the zones, the optimization is performed, according to the criterion of the maximum distance of the system from the dangerous level of contamination, i.e. choosing such mode of its operation, under which

$$T = ET(f_1, ..., f_n) \to \max,$$

where $T(f_1, ..., f_n)$ is the time for the value $A_1 \cdot f_1 + ... + A_n f_n$ to reach the critical level $F$, $f_1, ..., f_n$ – the characteristics of the industrial pollution of the zones, $A_1, ..., A_n$ – the weighting coefficients.
4. If the previous task is solved, but there are a number of strategies, when applied, haven't provided the results almost indistinguishable from the optimal one, then for these strategies they shall calculate: a) the average amount of harm from the system being in environmentally hazardous states; b) the average economic effect of the system operation, taking into account both profits from the operation of industry, positive socio-economic shifts, as well as the losses described above. It is clear that comparing the outcome of choosing one of the strategies, in this case, is necessary for decision making at the government level.

The control of the system can be carried out both by changing the phase space (revision of norms and levels of pollution) and by changing the matrix of transient probabilities, both of which can be combined in series. It should be noted that changing the transition probability matrix can require significant investment (e.g., improved reliability of industrial installations, reduced emissions levels, etc.), which should be taken into account when making appropriate management decisions.

**Conclusions.** The article proposes a technique for forecasting emergencies that may occur when a nature & engineering complex system operates. The use of the methodology presented above will allow to increase the efficiency of functioning of enterprises, generate the balanced informed management decisions and to create software and technologies to respond the emergencies.

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*Стаття надійшла 21.01.2020*

*Getun G., Butsenko Y., Labzhinsky V., Balina O., Bezklubenko I., Solomin A.*

**SITUATION FORECASTING AND DECISION-MAKING OPTIMIZATION BASED ON USING MARKOV FINITE CHAINS FOR AREAS WITH INDUSTRIAL POLLUTIONS**

The paper considers the issues of predicting the situations and optimizing decision-making to improve the environmental situations in the areas with industrial pollution based on the finite Markov's chains.

The article systematizes the existing approaches to forecasting technological risks. The problems associated with the search for optimal forms of environmental safety management and approaches for predicting anthropogenic impact on the environment using mathematical models are considered. To predict the state of the environment, stochastic modeling is proposed, the basis of which is the theory of finite Markov chains. A technique for predicting and optimizing the economic effect on a discrete set of strategies has been developed. The figures show: building system states graph, determining the basic characteristics of system states, finding transition probabilities of Markov chains for non-critical states, a typical cycle of checking the model's adequacy and system quality.

Based on the analysis of existing approaches to forecasting technological risks, a methodology has been developed for forecasting and optimizing the economic effect on a discrete set of strategies. The proposed methodology allows combining economic estimates with the ability to predict the situations and optimize decision-making to improve the environmental situation in the areas of possible chemical pollution.

Using the developed methodology will increase the efficiency of the industrial enterprises, facilitate generating informed management decisions, create software and hardware ways to respond the emergencies.

The methodology for modeling engineering within nature complex systems and the optimization of decision-making based on finite Markov chains in the areas with industrial pollution will be helpful to researchers and operators of complex technical systems in predicting emergencies using environmental monitoring systems.

**Keywords:** complex technical systems, engineering-within-nature complex systems, nature & engineering complex systems, environmental monitoring, emergency forecasting, statistical research, decision support.

УДК 681.518.5

*Гетун Г., Буценко Ю., Лабжинський В., Баліна О., Безклубенко І., Соломін А.* **Прогнозування ситуації та оптимізація прийняття рішень на основі використання кінцевих ланцюжків Маркова для районів з промисловими забрудненнями** // Опір матеріалів та теорія споруд: Наук.-техн. збірник. - К.: КНУБА, 2020. – Вип. 104. - С. 164-174.

*У статті пропонується методологія моделювання комплексних систем інженерії (далі, «системи»), яка буде корисною для дослідників та операторів складних технічних систем при прогнозуванні надзвичайних ситуацій за допомогою систем моніторингу навколишнього середовища.*

*Getun G., Butsenko Y., Labzhinsky V., Balina O., Bezklubenko I., Solomin A.* **Situation forecasting and decision-making optimization based on using markov finite chains for areas with industrial pollutions** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 164-174.

*The paper proposes a methodology for modeling engineering-within-nature complex systems (further, "systems"), which will be helpful for researchers and operators of complex technical systems in predicting the emergencies using environmental monitoring systems.*

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UDC 621.87

# MATHEMATICAL MODEL OF THE DYNAMICS CHANGE DEPARTURE OF THE JIB SYSTEM MANIPULATOR WITH THE SIMULTANEOUS MOVEMENT OF ITS LINKS

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DOI: 10.32347/2410-2547.2020.104. 175-190

An equation of motion of the manipulator is obtained taking into account the influence of the inertial component of each link of the boom system and the effect of the oscillatory movement of the cargo on the dynamic loads of the metalware elements and hydraulic drive elements. The influence of the simultaneous movement of the first jib section, the second jib section and the telescopic jib section on cargo oscillation, as well as the effect of cargo oscillation on dynamic loads that occur in the boom system and manipulator hydraulic drive elements, is determined.

**Keywords:** mathematical model, varying the radius, combination of movements, manipulator, Lagrange equations of the second kind, dynamic loads, load oscillations.

## Introduction

During the process of unloading and loading operations in the elements of the boom system and the elements of the drive of the manipulator considerable dynamic loads occur. These loads are the result of the oscillatory movement of the load and the uneven rotation of the boom system with the uniform movement of the rods of the hydraulic cylinders [1-2]. Dynamic loads depend on the kinematic parameters of the manipulator and the nature of the speed of movement of the links of the boom system with the cargo. According to the normative-technical documentation, which regulates the operation of manipulators, it is allowed to combine operations of simultaneous movement of several links of the boom system. Combining the operations of the simultaneous movement of the links of the boom system can significantly reduce the dynamic loads and accordingly increase the performance, reliability of the elements of the boom system and the hydraulic equipment of the manipulator. To determine the actual dynamic loads in the elements of the design of the manipulator when combined movements of the links of the boom system, it is necessary to have adequate mathematical models [3-8].

**Analysis of publications**

Known [3-11] methods for constructing a mathematical model of the manipulator. In these works, the boom system of the manipulator is presented as a holonomic mechanical system in which the centres of gravity of the links of the metal structure coincides with their geometrical parameters. The mathematical model of the manipulator is considered, the relation between the kinematic dependences of the drive link of the manipulator and the load. The influence of dynamic loads on the elements of metalwork of the boom system of the manipulator is analyzed. In the papers [12-15], an analysis of the solution of optimization problems for reducing load oscillations is considered. The analysis of the influence of dynamic loads on the elements of the boom system and the hydraulic drive is considered in the papers [16-19]. With a large amount of consideration of the problem of dynamic analysis of the combination of simultaneous movement of the links of the boom system, the solution of this problem for manipulators with hydraulic drive is not considered taking into account the load fluctuations at the end of the boom system.

**Purpose and research task statement**

The purpose of this work is to build a mathematical model of the dynamics of change of departure of the boom system of the manipulator when combining operations of simultaneous movement of the first jib section, movement of the second jib section and movement of the telescopic jib section with cargo oscillation at the end of the boom system.

**Research results**

When investigating the dynamics of change of departure of the boom system of the manipulator with simultaneous movement of the first jib section, movement of the second jib section, moving the telescopic jib section and the cargo oscillations, we accept the following assumptions:

- we believe that all links in the boom system are perfectly rigid except for the cargo, which oscillates in the plane of change of departure;
- semi-dry friction in moving elements of eye joint and viscous friction of fluid in pipelines is not taken into account;
- the compressibility of the working fluid in the elements of the hydraulic drive is not taken into account.

Based on the above assumptions, the boom system of the manipulator in the process of changing the departure of the cargo with the combination of the three main movements and oscillates of the cargo is presented as a holonomic mechanical system with four degrees of freedom. The angular and linear coordinates of the moving of the first jib section, the second jib section and the telescopic jib section are calculated from the $x$ axis, and the angular coordinate of the cargo deviation from the $y$ axis, (Fig. 1). For the generalized coordinates of the boom system we take the angular coordinates: rotate the first jib section, rotate the second jib section, the linear coordinate of movement of the telescopic jib section, and the vertical deviation of the cargo (Fig. 1).

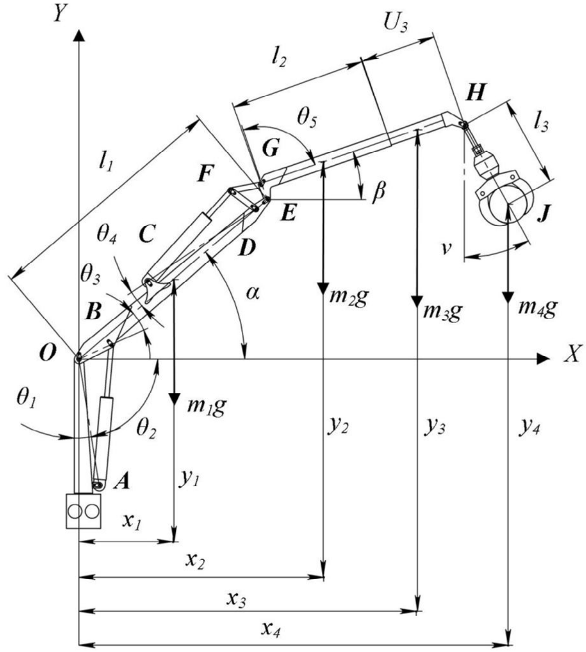



Fig. 1. Dynamic model of boom system of the manipulator

In Fig. 1 accepted the following designations: $l_1$ - length of the first jib section; $l_2$ - length of the second jib section; $l_3$ - the length of the suspension; $m_1, m_2, m_3, m_4$ - weight respectively of the first boom section, the second boom section, the telescopic section and the cargo; $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ - angles formed by the geometrical parameters of the elements of the boom system and the hydraulic cylinders of the manipulator; $x_1, x_2, x_3, x_4$ - horizontal coordinates of the centers of mass of the first jib section, the second jib section, the telescopic jib section and the cargo; $y_1, y_2, y_3, y_4$ - vertical coordinates of the centers of mass of the first jib section, the second jib section, the telescopic jib section and the cargo.

Expressed of the coordinates of the centres of mass for the first jib section, the second jib section, the telescopic jib section and the cargo in the generalized coordinates:

$$\begin{cases} x_1 = \frac{l_1}{2}\cos(\alpha); \\ y_1 = \frac{l_1}{2}\sin(\alpha); \end{cases} \tag{1}$$

$$\begin{cases} x_2 = l_1 \cdot \cos(\alpha) + \dfrac{l_2}{2} \cdot \cos(\beta); \\ y_2 = l_1 \cdot \sin(\alpha) + \dfrac{l_2}{2} \cdot \sin(\beta); \end{cases} \qquad (2)$$

$$\begin{cases} x_3 = l_1 \cdot \cos(\alpha) + \left( \dfrac{l_2}{2} + U_3 \right) \cdot \cos(\beta) + l_3 \cdot \sin(\nu); \\ y_3 = l_1 \cdot \sin(\alpha) + \left( \dfrac{l_2}{2} + U_3 \right) \cdot \sin(\beta) - l_3 \cdot \cos(\nu). \end{cases} \qquad (3)$$

$$\begin{cases} x_4 = l_1 \cdot \cos(\alpha) + (l_2 + U_3) \cdot \cos(\beta) + l_3 \cdot \sin(\nu); \\ y_4 = l_1 \cdot \sin(\alpha) + (l_2 + U_3) \cdot \sin(\beta) - l_3 \cdot \cos(\nu). \end{cases} \qquad 4)$$

To compile the equations of motion of the manipulator in the process of changing the departure of the boom system with the cargo, while simultaneously moving the first jib section, the second jib section, the telescopic jib section and the cargo, we use the second-order Lagrange equations, which for the system shown in Fig. 1, have the form:

$$\begin{cases} \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{\alpha}} - \dfrac{\partial T}{\partial \alpha} = Q_\alpha - \dfrac{\partial V}{\partial \alpha}; \\ \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{\beta}} - \dfrac{\partial T}{\partial \beta} = Q_\beta - \dfrac{\partial V}{\partial \beta}; \\ \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{U}_3} - \dfrac{\partial T}{\partial U_3} = Q_{U_3} - \dfrac{\partial V}{\partial U_3}; \\ \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{\nu}} - \dfrac{\partial T}{\partial \nu} = Q_\nu - \dfrac{\partial V}{\partial \nu}, \end{cases} \qquad (4)$$

where: $t$ - time; $T, V$ - respectively, the kinetic and potential energy of the boom system of the manipulator; $Q_\alpha, Q_\beta, Q_{U_3}, Q_\nu$ - non-conservative components of the generalized forces of the system corresponding to the generalized coordinates $\alpha, \beta, U_3, \nu$.

Then the kinetic energy of the boom system of the manipulator will take the form:

$$T = \frac{1}{2} \cdot J_1 \cdot \dot{\alpha}^2 + \frac{1}{2} \cdot m_2 \cdot (\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} \cdot J_2 \cdot \dot{\beta}^2 + \frac{1}{2} \cdot m_3 \cdot (\dot{x}_3^2 + \dot{y}_3^2) + \\ + \frac{1}{2} \cdot J_3 \cdot \dot{\beta}^2 + \frac{1}{2} \cdot m_4 \cdot (\dot{x}_4^2 + \dot{y}_4^2), \qquad (5)$$

where: $m_2, m_3, m_4$ - the masses respectively of the second jib section, the telescopic jib section and the cargo; $J_1, J_2, J_3$ - moments of inertia respectively of the first jib section, the second jib section and the telescopic jib section (Fig. 1).

The potential energy of the boom system of the manipulator in the process of change of departure will be as follows:

$$V = (m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4) \cdot g, \qquad (6)$$

where $g$ - free fall acceleration.

Take the derivatives of kinetic energy (5) that are included in the system of equations (4):

$$\frac{\partial T}{\partial \alpha}=m_2\left(\dot{x}_2\frac{\partial \dot{x}_2}{\partial \alpha}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial \alpha}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial \alpha}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \alpha}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial \alpha}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \alpha}\right);$$

$$\frac{\partial T}{\partial \beta}=m_2\left(\dot{x}_2\frac{\partial \dot{x}_2}{\partial \beta}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial \beta}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial \beta}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \beta}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial \beta}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \beta}\right);$$

$$\frac{\partial T}{\partial U_3}=m_2\left(\dot{x}_2\frac{\partial \dot{x}_2}{\partial U_3}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial U_3}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial U_3}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial U_3}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial U_3}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial U_3}\right);$$

$$\frac{\partial T}{\partial v}=m_2\left(\dot{x}_2\frac{\partial \dot{x}_2}{\partial v}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial v}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial v}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial v}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial v}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial v}\right);$$

$$\frac{\partial T}{\partial \dot{\alpha}}=J_1\cdot\dot{\alpha}+m_2\left(\dot{x}_2\frac{\partial x_2}{\partial \alpha}+\dot{y}_2\frac{\partial y_2}{\partial \alpha}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial \alpha}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \alpha}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial \alpha}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \alpha}\right);$$

$$\frac{\partial T}{\partial \dot{\beta}}=(J_2+J_3)\cdot\dot{\beta}+m_2\left(\dot{x}_2\frac{\partial x_2}{\partial \beta}+\dot{y}_2\frac{\partial y_2}{\partial \beta}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial \beta}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \beta}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial \beta}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \beta}\right);$$

$$\frac{\partial T}{\partial \dot{U}_3}=m_2\left(\dot{x}_2\frac{\partial x_2}{\partial U_3}+\dot{y}_2\frac{\partial y_2}{\partial U_3}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial U_3}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial U_3}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial U_3}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial U_3}\right);$$

$$\frac{\partial T}{\partial \dot{\nu}}=m_2\left(\dot{x}_2\frac{\partial \dot{x}_2}{\partial \nu}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial \nu}\right)+m_3\left(\dot{x}_3\frac{\partial \dot{x}_3}{\partial \nu}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \nu}\right)+m_4\left(\dot{x}_4\frac{\partial \dot{x}_4}{\partial \nu}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \nu}\right);$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}}=J_1\cdot\ddot{\alpha}+m_2\left(\ddot{x}_2\frac{\partial x_2}{\partial \alpha}+\dot{x}_2\frac{\partial \dot{x}_2}{\partial \alpha}+\ddot{y}_2\frac{\partial y_2}{\partial \alpha}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial \alpha}\right)+$$
$$+m_3\left(\ddot{x}_3\frac{\partial x_3}{\partial \alpha}+\dot{x}_3\frac{\partial \dot{x}_3}{\partial \alpha}+\ddot{y}_3\frac{\partial y_3}{\partial \alpha}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \alpha}\right)+m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial \alpha}+\dot{x}_4\frac{\partial \dot{x}_4}{\partial \alpha}+\ddot{y}_4\frac{\partial y_4}{\partial \alpha}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \alpha}\right);$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\beta}}=(J_2+J_3)\cdot\ddot{\beta}+m_2\left(\ddot{x}_2\frac{\partial x_2}{\partial \beta}+\dot{x}_2\frac{\partial \dot{x}_2}{\partial \beta}+\ddot{y}_2\frac{\partial y_2}{\partial \beta}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial \beta}\right)+$$
$$+m_3\left(\ddot{x}_3\frac{\partial x_3}{\partial \beta}+\dot{x}_3\frac{\partial \dot{x}_3}{\partial \beta}+\ddot{y}_3\frac{\partial y_3}{\partial \beta}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \beta}\right)+m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial \beta}+\dot{x}_4\frac{\partial \dot{x}_4}{\partial \beta}+\ddot{y}_4\frac{\partial y_4}{\partial \beta}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \beta}\right);$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{U}_3}=m_2\left(\ddot{x}_2\frac{\partial x_2}{\partial U_3}+\dot{x}_2\frac{\partial \dot{x}_2}{\partial U_3}+\ddot{y}_2\frac{\partial y_2}{\partial U_3}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial U_3}\right)+m_3\left(\ddot{x}_3\frac{\partial x_3}{\partial U_3}+\dot{x}_3\frac{\partial \dot{x}_3}{\partial U_3}+\right.$$
$$\left.+\ddot{y}_3\frac{\partial y_3}{\partial U_3}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial U_3}\right)+m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial U_3}+\dot{x}_4\frac{\partial \dot{x}_4}{\partial U_3}+\ddot{y}_4\frac{\partial y_4}{\partial U_3}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial U_3}\right);$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\nu}}=m_2\left(\ddot{x}_2\frac{\partial x_2}{\partial \nu}+\dot{x}_2\frac{\partial \dot{x}_2}{\partial \nu}+\ddot{y}_2\frac{\partial y_2}{\partial \nu}+\dot{y}_2\frac{\partial \dot{y}_2}{\partial \nu}\right)+m_3\left(\ddot{x}_3\frac{\partial x_3}{\partial \nu}+\dot{x}_3\frac{\partial \dot{x}_3}{\partial \nu}+\right.$$
$$\left.+\ddot{y}_3\frac{\partial y_3}{\partial \nu}+\dot{y}_3\frac{\partial \dot{y}_3}{\partial \nu}\right)+m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial \nu}+\dot{x}_4\frac{\partial \dot{x}_4}{\partial \nu}+\ddot{y}_4\frac{\partial y_4}{\partial \nu}+\dot{y}_4\frac{\partial \dot{y}_4}{\partial \nu}\right).$$

Take partial derivatives of potential energy (6):

$$\frac{\partial V}{\partial \beta}=\left(m_2\frac{\partial y_2}{\partial \beta}+m_3\frac{\partial y_3}{\partial \beta}+m_4\frac{\partial y_4}{\partial \beta}\right)g;$$

$$\frac{\partial V}{\partial \alpha}=\left(m_1\frac{\partial y_1}{\partial \alpha}+m_2\frac{\partial y_2}{\partial \alpha}+m_3\frac{\partial y_3}{\partial \alpha}+m_4\frac{\partial y_4}{\partial \alpha}\right)g;$$

$$\frac{\partial V}{\partial U_3}=\left(m_3\frac{\partial y_3}{\partial U_3}+m_4\frac{\partial y_4}{\partial U_3}\right)g;$$

$$\frac{\partial V}{\partial v}=m_4\frac{\partial y_4}{\partial v}\cdot g. \tag{8}$$

The variation of the displacement of the rods of the hydraulic cylinders is expressed by the variation of the generalized coordinates:

$$Q_\alpha=F_1\frac{\partial U_1}{\partial \alpha}+F_2\frac{\partial U_2}{\partial \alpha};$$

$$Q_\beta=F_2\frac{\partial U_2}{\partial \beta};$$

$$Q_{U_3}=F_3. \tag{8}$$

where: $F_1$ - efforts in lifting the first jib section; $F_2$ - efforts in the hydraulic cylinder of the second jib section; $F_3$ - efforts in the hydraulic cylinder to move the telescopic jib section.

Substituting expressions (7 - 9) into the system of equations (4), we obtain a system of differential equations of motion of the manipulator in the process of changing the departure of the boom system with the cargo when the three main motions of the boom system are combined:

$$\begin{cases} J_1\ddot{\alpha}+m_2\left(\ddot{x}_2\frac{\partial x_2}{\partial \alpha}+\ddot{y}_2\frac{\partial y_2}{\partial \alpha}\right)+m_3\left(\ddot{x}_3\frac{\partial x_3}{\partial \alpha}+\ddot{y}_3\frac{\partial y_3}{\partial \alpha}\right)+m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial \alpha}+\right.\\ \left.+\ddot{y}_4\frac{\partial y_4}{\partial \alpha}\right)=F_1\frac{\partial U_1}{\partial \alpha}+F_2\frac{\partial U_2}{\partial \alpha}-\left(m_1\frac{\partial y_1}{\partial \alpha}+m_2\frac{\partial y_2}{\partial \alpha}+m_3\frac{\partial y_3}{\partial \alpha}+m_4\frac{\partial y_4}{\partial \alpha}\right)g;\\ (J_2+J_2)\ddot{\beta}+m_2\left(\ddot{x}_2\frac{\partial x_2}{\partial \beta}+\ddot{y}_2\frac{\partial y_2}{\partial \beta}\right)++m_3\left(\ddot{x}_3\frac{\partial x_3}{\partial \beta}+\ddot{y}_3\frac{\partial y_3}{\partial \beta}\right)+\\ +m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial \beta}+\ddot{y}_4\frac{\partial y_4}{\partial \beta}\right)=-F_2\frac{\partial U_2}{\partial \beta}-\left(m_2\frac{\partial y_2}{\partial \beta}+m_3\frac{\partial y_3}{\partial \beta}+m_4\frac{\partial y_4}{\partial \beta}\right)g;\\ m_3\left(\ddot{x}_3\frac{\partial x_3}{\partial U_3}+\ddot{y}_3\frac{\partial y_3}{\partial U_3}\right)+m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial U_3}+\ddot{y}_4\frac{\partial y_4}{\partial U_3}\right)=-F_3-\left(m_3\frac{\partial y_3}{\partial U_3}+m_4\frac{\partial y_4}{\partial U_3}\right)g;\\ m_4\left(\ddot{x}_4\frac{\partial x_4}{\partial v}+\ddot{y}_4\frac{\partial y_4}{\partial v}\right)=-m_4\frac{\partial y_4}{\partial v}g. \end{cases} \tag{10}$$

Find the coordinates of the driving mechanisms that are part of the system of equations (11).

$$AB = U_1 = \sqrt{AO^2 + OB^2 - 2 \cdot AO \cdot OB \cdot \cos(\theta_2 + \alpha - \theta_3)}. \qquad (11)$$

$$CF = U_2 = \sqrt{CD^2 + DF^2 - 2 \cdot CD \cdot DF \cdot \cos(\angle CDF)}. \qquad (12)$$

To determine, $\angle CDF$ first consider the four link mechanism *EDFG* (Fig. 2), and define the diagonal *DG*:

$$DG = \sqrt{EG^2 + DE^2 - 2 \cdot EG \cdot DE \cdot \cos(\angle DEG)}. \qquad (13)$$

$$\angle DEG = \pi - (\theta_5 + \beta - \alpha). \qquad (14)$$

After substitution of expression (13) in dependence (14) we obtain:

$$DG = \sqrt{EG^2 + DE^2 - 2 \cdot EG \cdot DE \cdot \cos(\theta_5 + \beta - \alpha)}. \qquad (15)$$

Using the sine theorem, we write:

$$\frac{Sin\angle DEG}{DG} = \frac{Sin\angle EDG}{EG}. \qquad (16)$$

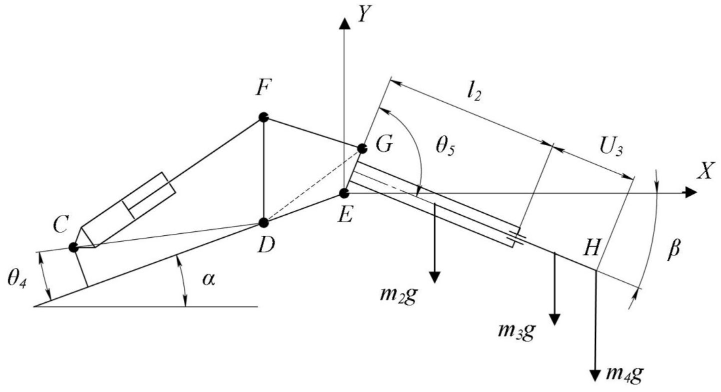



Fig. 2. The kinematic scheme of the drive of the second jib section

From equation (16) we find:

$$\angle ADE = ArcSin\left(\frac{EG \cdot \sin(\theta_5 + \beta - \alpha)}{DG}\right). \qquad (17)$$

We find the angle $\angle FDG$ from the expression:

$$FG^2 = DF^2 + DG^2 - 2 \cdot DF \cdot DG \cdot \cos\angle FDG. \qquad (18)$$

Then:

$$\angle FDG = \arccos\left(\frac{DG^2 + DF^2 - FG^2}{2 \cdot DF \cdot DG}\right). \qquad (19)$$

Adding expressions (18) and (20), we find the angle $\angle EDF$ :

$$\angle EDF = \arcsin\left(\frac{EG \cdot \sin(\theta_5 + \beta - \alpha)}{DG}\right) + \arccos\left(\frac{DG^2 + DF^2 - FG^2}{2 \cdot DF \cdot DG}\right). \qquad (20)$$

Now can find the angle $\angle CDF$ :

$$\angle CDF = \pi - \angle EDF - \theta_4. \tag{21}$$

After substitution of expression (21) in dependence (12) we obtain:

$$U_2 = \sqrt{CD^2 + DF^2 + 2 \cdot CD \cdot DF \cdot \cos\left(\angle EDF - \theta_4\right)}. \tag{22}$$

To move the links of the boom system, hydraulic cylinders develop driving forces, which are determined by mechanical characteristics.

The characteristics are presented in the form of quadratic relationships between the acting forces and the displacement rates of the rods of the hydraulic cylinders:

$$F_1 = P_n \cdot A_1 \cdot \sqrt{1 - \frac{A_1 \cdot \dot{U}_1}{Q_1}},$$

$$F_2 = P_n \cdot A_2 \cdot \sqrt{1 - \frac{A_2 \cdot \dot{U}_2}{Q_2}},$$

$$F_3 = P_n \cdot A_3 \cdot \sqrt{1 - \frac{A_3 \cdot \dot{U}_3}{Q_3}}, \tag{23}$$

where: $P_n$ - fluid pressure in the hydraulic system; $A_1, A_2, A_3$ - respectively, the piston area of the hydraulic cylinders of the first jib section, the second jib section and the telescopic jib section; $\dot{U}_1, \dot{U}_2, \dot{U}_3$ according the speed of the cylinder rods of the first jib section, the second jib section and the telescopic section. The flow of the working fluid flowing through the hydraulic distributor to provide the hydraulic cylinders with the required start mode and speed of movement of the boom system is determined by the following dependencies, respectively, for the hydraulic cylinder of the first jib section, the second jib section and the telescopic jib section:

$$Q_1 = \mu \cdot f_1 \cdot \sqrt{\frac{2 \cdot \Delta P_1}{\rho}};$$

$$Q_2 = \mu \cdot f_2 \cdot \sqrt{\frac{2 \cdot \Delta P_2}{\rho}};$$

$$Q_3 = \mu \cdot f_3 \cdot \sqrt{\frac{2 \cdot \Delta P_3}{\rho}}, \tag{24}$$

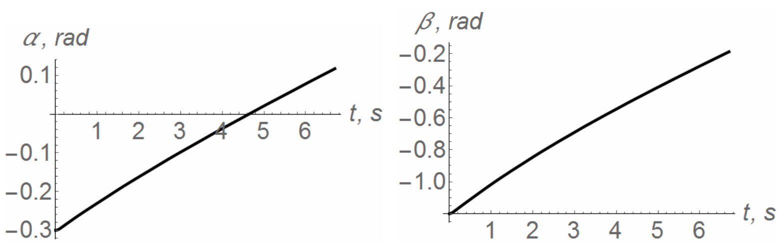
where: $\Delta P_1, \Delta P_2, \Delta P_3$ - respectively the pressure drop in the cylinders; $\mu$ - coefficient of consumption of working fluid; $f_1, f_2, f_3$ - according of the area cross-sectional of the hydraulic distributor; $\rho$ - the specific gravity of the liquid.

To calculate the dynamics of change of departure of the boom system of the manipulator we use the following output parameters: $m_1 = 350$ kg, $m_2 = 155$ kg, $m_3 = 65$ kg, $m_4 = 500$ kg, $l_1 = 4$ m, $l_2 = 2$ m, $l_3 = 0{,}8$ m, $AO = 1{,}6$ m, $OB = 0{,}5$ m, $CD = 1{,}6$ m, $DF = 0{,}425$ m, $FG = 0{,}425$ m, $EG = 0{,}425$ m,

$DE = 0,255\,\text{m}$, $P_n = 20\cdot 10^6\,\text{Pa}$, $A_1 = 0,012265\,\text{m}^2$, $A_2 = 0,00915\,\text{m}^2$, $A_3 = 0.00185\ \text{m}^2$, $\theta_1 = 0,192$ rad, $\theta_2 = 1,378$ rad, $\theta_3 = 0,384$ rad, $\theta_4 = 0,157$ rad, $\theta_5 = 1,57$ rad, $\rho = 850\,\text{kg/m}^3$. Initial conditions of manipulator movement: $\alpha[0] = -0.3$, $\dot{\alpha}[0] = 0$, $\beta[0] = -1.2$, $\dot{\beta}[0] = 0$, $U_3[0] = 1$, $\dot{U}_3[0] = 0$, $\nu[0] = 0$, $\dot{\nu}[0] = 0$.

Substituting the initial parameters and initial conditions into the system of equations (10) and solving it, the graphical dependences of the kinematic and force characteristics of the manipulator with the cargo are determined and constructed (Fig. 3 - Fig. 7). The following assumptions were made when solving the equation system:

- switching time of the hydraulic distributor is 0,1 s;
- the cross-sectional area of the hydraulic distributor varies according to linear law.




(a) (b)

Fig. 3. Graphical dependencies of angular displacement:
(a) the first jib section; (b) the second jib section

From the system of equations (10) and expressions (11 - 22), the displacement of the rods of the hydraulic cylinders, respectively, of the first jib section, the second jib section, and the telescopic jib section were determined. Depending on the movement of the rod of hydraulic cylinders, the angular movement of the first jib section and the second jib section is determined. Analyzing the graphical dependences of movement of the units of the boom system (Fig. 3), it is possible to determine, in accordance with the geometric and kinematic characteristics, the functional dependence of the angular movement of the boom system units in accordance with the linear movement of the rods of hydraulic cylinders.

Having solved the system of equations with initial parameters and initial conditions, graphical dependences of speeds of movement of rods of drive hydraulic cylinders (Fig. 4) and respectively elements of the boom system (Fig. 5) were constructed with the condition of simultaneous movement of the first jib section, the second jib section, the telescopic jib section and cargo.

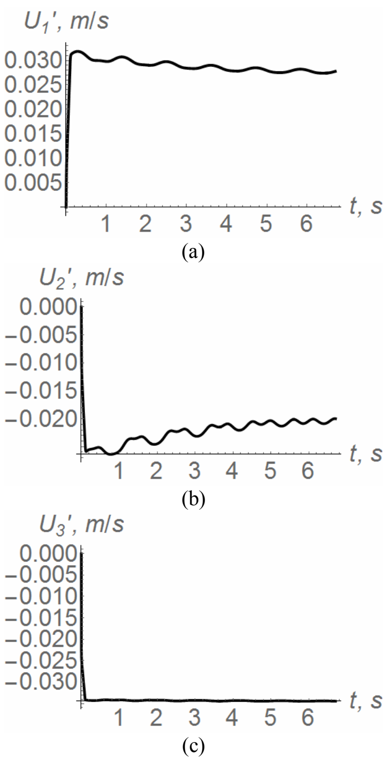



Fig. 4. Graphical dependences of speeds of movement of rods of hydraulic cylinders: (a) the first jib section; (b) the second jib section; (c) telescopic jib section

Acceleration of rod of the hydraulic cylinder of the telescopic jib section (Fig. 4(c)) occurs within 0.1 s, which corresponds to the time of movement of the hydraulic control valve spool. The speed of movement of the rod of the hydraulic cylinder is 0.058 m/s, and with further movement is accompanied by dynamic loads.

Exit to on the steady motion of the first boom section and the second boom section (Fig. 5), occurs in accordance with of the rod steady motion of hydraulic cylinder. The angular speed of movement of the first section of the jib is 0.077 rad/s, at the beginning of steady motion, with further movement has a slight decrease.

The decrease in the angular speed of movement coincides with the decrease in the linear velocity of the rod of the hydraulic cylinder (Fig. 4(a)). The angular speed of movement of the second jib section at the beginning of steady motion is 0.2 rad / s, with further movement has a slight decrease. A gradual decrease in the angular speed of the first jib section and second jib section with steady motion, caused by the kinematic parameters of the manipulator and, accordingly, the oscillatory movement of the cargo (Fig. 7).

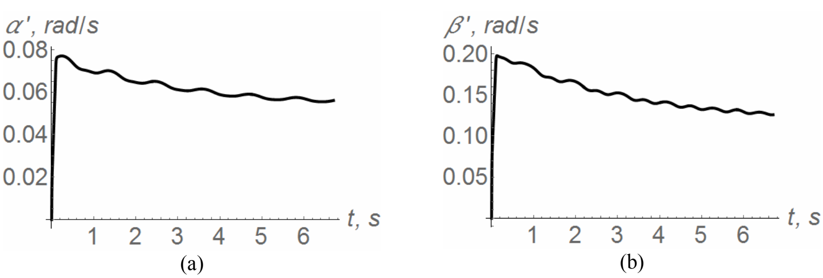



Fig. 5. Graphical dependences of the angular speed of movement of the links of the boom system: (a) the first jib section; (b) the second jib section

As can be seen from the graphical dependencies, the pressure at the beginning of the motion is equal $2 \cdot 10^7 \text{Pa}$, which corresponds to the pressure of the working fluid in the hydraulic system. Upon further movement of the boom system and their exit in a steady motion, the pressure in the hydraulic cylinder of the first jib section and the second jib section is equal to $5 \cdot 10^6 \text{Pa}$, and is accompanied by dynamic loads. This is caused by the inertial component of the units of the boom system and, accordingly, the occurrence in it and the elements of the hydraulic drive of dynamic loads caused by the oscillatory movement of the cargo (Fig. 7).

Taking into account the inertial components of the units of the boom system and the fluctuations of the working fluid pressure in the hydraulic cylinders, the dependence of the cargo oscillation at the end of the jib was constructed (Fig. 7). From the graphical dependence you can see the characteristic correspondence of the deviation of the cargo from the vertical (Fig. 7), which coincides in time with the dynamic loads in the elements of the hydraulic drive (Fig. 4), links of the boom system (Fig. 5) and fluctuations in the working fluid pressure (Fig. 6).

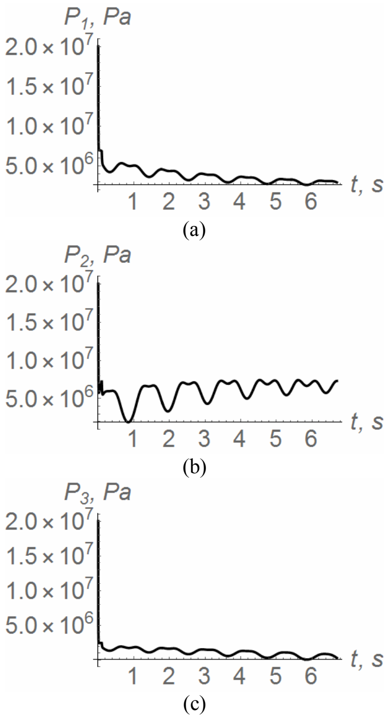



Fig. 6. Graphic dependencies of pressure change in the hydraulic cylinder: (a) the first jib section; (b) the second jib section; (c) the telescopic jib section

Based on the initial conditions, there is no deviation of the cargo at the beginning of the movement. At the beginning of the steady movement of elements of the boom system (Fig. 4 - Fig. 5) the cargo deviation is maximized and equals $0,34\text{rad}$.

Such high load variability results in considerable dynamic loads in the elements of the boom system and the elements of the hydraulic drive of the manipulator. Thus reducing its

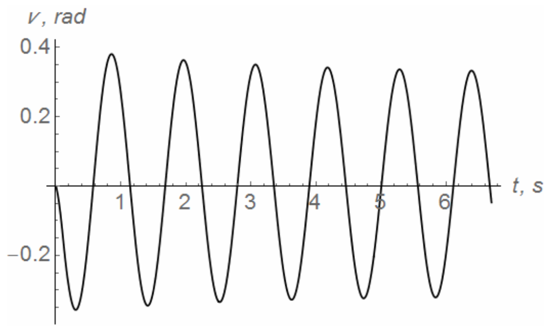



Fig. 7. Graphical dependence of the cargo oscillation

reliability and speeding up the failure of the mechanical system as a whole.

**Conclusions.** As a result of the research, a mathematical model of the dynamics of change of departure in the plane of movement of the boom system of the manipulator was constructed, provided that the motions of the elements of the boom system were combined with the cargo. Dynamic analysis of the mechanism of simultaneous movement of the first jib section, the second jib section, the telescopic jib section, and the cargo oscillations is performed. Graphical dependences of dynamic loads in the boom system and elements of the hydraulic drive of the manipulator were obtained. The proposed mathematical model makes it possible to determine the actual dynamic loads in the elements of the manipulator design and drive mechanisms. The results obtained can be used in further practical design of hydraulic manipulators.

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*Стаття надійшла до редакції 10.12.2019 р.*

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**МАТЕМАТИЧНА МОДЕЛЬ ДИНАМІКИ ЗМІНИ ВИЛЬОТУ СТРІЛОВОЇ СИСТЕМИ МАНІПУЛЯТОРА ЗА ОДНОЧАСНОГО ПЕРЕМІЩЕННЯ ЇЇ ЛАНОК**

З метою підвищення продуктивності та надійності маніпулятора згідно з нормативно-технічною документацією, яка регламентує безпечну експлуатацію маніпуляторів допускається суміщення рухів з одночасним переміщенням декількох ланок стрілової системи. В результаті в роботі розглянута методика побудови математичної моделі в площині зміни вильоту стрілової системи маніпулятора із вантажем. Математична модель побудована із врахуванням трьох одночасних рухів, а саме, одночасного переміщення першої секції стріли, другої секції стріли, телескопічної секції стріли та коливання вантажу. Розраховано функції зміни кінематичних та динамічних характеристик стрілової системи за одночасного переміщення її ланок. Побудова математичної моделі виконана із застосуванням рівнянь Лагранжа другого роду. При цьому за узагальнені координати моделі маніпулятора прийнято, кутові координати положення ланок стрілової системи та кутове відхилення від вертикалі вантажу. А механічні характеристики гідравлічного приводу, представлені у вигляді квадратичних залежностей між діючими зусиллями та швидкостями переміщень штоків гідроциліндрів. Керування елементами приводу представлено у вигляді рівнянь витрати робочої рідини зі зміною площі прохідного перерізу гідравлічного розподільника за лінійним законом. В результаті отримано рівняння руху маніпулятора з врахуванням впливу інерційної складової кожної ланки стрілової системи та впливу коливань вантажу на динамічні навантаження елементів металоконструкції та елементів гідравлічного приводу. Розроблена математична модель дозволяє теоретично визначити вплив одночасного переміщення першої секції стріли, другої секції стріли та телескопічної секції стріли на коливання вантажу, а також вплив коливання вантажу на динамічні навантаження, які при цьому виникають в стріловій системі та елементах гідравлічного приводу маніпулятора.

**Ключові слова:** математична модель, зміна вильоту, суміщення рухів, маніпулятор, рівняння Лагранжа другого роду, динамічні навантаження, коливання вантажу.

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**MATHEMATICAL MODEL OF THE DYNAMICS CHANGE DEPARTURE OF THE JIB SYSTEM MANIPULATOR WITH THE SIMULTANEOUS MOVEMENT OF ITS LINKS**

In order to increase the productivity and reliability of the manipulator according to the normative and technical documentation, which regulates the safe operation of the manipulators, it is allowed to combine movements with the simultaneous movement of several elements of the boom system. As a result, the paper methodology reviewed for constructing a mathematical model in the plane of the departure change of the boom system of a manipulator with a load. The mathematical model is constructed from the calculation of three simultaneous movements, namely, the simultaneous movement of the first jib section, the second jib section, the telescopic jib section and the oscillation of the cargo. The functions of changing the kinematic and dynamic characteristics of the boom system with the simultaneous movement of its links are calculated. The construction of a mathematical model is carried out using Lagrange equations of the second kind. Moreover, the generalized coordinates of the manipulator model are taken as the angular coordinates of the position of the links of the boom system and the angular deviation from the vertical of the cargo. And the mechanical characteristics of the hydraulic drive are presented in the form of square dependencies between the acting forces and the speeds of movement of the hydraulic cylinder rods. The control of the drive elements is presented in the form of equations of the flow rate of the working fluid with a change in the area of the flow cross-section of the hydraulic distributor according to a linear law. As a result, the equation of motion of the manipulator is obtained, taking into account the influence of the inertial component of each link of the boom system and the influence of cargo oscillations on the dynamic loads of metalware elements and hydraulic drive elements. The developed mathematical model allows one to theoretically determine the effect of simultaneous movement of the first jib section, the second jib section and the telescopic jib section on cargo oscillation, as well as the effect of cargo oscillation on dynamic loads that occur in the boom system and manipulator hydraulic drive elements.

**Keywords:** mathematical model, varying the radius, combination of movements, manipulator, Lagrange equations of the second kind, dynamic loads, load oscillations.

*Ловейкин В.С., Ромасевич Ю.А., Сподоба А.А., Ловейкин А.В., Почка К.И.*

**МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДИНАМИКИ ИЗМЕНЕНИЯ ВЫЛЕТА СТРЕЛОВОЙ СИСТЕМЫ МАНИПУЛЯТОРА ПРИ ОДНОВРЕМЕННОМ ПЕРЕМЕЩЕНИИ ЕЕ ЗВЕНЬЕВ**

С целью повышения производительности и надежности манипулятора согласно нормативно-технической документации, которая регламентирует безопасную эксплуатацию манипуляторов, допускается совмещение движений с одновременным перемещением нескольких элементов стреловой системы. В результате в работе рассмотрена методика построения математической модели в плоскости изменения вылета стреловой системы манипулятора с грузом. Математическая модель построена из расчета трех одновременных движений, а именно, одновременного перемещения первой секции стрелы, второй секции стрелы, телескопической секции стрелы и колебания груза. Рассчитаны функции изменения кинематических и динамических характеристик стреловой системы при одновременном перемещении ее звеньев. Построение математической модели проводится с использованием уравнений Лагранжа второго рода. При этом за обобщенные координаты модели манипулятора принято, угловые координаты положения звеньев стреловой системы и углового отклонения от вертикали груза. А механические характеристики гидравлического привода, представлены в виде квадратных зависимостей между действующими усилиями и скоростями перемещения штоков гидроцилиндров. Управление элементами привода представлено в виде уравнений расхода рабочей жидкости со сменой площадью проходного сечения гидравлического распределителя за линейным законом. В результате получено уравнение движения манипулятора с учетом влияния инерционной составляющей каждого звена стреловой системы и влияния колебаний груза на динамические нагрузки элементов металлоконструкции и элементов гидравлического привода. Разработанная математическая модель позволяет теоретически определить влияние одновременного перемещения первой секции стрелы, второй секции стрелы и телескопической секции стрелы на колебания груза, а также влияние колебаний груза на динамические нагрузки, которые при этом возникают в стреловой системе и элементах гидравлического привода манипулятора.

**Ключевые слова:** математическая модель, изменение вылета, совмещение движений, манипулятор, уравнение Лагранжа второго рода, динамические нагрузки, колебания груза.

УДК 621.87
*Ловейкін В.С., Ромасевич Ю.О., Сподоба О.О., Ловейкін А.В., Почка К.І.* **Математична модель динаміки зміни вильоту стрілової системи маніпулятора за одночасного переміщення її ланок** // Опір матеріалів та теорія споруд: Наук.-техн. збірник. - К.: КНУБА, 2020. – Вип. 104. - С. 175-190.

*Отримано рівняння руху маніпулятора з врахуванням впливу інерційної складової кожної ланки стрілової системи та впливу коливань вантажу на динамічні навантаження елементів металоконструкції та елементів гідравлічного приводу. Встановлено вплив одночасного переміщення першої секції стріли, другої секції стріли та телескопічної секції стріли на коливання вантажу, а також вплив коливання вантажу на динамічні навантаження, які при цьому виникають в стріловій системі та елементах гідравлічного приводу маніпулятора.*
Іл. 7. Бібліогр. 19 назв.

UDC 621.87
*Loveikin V.S., Romasevich Yu.O., Spodoba O.O. Loveikin A.V., Pochka K.I.* **Mathematical model of the dynamics change departure of the jib system manipulator with the simultaneous movement of its links** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 175-190.

*An equation of motion of the manipulator is obtained taking into account the influence of the inertial component of each link of the boom system and the effect of the oscillatory movement of the cargo on the dynamic loads of the metalware elements and hydraulic drive elements. The influence of the simultaneous movement of the first jib section, the second jib section and the telescopic jib section on cargo oscillation, as well as the effect of cargo oscillation on dynamic loads that occur in the boom system and manipulator hydraulic drive elements, is determined.*
Fig. 7. Ref. 19.

УДК 621.87
*Ловейкин В.С., Ромасевич Ю.А., Сподоба А.А., Ловейкин А.В., Почка К.И.* **Математическая модель динамики изменения вылета стреловой системы манипулятора при одновременном перемещении ее звеньев** // Сопротивление материалов и теория сооружений. – 2020. – Вып. 104. – С. 175-190.

*Получено уравнение движения манипулятора с учетом воздействия инерционной составляющей каждого звена стреловой системы и воздействия колебательного движения груза на динамические нагрузки элементов металлоконструкции и элементов гидравлического привода. Определено влияние одновременного перемещения первой секции стрелы, второй секции стрелы и телескопической секции стрелы на колебания груза, а также воздействие колебаний груза на динамические нагрузки, которые при этом возникают в стреловой системе и элементах гидравлического привода манипулятора.*
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UDC 624.014

# A NEW APPROACH TO THE DESIGN OF SUSPENSION ROOF SYSTEMS

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DOI: 10.32347/2410-2547.2020.104.191-220

Over the last century, the suspension roofs design has progressed until the advent of the shells theory in the first half of the 20th century, due to a rapid pace in technological advancement. A paradigm shift emerged with the new trend in structural design towards a new design process that cooperatively integrated economy, efficiency, and elegance. Different approaches in computation, design and reliability assessment of roof structures are discussed in this work to identify the key conditions that have significantly contributed to modern suspension roof design principles. A new algorithm to assess the reliability of suspension roofs at the design stage is proposed and a novel method for computational design and reliability evaluation of suspension roofs is presented in this paper. This method enables to solve some topical issues, such as assignment of initial geometric roof parameters and relevant problems, like numerical determination of reliability indices for statistically non-determined systems of suspension roofs with a big cut on elliptical plan.

**Keywords:** suspension roof, stress-strain state, computational methods, reliability indices, roof failure.

## 1. Introduction

The esthetic superiority and the overall structural performance of suspension roofs are well known by Otto [1] and Rabinovich [2], since these structural systems combine stability, economy and satisfaction of special architectural demands, while their application is closely related to major engineering challenges. Based on inspiration and intuitive conception quality, distinguished pioneer engineers have designed and realized numerous buildings with suspension roofs as their main structural component (Mies van der Rohe [3], Tange and Nervi [4], etc. Starting from the famous Crown Hall at the Illinois Institute of Technology between 1950 and 1956 [5, 6], the Tokyo Small Olympic Arena from the early sixties [7, 8] and the Paper Mill at Mantua, Italy [9], there are many applications of suspension roofing system, like that in Dulles

International Airport (Washington DC 1962), Stadthalle (Bremen 1964), Europahalle (Karlsruhe 1983), PA Tech Laboratories (Princeton 1986) and Church of Fatima (Brasilia 1988) must be quoted [4].

A valuable contribution to the theory and design of large-span spatial shells was done by many scientists [10]. In the last 15 years the advent of powerful computers and development of sophisticated nonlinear computer-aided design (CAD) software [11, 12, etc.] have enabled engineers to utilize suspension roofs in complicated large scale structures, some of which can be classified among unique examples of engineering excellence. A comparative presentation of earlier and recent applications of suspension roofs is shown in Fig. 1, where the last image (Fig. 1(c)) refers to the Oquirrh Park Speed Skating Oval, belonging to the facilities of the Salt Lake City cite for the XIX Olympic Winter Games of 2002 [4].

(a)

(b)

(c)

Fig. 1. Four classical applications of suspension roofs: (a) DullesAirport, UnitedStates[13]; (b) PA TechLab, UnitedStates [14]; (c) Oquirrh Park Speed Skating Oval, UnitedStates [15]

Supporting elements of suspension roofs are most often made by flexible or rigid threads (guys). The guys can be made from conventional rolled steel, trusses, rebar rods, strand of high-tensile wire and cables, etc. These roofs are different from others due to the lack of prestressing process. It is also possible to obtain the necessary roof rigidity with a low constant and high temporary load. The material in suspension shells works in two directions. In addition, shells can also withstand shear forces.

Large-span roofs have a higher responsibility level as their failure can lead to serious economic and social consequences. In this context, design of these unique structures should be based on an integrated approach of rational choice of design solutions [16, 17, 18]. These decisions are related to functionality, architectural design, manufacturing and installation techniques as well as operating conditions. The requirements of reliability, manufacturability and cost efficiency as well as accounting the environmental and social factors should be fully implemented.

Probabilistic assessment of reliability is one of the most important tasks in roofs researches in the objects with high responsibility. The main property that determines the reliability of these structures is their performance and ability to save the pre-defined operational quality during the lifetime. Quantitative characteristic of this property is the failure-free operation probability [18, 19].

Existing design codes [20-22] do not include large-span roofs reliability quantification requirements. It is assumed that the computational requirements implementation provides a sufficient reliability level. However, this level can vary within very wide limits depending on load variation characteristics and material properties; constructive scheme; number of structural elements; type of relations between the forces in the elements and the loads.

Reliability of metal structures in buildings and constructions represent statistically determined and non-determined systems that were investigated by many researchers. The major problems and samples were considered [23, 16] and reliability assessment of the large-span suspension roofs and cable-stayed structures was investigated [24, 25, 26].

It should be mentioned that joints between the supporting elements of bearing structures, such as trusses or beams, are a significant issue in the supporting elements composition, because their failure could lead to the start of the damage of the whole roof. Application of modern numerical simulation techniques, like ADINA and ABAQUS [11, 12] enables to investigate the effect of structural schemes on the joints operation and to accumulate necessary statistical data on stressed-strained state of joints. At the same time, contemporary status of computer engineering development opens possibilities for estimating the reliability of joints in suspension roofs, bearing in mind parameters of stressed-strained state and correlation links between functions of joints elements supporting capacity. Investigations of stressed-strained state of joints in steel structures was performed [27, 28], however lot of issues related to stress-strain state (SSS) of suspension roofs joints should steel be studied.

## 2. Aims and Scope

The main objective of this study is to present a broad perspective of using a novel method for computational design and reliability evaluation of suspension roofs, as they are often used in construction of modern stadiums. It is related to the fact that UEFA puts forward strict requirements to ensure comfort for spectators of world or continental football championships. One of the requirements is high-quality roofs over the stadium tribunes for protection against external influences, such as rain, snow, wind, sun, etc. The choice of this type of the stadium roofs is due to their numerous advantages. Thus, the present paper reviews the existing modern design approaches and ways for ensuring proper reliability of long span suspension roofs.

Consequently, the main objective at the given stage is developing fundamental approaches for determining reliability of stadium roofs joints by numerical methods, using microsimulation in modern CAD.

**3. Structures as art: from concept to design**

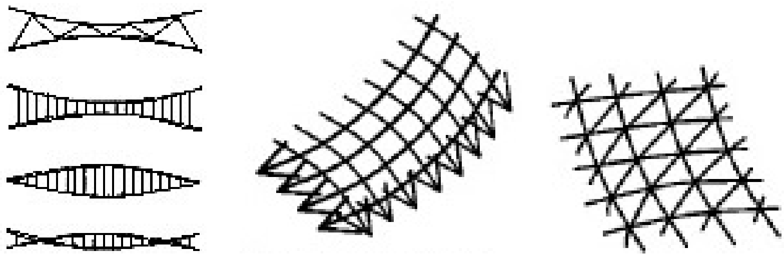
Over the past few decades, a lot of large-span shells have been designed and constructed all over the world. Each of them represents a unique architectural form and based on different approaches in design of supporting structures roof.

**3.1. Types of tension roof structures**

In order to achieve adequately stiff and stable tensile cable or membrane roof structures the following possibilities are known [24]:

1. Using gravity stiffness
2. Pre-stressing
3. Using air pressure support
4. Combination with rigid elements such as beams, trusses, arches, plates, grids, columns.

Simple suspension cables, synclastic shells and membranes are otherwise flexible. If loaded by heavy slab or shell elements, the resulting tension system can be imparted adequate stability and stiffness though it is an antithesis to the tension structures 'lightness'. An alternative concept is to design pre-stressed or 'counter-stressed' tension systems as shown in Fig. 2 [25].



(a) (b) (c)

Fig. 2. Typical roof supporting tension systems[29]:
(a) trusses, (b) two-way net; (c) tri-diagonal net.

It is recommended to use rigid threads without pre-stressing to improve the stability in suspension roofs [24]. This decision significantly reduces the roof weight, applies light prefabricated flooring and simplifies the construction. Rigid thread is easiest to produce rolled, preferably from the high-strength steel.

An important element of suspension roofs is the supporting contour [10]. Normally a supporting contour has a rectangular cross section and is made of reinforced concrete in a monolithic and/or modular form. The supporting contour is used for fastening the suspension roof that transmits the tensile forces from the trusses fixed by hinges to the supporting contour. These contours may be designed either closed or opened. In case when the contour is opened, the thrust force is transmitted by struts, buttresses, delays with anchors and other to foundations. Big forces appear in these elements from the guys thrust and they require higher material consumption. Therefore, the closed contours are more economical. At the same time, the supporting contour has the highest material consumption in the roof (about 35 ... 60%).

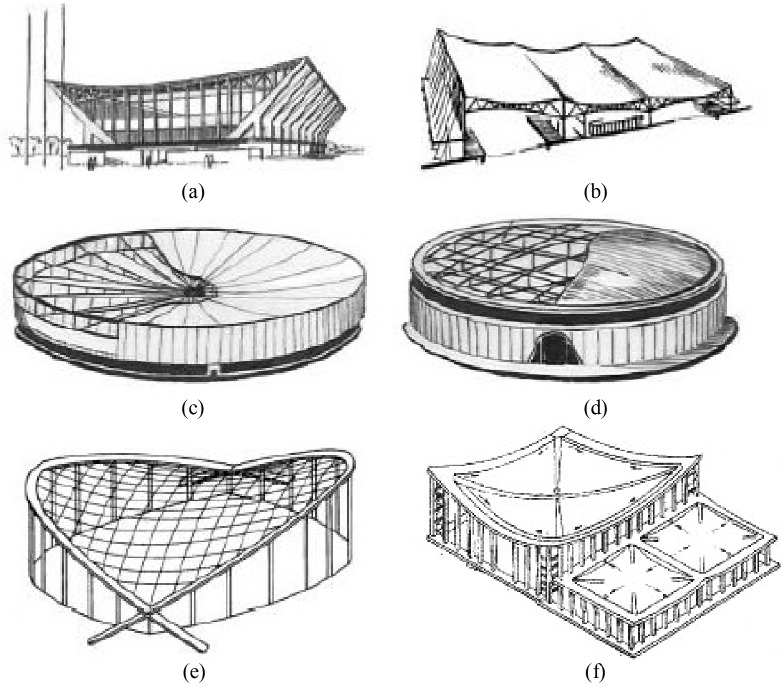
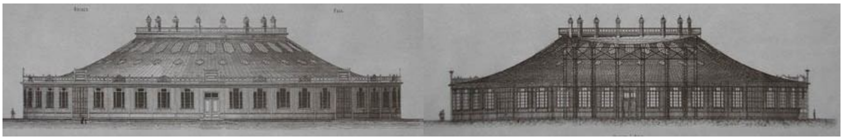



Fig. 3. Typical tension roof systems [31, 29]: (a) one-chord system; (b) two-chord system on rectangular plane; (c) two-chord system on round plane with cut; (d) two-chord system on round plane without cut; (e) typical saddles strained nets; (f) metal membrane

There are planar or spatial covering shells which together with a membrane or cladding form the basis of a roof system. Both gravity loaded or pre-stressed roofs have to be combined with elements such as beams, trusses, arches, plates, grids and columns. Typical structures [25] are shown in Fig. 3. Some known examples are worthy of discussion such as the roof of the Munich Olympic stadium in Germany, roof of the ice-skating rink in Munich, Germany, convertible roof over the Roman arena in Nimes, France, convertible roof of the Montreal Olympic stadium, Canada, the glass-grid dome of the Neckarsulm indoor swimming pool, the Olympiakos Stadium in Athens, Greece, the new suspended cable roof of Braga Stadium, Portugal, the Thessaloniki Olympic sport complex, Greece, the new Juventus Stadium in Turin, Italy etc. [30, 25, 26]. The present review is focused on landmark buildings that have a determined suspension roofs shape on a rectangular, square, elliptical or circular planes.

### 3.1.1. Suspension roofs of round or elliptical plans

Shukhov Rotunda (Fig. 4 (a)) is a unique round steel pavilion, constructed by V. G. Shukhov for the Russian Industrial and Art Exhibition in Nizhny Novgorod in 1896 [4]. Rare structure with a diameter of 68 m has a roof in a form of strongly stretched suspension annular mesh shell with a steel membrane and has a diameter of 25 m in the central part. It was the world's first membrane (suspension) structure of roof buildings.



(a)

(b) (c)

Fig. 4. Suspension roofs with round and elliptical plans:
(a) Shukhov Rotunda, Russia [32]; (b) Olympic pool in Mockow, Russia [33]; (c) Olympiastadium in Munich, Germany [34]

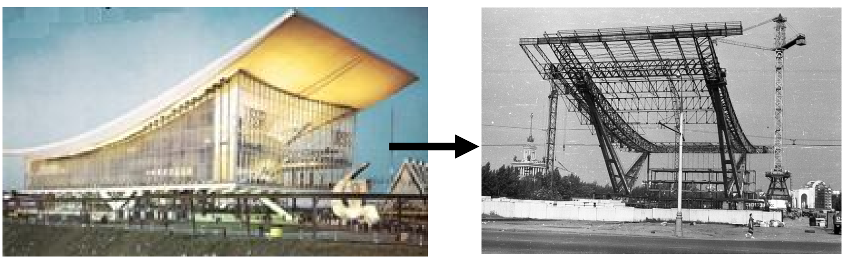
As example of an external thrust systems, where the thrust forces are transmitted to the foundation or anchor, is the roof of the Olympic pool [35] on the Mira-street in Moscow, Russia (Fig. 4 (b)). It has a rigid supporting contour in a form of two arches. The bases for the roof construction solution are rigid threads made in a form of trusses and parallel to the short axis of the building with a step of 4.5 m. Profiled sheeting panels are installed on the trusses to provide suitable stiffness. Insulation, screed and roof waterproofing are placed on the panels. Another example of rigid threads functional application is their using as stabilizing system components of large-span membrane roof (for example, that of the covered stadium on Mira street in Moscow, Russia [35], where the radial elements of the stabilizing system were designed as 2.5 m height suspension trusses).

Olympia stadium [36] (Fig. 4 (c)) is a unique multifunctional stadium in the heart of the Olympic Park in Munich, Germany, that was constructed in 1972. The tribunes are covered by a giant suspension shell that was designed by architect Frei Otto. Large canopies of acrylic glass and steel cables were used for the roof construction. Moreover, they were used for the first time in such a quantity for construction of sports facilities.

**3.1.2. Suspension roofs of square and rectangular plans.**

As a roof with rigid threads, one can distinguish the pavilion of USSR at the World Exhibition in Montreal, Canada, constructed in 1967 [25, 36], which was later dismantled and imported to the USSR in 1975. The structure of the pavilion (Fig. 5 (a)) is represented by stretched corner trusses that transmit the supporting forces to the powerful V-shaped pylons-spacers. At the bottom of the structure supported on hinges are installed special vertical ties that are loaded from the opposite side by the mass of the structure and pre-stressing for providing structural stability.

A modern stadium with a unique suspension roof on a rectangular plan and flexible threads is the stadium in Braga, Portugal. It is included in the top 20 unique stadiums of the world [39]. The Braga Stadium was one of the stadiums constructed in Portugal for the 2004 European Championship (Fig. 5 (b)). The most outstanding element of the structure is its very flexible suspension roof, which is formed by pairs of full locked coil cables spaced 3.75m apart from each other, supporting two concrete slabs over the stands of the stadium. This infrastructure was built for the 2004 European Football Championship. The stadium was designed by Eduardo Souto Moura in conjunction with the "Afassociados" consultancy office and has been considered by many a masterpiece of modern architecture. In the west stand, the concrete walls are anchored in the rock and the roof cable forces are transmitted to the foundation by pre-stressing tendons embedded in the concrete [40].



(a)

(b)

Fig. 5. Suspension roof on the square and rectangular plans:
(a) Pavilion of USSR in Montreal, Canada [37]; (b) The Braga Stadium, Portugal[38]

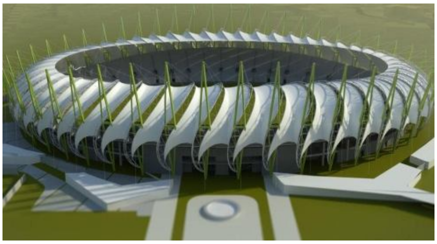
After considering all the variety of suspension roof design solutions, one can conclude the following remarks about their advantages:

- architectural and structural expressiveness;
- constructive form of suspension roof is quite common in large -span structures;
- materials consumption in metal suspension roofs is much less than in arched type structures, where the main part of the metal is consumed in arch supporting;
- the opportunity of constructing roof structures by parts, is important for the financing big construction projects.

**3.2. Necessary conditions**

Current trends should be taken into account in the design and construction of new roofs. There is a large number of both realized and pending projects of stadiums with suspension roofs in the world [41]. Figure 6 illustrates some of the new projects in different countries around the world.

(a)



(b)

(c)

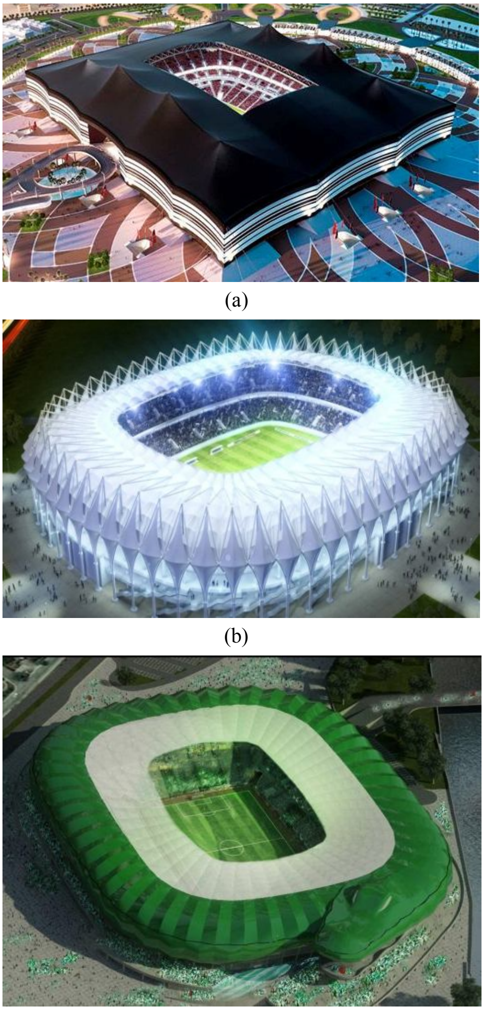
Fig. 6. Modern design solutions of suspension roofs of stadiums: (a) Vladikavkaz, Russia [42]; (b) Basra, Iraq[43]; (c) Durban, South Africa[44]

**3.2.1. Structural principles**

The history of architecture has involved countless styles and trends, but the rationale behind the structural art has not changed significantly no matter the scale of a structure; it always features a search for a cost-effective and performance-efficient design without losing elegance [5, 7, 10]. The aesthetic expression of a structural form is neither a pure desire to find a shape for decoration nor a subordination of its function; otherwise a structure would be overdesigned without any appearance of structural art [45]. Stadiums and their roofs exteriors should comply with the culture and the area where they are located. Examples of roof projects that consider the local culture are shown in Fig. 7. The Muslim Qatar stadium roof for the World Cup 2022 in Qatar is fully integrated into the traditional architecture ( see Fig. 7(a)); similarly, the Muslim style arches were used in Uzbekistan (Fig. 7(b)), stadium roof in Turkey (Fig. 7(c)) that looks as a crocodile [46].

Studies from the historical point of view have shown how design evolved to achieve an efficient and economic structure by understanding structural principles. Further, bio-inspired or biomimetic design (Fig. 8) inspires engineers to find a cost-effective structural form with elegant appearance and also presents emerging challenges, including design of smart and intelligent structures. Review on these topics is available [50–52].

F. Leonhardt [56] formulated ten rules for structural design and M. Troitsky [57] mentioned ten requirements for structures aesthetics. These rules could be sorted into two groups: to improve the elegance of structures and to improve their harmony with the environment. Although the rules cannot guarantee the elegance of a structure, but at least they can help designers to avoid certain kinds of unattractive designs.



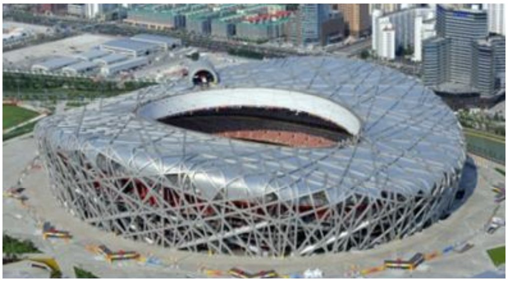
(a) (b) (c)

Fig. 7. Exteriors of stadiums roofs: (a) Al Khor, Qatar [47]; (b) Tashkent, Uzbekistan [48]; (c) Bursa, Turkey[49]

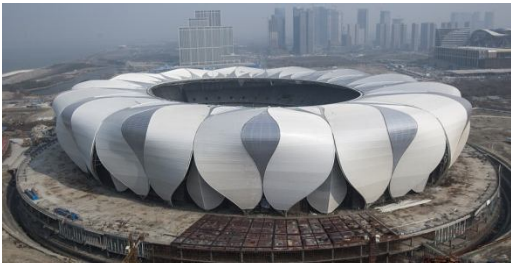
**3.2.2. Competitive environment**

Quality function deployment (QFD) has been widely used as a multi-functional design tool to translate customer requirements to a product's technical attributes. QFD originated in the late 1960s and early 1970s in Japan [58] and the topic is investigated till today [59]. At the beginning of QFD development, the primary QFD functions were product development, quality management, and customer needs analysis. Thus, QFD was used to help design teams to develop products with higher quality to meet or surpass customer requirements. With the development and widespread use of QFD, its application areas expanded to much wider fields, including design, planning, decision-making, engineering, management, teamwork, timing, costing and so on. The inherent incentive of the widespread use of QFD is its benefits to practitioners. Researchers have mentioned the benefits of QFD correctly rating the importance of every customer requirement is essential to the QFD process because it will largely affect the final target value of a product's technical attributes [60 - 63]. Traditionally, capturing customer requirements involves three steps in QFD:

1. Identifying customer requirements;
2. Structuring customer requirements;
3. Determining the importance weight for the individual customer requirements.



(a)



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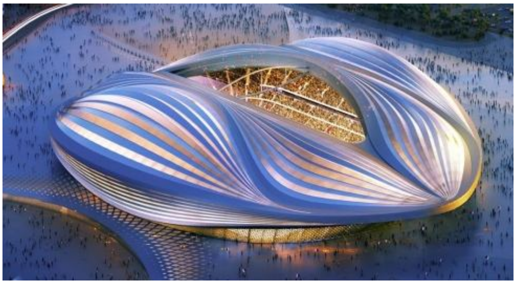


(c)

Fig. 8. Biomimetic architecture of stadiums. (a) Beijing, China [53]; (b) Hangzhou, China[54]; (c) Al Wakrah, Qatar[55]

The first two steps are usually accomplished via market survey, combined with expert opinion. Many researchers have proposed several mature methods on this topic. Presently, the success of a product in a competitive market place depends either on how well it meets the customers' requirements, or how it compares with competitors' products. Therefore, it is important to integrate competitive analysis into product design and development.

A new customer requirement ranking method that takes competitor information into account was proposed [64]. This method focuses on the voice of the customer and also considers the competitive environment. The method helps in finding out the most important customer requirements, and provides a way to combine them with the importance weights from a customer's viewpoint. The proposed rating method will provide the final weight from three perspectives: competition, performance and customers. The conceptual process of this model is presented in Fig. 9.

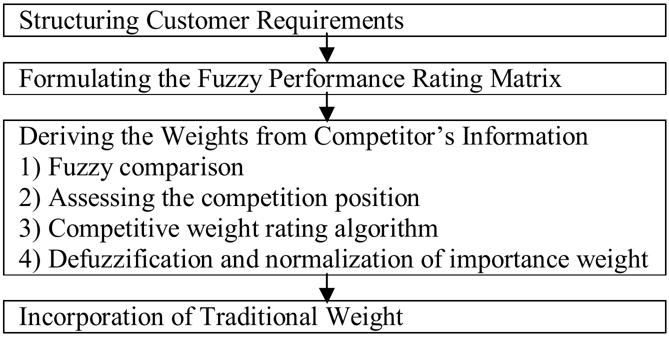



Fig. 9. Conceptual process according to the proposed model

Competition is a factor that can improve the result in works on structural art. European countries such as Switzerland and Germany have elevated modern structural design to an artistic level by making the design process itself competitive. It is reported that competition between structural artists pushed designs to become works of art, such as Thomas Telford and John Rennie in the age of iron, Isambard Brunel and George Stephenson in the design of unique structures, or Othmar Ammann and David Steinman in the design of long-span suspension bridges [65]. Hines and Billington have studied the design process of the winning bridge for the competition of the Ingolstada Bridge in 1998 and showed how design competition pushed structural engineers to exceed the norms of practice and design a work of art [66]. Overall, structural form based on competition can drive the conceptual design process, leading to a highly developed vision of the design and a form that could not have been conceived by structural theory alone.

**3.2.3. Materials efficiency**

The development in structural materials is a prime factor, leading to a revolution that has taken place in suspension structure development and use. The roof cladding which may have used animal skins in the primitive applications can today be chosen from a wide variety of possibilities. Corrugated sheeting from metals—galvanized iron, aluminum alloys, stainless steel-plain or corrugated, and sheets from non-metals such as fibre reinforced glass or plastic, timber planks, concrete slabs, and fabrics of different type, and, produced to a high degree of sophistication are available. The development in the capacity to carry direct forces or flexure is even more dramatic.

The basic tenet is the enormous increase in the strength-weight ratio. In compression and flexure elements, timber and stone have given way to high strength steels [67], high performance concrete [67], pre-stressed concrete [67], corrosion resistant high yield reinforcing steels [67], etc. For tension elements which initially used natural vines and creepers, and then cast iron chains, there is now abundant use of high tensile strength steel wire ropes and strands [67] and opportunity of using carbon reinforced plastic fiber [68].

Increase in the strength–weight ratio has enabled a substantial growth in the capacity of a structure or its elements to carry live and superimposed loads [67]. Furthermore, improvements in technology have led to enhanced corrosion resistance of metals and their products, as well as development of high strength non-metallic materials which are inert to the effect of corrosion [67]. Thus from the humble beginnings of small exotic suspension systems, the way has been paved for the large scale application of suspension structures.

Alongside typical construction materials such as steel and concrete, todays' innovative materials are finding their way into the designs of new structural form, such as fibre reinforced polymer (FRP) composites. With its high strength and durability, it has been recognized as a competitive material for suspension construction [69]. However, its optimal structural forms have not yet been fully explored, nor has its material potential been achieved. Attempts to use laminated composite in the shape of thin shells, membranes and woven webs were investigated [17, 68, 70]. Keller [71] reviewed the multifunctional use of FRP composites that offer the potential to meet the need of a new generation of infrastructures, such as lightweight and high sustainability. It is important to develop material-adapted forms, meaning efficient forms that exploit the unique properties of a material [72].

The requirements for the roof material are described in the International Building Code 2006 (IBC), chapter 15, [73].

**3.3. Failures of the large-span roof structures due to design mistakes**

Collapse examples of different large -span roofs types can be used to learn the errors in design or improper construction. Putting forward ideas should be technically and economically justified in the design of unique large-span structures. Requirements for reliability, manufacturability and cost-effectiveness, environmental and social factors should be performed in full.

The modern building codes are based on structural reliability theory [22]. Theoretically, the structural failure probability should be in the order of $10^6$ per year. However, failures occur and are in general caused by:

- Unfavorable combinations of circumstances like extreme snow load on a roof structure in combination with an unfortunately low structure strength.
- Unforeseen load conditions like for example explosions. It is quite rare events and precautions can be done to reduce the consequences like design against progressing collapse.
- Gross human error in the design, material production or construction phases. To minimize or avoid human errors, different actions can be taken like for example education, good working environment, complexity reduction, self-checking and inspection.

Violation of existing rules can lead to accidents. Some illustrative examples of large -span roofs are shown below.

In December 2010, 50 centimeters of snow fell in the western part of USA and the temperature dropped to minus 18 degrees [74]. Therefore, a huge inflatable roof collapsed over the stadium Metrodome in Minneapolis. The unique domed roof was made of fiberglass. The whole structure was maintained by overpressure of air. During the collapsing process first, a hole formed in the roof; second, the roof crushed under the snow weight on the field of the stadium (Fig. 10 (a)).

Roof of one of the FC Twente, Netherlands, stadium tribunes collapsed in 2011 [78]. The roof has collapsed during the stadium reconstruction in order to expand its capacity. The collapse was due to mistakes during construction and installation works. As a result, two supporting beams lost their load-supporting capacity (Fig. 10 (b)).

Siemens Super Arena is a multifunction sports hall. The arena was formally run by the Danish Bicycle Union and a cycle-racing track was one of the main features. The structure was built in 2001 and inaugurated in February 2002. The ‘‘fish-shaped’’ main trusses with a spacing of 10.1 m have a span of 72 m. On the morning of 3 January 2003 the truss in line 4 (Fig. 10 (c)) suddenly fell to the floor [79]. The Siemens Arena case differs from many other cases in that it was due only to design errors made by Albeit Many that was working under a tight time pressure and without supervision. Formally third control party by an approved independent structural engineer was hired, but what he did apart from putting his signature on the front page of the computations and his bill is difficult to see [79].

Nowadays, civil structures become more and more wind and snow sensitive, because of the trend towards lightweight

(a)

(b)

(c)

Fig. 10. Collapse of stadiums roofs
(a) Minneapolis, USA [75]; (b) Twente, Holland[76]; (c) Ballerup, Denmark [77]

construction and the evaluation of exact wind loads acting on such structures, frequently characterized by complex geometries, requiring expensive experiments in wind tunnels or semiempirical methods. The dynamic nature of wind action can cause oscillations and deformations, which can compromise the performance of the roof and, in the worst cases, its structural stability. On the other hand, the static effect of snow represents a dominant load for this type of structure, even reaching as high as 70%-80% of the total load. One of the primary collapse causes (corresponding to approximately 45% of the cases analyzed) lies in an erroneous evaluation of the loading conditions and of the structural response [18]. A number of studies and analyses have been carried out on structures that have completely or partially collapsed:

- due to snow, e.g. the Hartford Coliseum (1978), the Pontiac Stadium (1982), the Milan Sports Hall (1985) and the Montreal Olympic Stadium (1992);
- due to wind, e.g. the Montreal Olympic Stadium (1988).

From the observation of such collapse events significant information has been collected, and design specifications have been obtained for verification of these structures in ultimate and serviceability limit states. In particular, the great difficulties in assessing and simulating real load conditions have emerged and some considerations about such problems are described [80, 81].

Among the facts, related to the work of suspension threads and hard errors in their design, one can identify the following examples:

1. Swimming pool of Olympic sports complex in Moscow, Russia (Fig. 4b) [35]. Following the publications of the appropriate commissions that have studied the reasons of these events, the influence of concrete creep and changes in the shell geometry on buckling of RC thin-walled shells was not properly considered in the design. Iskhakov and Ribakov [82] focused on buckling of such shells, taking into account geometrical and physical nonlinear behavior of compressed concrete. The critical buckling loads for the shells are obtained. It was shown that these loads are lower than the actual ones and therefore the shells’ buckling was unavoidable. To prevent brittle shell failure, they should be designed using other dominant failure modes that appear before the buckling. It was concluded that possible failure schemes of real RC shells can be predicted using dominant failure modes obtained by laboratory testing of scaled models. Rigid threads (suspension trusses) are the main load-supporting structural elements of the roof mounted on the support contour. The error in the geometry of the support contour at this facility led to significant changes in the stress-strain state of threads. It has led to violations of their design geometry and operating problems of the roof.

2. Covered stadium on Mira street in Moscow, Russia, [35]. Rigid threads (suspension trusses) are part of the radial-ring stabilizing system. It is working together with the membrane roof in perception of non-equilibrium loads. Considering of geometric nonlinearity was performed incorrectly. It has led to necessity correction of the roof geometry in the final stages and cutting

(essentially it is generate failures) of the separate elements of the lower chords of suspension trusses.

Consider addition of the case regarding the airport terminal roof collapse in Paris.

Terminal 2E of the Charles De Gaulle International Airport (Paris) collapsed unexpectedly in the early morning of Sunday, 23-May-2004. According to an initial enquiry, the metal support structure had pierced the concrete roof, causing it to split and fall in. The new terminal collapse was "linked to the perforation of the vault by thes, that was a consequence of a design errors" [83].

Long span coverings were subjected to partial and global failures as that of the Hartford Colisseum (1978), the Pontiac Stadium (1982) and the Milan Sport Hall (1985) due to snow storms, the Montreal Olympic Stadium due to wind excitations of the membrane roof (1988) and snow accumulation (1995), the Minnesota Metrodome (1983) air supported structure that deflated under water ponding, the steel and glass shell sporthall in Halstenbeck (2002).

These examples describe the importance of considering the nonlinear effects (geometrical, physical, structural, genetic) in more accurate computation and proper design of similar roofs, and as result, ensuring the reliability of rigid threads.

The problems with general poor quality in the building industry are not new and were investigated [84]. It was identified that there are five main drivers for change, four processes on which to focus improvement and seven targets for improvement with for example an annual targets for capital cost of 10% (see Fig. 11.). In Sweden, a commission has published a 400-page report, describing a building sector with many problems, like illegal cartels and other market manipulating activities, macho culture and short time for planning design even for very complex structures [85]. According to the report more than 60% of the shortcomings in the finished structure are related to shortage in documents that the client has the responsibility for. These problems have resulted in poor quality and high prices.

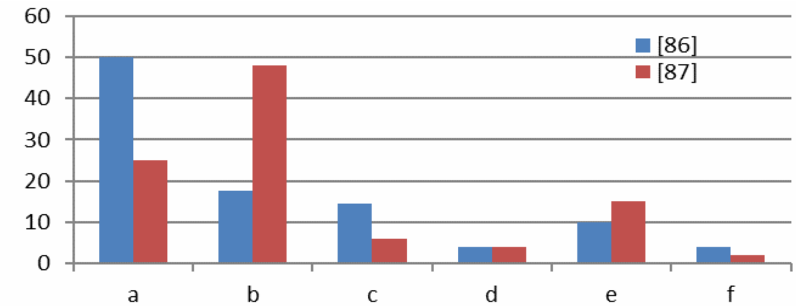



Fig. 11. Cause of accidents in building in percentage following [86, 87]:
(a) design errors; (b) manufacturing and installation errors; (c) poor materials quality;
(d) shortcomings in codes; (e) incorrect operation; (f) other

After considering various examples of accidents it can be concluded that there are different causes of accidents [88]. The available data are presented in Fig. 11. The difference of these data can be explained, apparently, by lack of statistical data, imperfect methodology for assessing the accidents causes, etc. However, a large proportion of design errors is alarming.

These cases are the lessons that should be learned as the causes and mechanisms of accidents. Methods of structural analysis of roofs to eliminate the possibility of structures failures in the future should also be improved.

## 4. Modern methods of computation and providing reliability in design of suspension roofs

Effective solution of the designing problem in modern construction is highly dependents on considering of real construction work in the computation and construction.

### 4.1. Computation methods for stress-strain state evaluation in spatial rod shells formed by threads

There are various methods for computation of stress-strain state (SSS) of spatial rod shells. Detailed information about it is given in [10].

Analytical computation methods of spatial rod shells lead to solution of problems described by a system of nonlinear differential equations. Solutions of these problems can be implemented using the following methods [23]:

- methods of solution of boundary problem together with the boundary conditions;
- methods of energy functional minimization;
- linearization method.

Particular attention should be paid to the computation of shell structures considering the geometrical and "structural" nonlinearity, because considering of the design scheme that changes during the construction (as built) is required for computation and design.

Numerical computation methods of spatial rod shells have been widely applied due to the rapid development of computer technology. Among them are the finite difference method, finite element method (FEM), the variational-difference method, and others [35]. FEM allows solving problems with the changes in the design scheme. It is important for the large -span structures, which during the construction change the distribution of internal forces, and also the direction of displacements in the main load-supporting elements.

Currently, particular wide application among numerical methods has the finite element method. This is due to a number of important advantages for this case:

- the ability to consider a large number of structural elements with specified analytical models (finite elements) and a wide range of analytic representation;
- boundary conditions and random load can be considered;
- the size and stiffness characteristics of the finite element may be variable, depending on the geometry of the structure, as well as operational and technological characteristics;

- properties of structural elements and materials may be different, that allows to analyze structure dwith multi-modulus materials.

There are different manufacturing and computation methods for estimating actual design performance of rigid threads with varying degrees of accuracy [10, 24, 25, 89], etc. However, all the above methods do not consider the difference between computation of solid threads as well as threads with through-section. Furthermore, they do not consider the type of lattice, and its stiffness characteristics. It yields forces redistribution in the elements, but particularities in structural behavior under dead and live loads are not fully considered.

**4.2. Modern methods for providing reliability of suspension roofs in design stage**

The problem of reliability especially concerns unique large -span structures. Among these are suspension shells that have increased level of responsibility on application denial that may lead to severe economic results and social consequences (as it was discussed in section 3.3). During design there are problems exceeding the limits of existing regulatory documents. Novelty of technical concepts, specific knowledge and experience in design of such kind of structures is required from a structural engineer. Requirements of reliability, technological and economic efficiency should be fully realized in this case, as well as environmental and social factors should be considered.

**4.2.1. Requirements of modern codes for providing strength, rigidity and stability of roofs structures and their elements**

Strength, stiffness and stability of suspension roofs and their elements are regulated by current standards [20-22] and provided by computation, performed assuming that entire load is perceived by threads and transmitted to supports. Considering the strength and stiffness of beams is required for determining the roofs deformations and displacements, caused by temporary load.

Computation of the suspension system should include:

- finding the maximum force for all elements under any possible loads combination;

- finding the cross-sections of all elements in the suspension system and supporting structures;

- finding the deformation of the roof and supporting structures under the possible loads combinations;

- verification, if necessary, to special effects: thermal stresses, supports displacement, seismic loads, fatigue, dynamic stability, etc.

The computation is recommended in the following order:

A. Selecting the type and the main parameters of the roof system: spans, supports location, etc. The shape taken by the system under the action of the full design load is determined.

B. Computation of strength.

Forces in the supporting threads and the supporting structures under the full load and the actual roof geometry are calculated. Cross sections of threads are found accordingly. After that, the dimensions of the support structure are assigned.

C. Computation of deformation.

The computation is required to determine the sections of stabilizing structures that together with the supporting threads provide the necessary stiffness and stability of the suspension roof. It is advisable to separate the deformation to elastic and kinematic. The suspension roof stiffness is achieved by increasing the supporting threads sag and cross section.

The type and section of the stabilizing structures are determined by the terms of the maximum allowable kinematic displacements of the roof under the uneven payloads. After that, verification of the shell stiffness is performed (elastic deflection under the temporary load is determined).

Verification of the supporting structures with the unfavorable mounting load is performed. Computation of the systems and support contour under the uneven temporary load is performed. The analysis of rigid threads is performed in a geometrically nonlinear formulation.

Despite the demands of modern codes for providing strength, rigidity, stability of roof structures and their components, the system of partial reliability coefficients for structures with high responsibility level is not normalized [22]. Therefore, it is most logical to analyze the roof structure using direct reliability theory methods, which can later become a basis for normalization of reliability coefficient for the required level. This method is described in paragraph 4 below.

**4.2.2. The methods of reliability theory of building construction in providing reliability requirements at the design stage**

Uncertainty analysis in engineering should ideally be a part of routine design because the variables and supposedly constant parameters are either random or known with imprecision. In some cases the uncertainty can be very large, such as in case of natural actions provoking disasters or modeling errors leading to technological disasters. To estimate the risk of a given engineering problem, cumulative distributions functions (CDFs), defining the input variables, are traditionally used and then, by means of analytic or synthetic methods (i.e. Monte Carlo) the probability of not exceeding undesirable thresholds, is computed [90, 91].

One of the main problems in applying the probabilistic approach is that the CDFs of the input variables are usually known with imprecision. This is normally due to the lack of sufficient data for fitting the model to each input random variable. For this reason, the parameters of the input distributions are commonly known up to confidence intervals and even these are not wholly certain. This hinders the application of the probability-based approach in actual design practice [19]. Even if the information is abundant, there remains a problem of high sensitivity to usually small failure probabilities to the distribution functions parameters [86]. Such a sensitivity is due to the fact that the probability density function estimation from empirical data is an ill-posed problem [87, 92]. This means that small changes in the empirical sample affect the parameters, defining the model being fitted, with serious consequences in the tails, which are just the most important zones of the distribution functions for probabilistic reliability methods [93, 94].

These and other considerations have fostered the research on alternative methods for incorporating uncertainty in the structural analysis, such as fuzzy sets and related theories [95], anti-optimization or convex-set modeling [96], interval analysis [97], random sets [98], ellipsoid modeling [99,100] and worst-case scenarios [101]. Comparisons have been also made between probabilistic and alternative methods [102] or their combination has been explored [103, 104]. Analytical and computational methods used in the technical reliability theory for computation of complex systems, which can be used for reliability analysis of statically indeterminate systems are shown in Fig. 12.

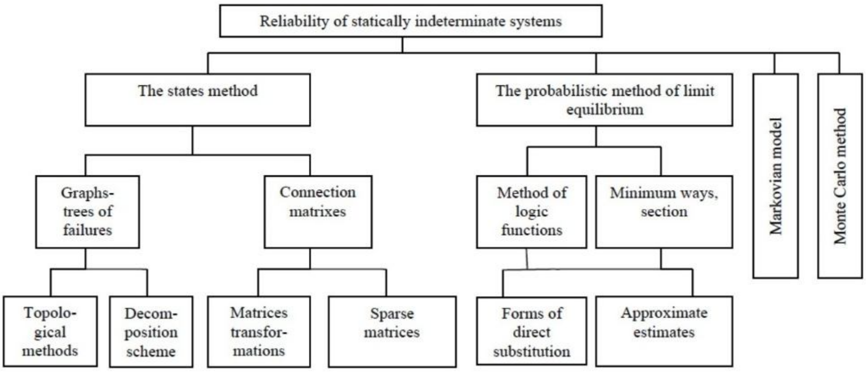



Fig. 12. Methods for assessing the reliability of statically indeterminate systems

In the engineering theory, reliability evaluation of complex systems is usually reduced to examination and analysis of two principal kinds of joints [23]:

a) series connection, failure-free performance probability of which at independent components is determined as

$$P_m = \prod_{i=1}^{m} P_i, \quad (1)$$

where Pi is probability of failure-free performance of component i;

b) parallel connection

$$P_m = 1 - \prod_{i=1}^{m} (1 - P_i). \quad (2)$$

Series connection in probabilistic meaning can be used for description of statistically determined system, e.g. trusses.

But practical evaluation of real structures reliability cannot be reduced to application of simple equation (1) in consequence of availability of correlation between resistance conditions of components.

Activities of statically non-determined systems is definitely associated with parallel connection, but evaluation of their reliability cannot be done according to Eq. (1) because redistribution of forces in the system after failure of separate components, which are dependent. Thus, reliability evaluation of statically non-determined structures requires thorough and careful analysis of their activities character and failure under load and discount of distinguishing features of failures of the components and the system on the whole.

A special technique of construction reliability level estimation has been developed [10]. This technique is a good example of computing the reliability of large -span roofs, as systems with series of elements connections.

Failure probability of was adopted as the quantitative characteristics to evaluate the reliability of suspension structures [105].

The survivability term should be noted together with reliability. Survivability is the ability of an object to keep (perhaps with a degradation of performance) working condition, even with damage of some parts. The term of construction survivability is directly related with that of sensitivity. As a rule, the last one is used for design purposes. There are various methods for sensitivity analysis, based on certain restrictions and conditions in the algorithm for design of structures using FEM [23]. Therefore, finite element computation algorithm is needed to analyze the survivability and sensitivity of construction. In the investigation of the survivability of the Ice Palace "Luzhniki" roof in Moscow, Russia, failure of individual components was simulated and construction functionality was estimated [10].

**5. Method of determining the reliability indices at the design stage**

It can be concluded, that current design standards do not contain demands on quantitative estimation of reliability of large -span roofs. Various approaches and structural design concepts, particularly for metal structures, have been significantly developed and justified however, nevertheless, there are no convenient and sufficiently simple methods for determining reliability of parameters for a structures in evident form. Known design solutions, based on ultimate state method, cannot be properly compared by the design concepts' reliability.

The hot topic of providing the required reliability level in design of large -span roofs, in particular suspension roofs and rod shells, in many aspects, determining the efficiency of large -span roofs construction, was considered and approved [24]. The design method of and design work of rigid through section threads based on determining numerical exponents of designed structure reliability has been made (Fig. 14).

The developed calculation and design technique of three-dimensional rod roof with a cut on elliptical basis can consider such problems as assigning the initial geometric parameters of the roof, initial selection of the section with further refinement using a finite elements model under different loading combinations. It also allows numerical calculation of reliability indices for statistically non-determined systems in a form of suspended shell with a big cut on elliptical plane – a problem that was not considered previously in design methodologies, based on the ultimate state method.

This type of roof corresponds to modern requirements of aesthetics and biomimetic architecture. Suspension rigid threads may be compared with the lianas, whole roof system with the bird's nest, where these lianas are the supporting elements (Fig. 13) [106]

The main load-supporting elements of the roof system: a) external contour, supported by columns or walls of the stadium; b) internal unsupported contour, supported by thrust; c) rigid thread with the form of trusses (Fig. 13).

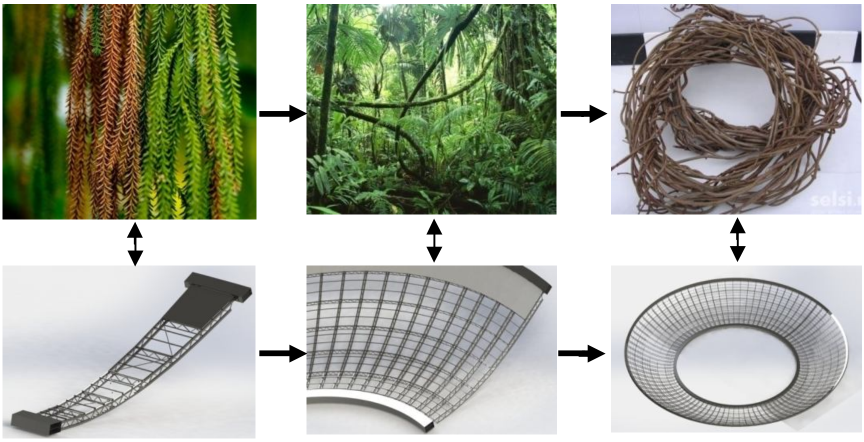


Fig. 13. Long-span suspension roof with rigid threads

The proposed method provides solution for the following problems:

- finding rational geometric parameters of a structure;
- obtaining appropriate rigidity characteristics of basic supporting elements;
- finding a track of elements failure for typical roof diagram with following evaluation of stressed and strained state of the object;
- finding of numerical safety indices of a structure (finding the lower and upper safety limits).

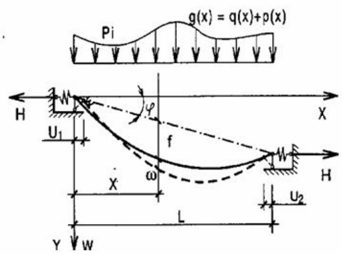
The described method has been applied to obtain the reliability indices of the stadiums roofs of FC «Schakhtar» in Donetsk, Ukraine [24]. It was shown that the obtained values of reliability indices for a roof structure satisfy the European codes requirements [20,22].

A design procedure of roof reliability can be described by a scheme presented in Fig. 14. The values designations of this scheme are shown in Table 1.

The above-described method has also shortcomings. It ignores issues related to the joints' action as part of the shell when calculating the roof reliability level. This issue opens a new research areas. The first steps in this direction were already made [82]. Fundamental approaches for providing reliability of suspension roof joints by numerical methods were determined. The authors suggest that creation of new computation and design methods for suspension roofs based on recently developed technique [24] will enable to improve the quality of computation methods and increase the reliability and durability of such type of structures. It will be based on the definition of numerical reliability indicators of the designed structure taking into account the performance of joints in the roof.

**1. Engineering analysis of the roof**

**1.1. Analysis of the bent and rigid thread**




Equation of thread equilibrium

$$EI\frac{dw}{dx^4} - H\frac{d^2(f+\omega)}{dx^2} + q(x) = 0$$

Forces in thread components

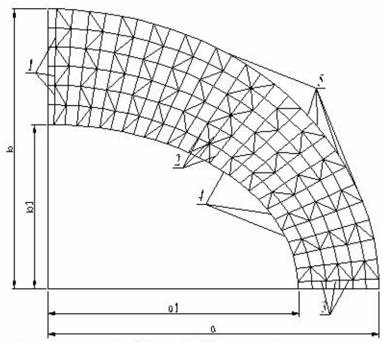
$$N_{UB} \approx N \cdot \frac{a_{UB}}{h} + \frac{M}{h} \cdot \alpha \cdot k$$

$$N_{LB} \approx N \cdot \frac{a_{LB}}{h} + \frac{M}{h} \cdot \alpha$$

Improved values of forces

$$N_{UB}^{y} = k'_{\sigma} k''_{\sigma} N_{UB} \quad N_{UB}^{y} = k'_{\sigma} k''_{\sigma} N_{UB}$$

**1.2. Structural analysis of the roof**




1. Assignment of the shell geometry parameters;
2. Snow load S requirement of $\overline{D}_1, \overline{D}_4, \overline{W}, \overline{U}$;
3. $\overline{D}_1, \overline{D}_4, \overline{W}, \overline{U}$ determination;
4. Determination of N and M forces from dependences of $\overline{D}_1 = f(\overline{M}, \overline{W})$, $\overline{D}_1 = f(\overline{N}, \overline{W})$ $\overline{D}_1 = f(\overline{M}, \overline{U})$, $\overline{D}_1 = f(\overline{N}, \overline{U})$

↓

**2. Numerical analysis of the shell in the software SCAD**

(determination of trace to failure)

↓

**3. Probabilistic design of the roof**

**3.1. Lower limit of reliability**

– Series connection (brittle model);
– Random quantities: $\widetilde{A}$, $\widetilde{\sigma}_y$;
– Computation of the reliability indices based on using the unique Monte Carlo method [16];
– Identification of the random quantity density distribution law and determination of its boundaries.

**3.2. Higher limit of reliability**

– Parallel connection (plastic model);
– Random quantities: $\widetilde{A}$, $\widetilde{S}$, $\widetilde{M}, \widetilde{N}$, $\widetilde{\sigma}_y$,

$$\widetilde{\sigma}(\widetilde{M}, \widetilde{N}, \widetilde{A}) = \overline{\sigma}_N \cdot (1 + \nu_N - \nu_A) + \overline{\sigma}_M \cdot (1 + \nu_M - \nu_A);$$

– Safety characteristics of Rzhanitsyn [16]

$$\gamma = (\overline{\sigma}_y - \overline{\sigma}) / \sqrt{\bar{\sigma}_y + \bar{\sigma}};$$

– Computation of the reliability indices of the roofing

$$1\text{-}P_{sys} = (1\text{-}P_{span}) \times (1\text{-}P_{ext.con.}) \times (1\text{-}P_{int.con.})$$

Fig. 14. A scheme for finding the numerical indices of suspension roof reliability

### 6. Results and Discussion

From the above-mentioned procedure the reliability indices of a spatial structural block of shell “G” as a constituent of the section NC of the “Donbas-Arena” Stadium in the city of Donetsk were determined according to the actual fact of the construction [24]. The span of the roof block is a spatial bar shell of a double curvature and variable height. The basic dimensions are: the shell external profile – 59.9 m, the shell internal profile – 31.7 m, the span – 61.2 m. The structure height varies and is 1.99 m – 5.3 – 3.1 m.

From the results of analyses carried out at the maximum normative snow load 160 kg/m$^2$ there was fixed the most stressed element, but stress σ in it was therewith 108 MPa < $\sigma_т$ = 440 MPa. To reach the limiting state of the elements this load was being increased step-by-step and reached 535 kg/m$^2$ when the failure tracing of the shell block elements was fixed, see Fig. 15.

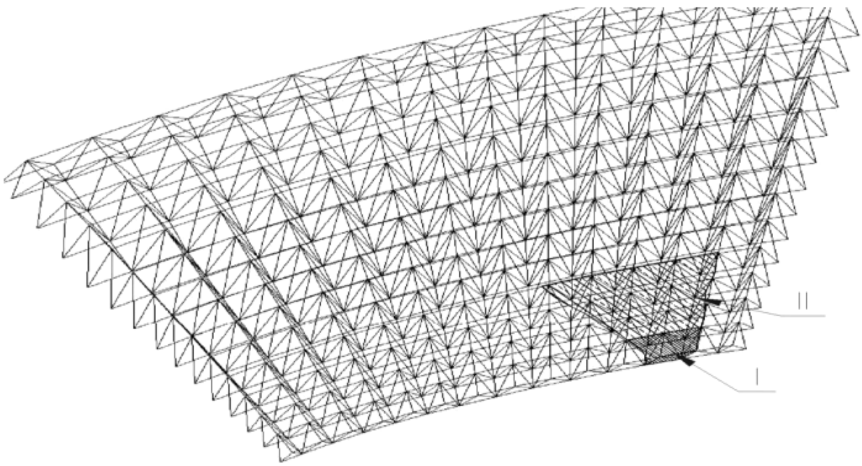



Fig. 15. A shell section being failed [24]:
(I) a raptured zone of the shell; (II) a predicted zone of the next failure of the shell

### 7. Conclusions

The paper reviews the latest issues and developments in suspension roofs, touches upon the historical perspective. It particularly addresses the latest design trends and concepts as well as ways for providing reliability of suspension roofs.

The proposed method provides solutions for the following problems: obtaining rational geometric parameters of a structure; finding appropriate rigidity characteristics of basic supporting elements; determining the elements failure trajectory for typical roof diagram with the following evaluation of stress - strain state of a structure; calculating numerical safety indices of a structure (determining the lower and upper safety limits).

The method enables to find the zones, where failure will be initiated. It offers an opportunity to create additional strength and reliability of structures, located in dangerous places, such as bearing joints of the connecting trusses to external contour and internal contour, the braces to the lower chord of the trusses, intermediate joints of upper and lower chords of supporting trusses etc., at the stage of design and construction.

Large-span roofs have increased liability level, since their failure can lead to severe economic and social consequences. In this case, the design of these unique structures should be based on complex approach for selecting the rational structural concept related to the structure's function, architectural concept, manufacturing methods, construction, etc. Reliability requirements, adaptability to manufacture, economic efficiency, ecological and social factors should be also fulfilled.

Young engineers should be inspired by the great structural forms of the past and be encouraged to study more works from our generation to spark improved designs in the future.

Based on the above, we can recommend to young researchers the universal algorithm, based on:

- preliminary computation;
- analysis of survivability;
- design according to the limit states requirements;
- design based on the numerical reliability indicators.

It will allow improvement of computation methods quality and more accurate analysis of roof structures. Using this approach also leads to increasing of reliability and durability of such types of structures and minimizes mistakes in designin and computation.

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*Стаття надійшла 20.12.2019*

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**A NEW APPROACH TO THE DESIGN OF SUSPENSION ROOF SYSTEMS**

Over the last century, the suspension roofs design has progressed until the advent of the shells theory in the first half of the 20th century, due to a rapid pace in technological advancement. A paradigm shift emerged with the new trend in structural design towards a new design process that cooperatively integrated economy, efficiency, and elegance. Different approaches in computation, design and reliability assessment of roof structures are discussed in this work to identify the key conditions that have significantly contributed to modern suspension roof design principles.

A new algorithm to assess the reliability of suspension roofs at the design stage is proposed and a novel method for computational design and reliability evaluation of suspension roofs is presented in this paper.

The proposed method provides solutions for the following problems: obtaining rational geometric parameters of a structure; finding appropriate rigidity characteristics of basic supporting elements; determining the elements failure trajectory for typical roof diagram with the following evaluation of stress - strain state of a structure; calculating numerical safety indices of a structure (determining the lower and upper safety limits).

The method enables to find the zones, where failure will be initiated. It offers an opportunity to create additional strength and reliability of structures, located in dangerous places, such as bearing joints of the connecting trusses to external contour and internal contour, the braces to the lower chord of the trusses, intermediate joints of upper and lower chords of supporting trusses etc., at the stage of design and construction.

Large-span roofs have increased liability level, since their failure can lead to severe economic and social consequences. In this case, the design of these unique structures should be based on complex approach for selecting the rational structural concept related to the structure's function, architectural concept, manufacturing methods, construction, etc. Reliability requirements, adaptability to manufacture, economic efficiency, ecological and social factors should be also fulfilled.

Young engineers should be inspired by the great structural forms of the past and be encouraged to study more works from our generation to spark improved designs in the future.

Based on the above, we can recommend to young researchers the universal algorithm, based on:
- preliminary computation;
- analysis of survivability;
- design according to the limit states requirements;
- design based on the numerical reliability indicators.

It will allow improvement of computation methods quality and more accurate analysis of roof structures. Using this approach also leads to increasing of reliability and durability of such types of structures and minimizes mistakes in designin and computation.

**Keywords:** suspension roof, stress-strain state, computational methods, reliability indices, roof failure.

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**НОВИЙ ПІДХІД ДО ПРОЕКТУВАННЯ СИСТЕМ ПОКРИТТІВ ВИСЯЧОГО ТИПУ**

Протягом останнього століття проектування підвісних покриттів інтенсивно розвивалося до появи теорії оболонок у першій половині 20-го століття, завдяки швидким темпам технологічного прогресу. Зміна парадигми виникла з новою тенденцією в структурному дизайні до нового процесу проектування, який швидко об'єднав економіку, ефективність та елегантність. У цій роботі розглядаються різні підходи до обчислення, проектування та оцінки надійності конструкцій висячих покриттів, щоб визначити ключові умови, які суттєво сприяли появі сучасних принципів утворення конструктивної схеми висячого покриття.

В цій роботі запропоновано новий алгоритм оцінки надійності висячого покриття на стадії проектування, а також новий метод розрахунку та оцінки надійності висячого покриття.

Запропонований спосіб пропонує рішення наступних завдань: отримання раціональних геометричних параметрів споруди; знаходження відповідних характеристик жорсткості основних опорних елементів; визначення траєкторії відмови елементів для типової схеми покриття з певною оцінкою напружено-деформованого стану конструкції; обчислення числових показників безпеки споруди (визначення нижньої та верхньої меж безпеки).

Метод дозволяє знайти зони, де буде починатися руйнування. Пропонується створити додаткову міцність і надійність конструкцій, розташованих в небезпечних місцях, таких як вузли з'єднання несучих ферм до зовнішнього контуру і внутрішнього контуру, в’язи до нижнього поясу ферми, проміжні вузли з'єднання верхніх та нижніх поясів несучих ферм тощо, на етапіпроектування та будівництва.

Великопрольотні покриття мають підвищений рівень відповідальності, оскільки їх відмова може призвести до серйозних економічних та соціальних наслідків. У цьому випадку проектування цих унікальних споруд має базуватися на комплексному підході до вибору раціональної конструктивної схеми покриття, пов'язаної з функцією конструкції, архітектурною концепцією, способами виготовлення, будівництвом та інше. Вимоги до надійності, адаптованості до виробництва, економічної та екологічної ефективності, а також соціальні фактори повинні виконуватися.

Молодих інженерів слід надихати великими конструктивними формами минулого і заохочувати вивчати більше робіт нашого покоління, щоб ініціювати вдосконалені проекти в майбутньому.

Виходячи з вищесказаного, ми можемо рекомендувати молодим вченим універсальний алгоритм, заснований на наступному:

- попереднє обчислення;
- аналіз живучості;
- проектування відповідно до вимог граничних станів;
- проектування на основі числових показників надійності.

Це дозволить покращити якість обчислювальних методів та отримати більш точний аналіз конструкцій покриттів. Використання цього підходу також призводить до підвищення надійності та довговічності таких типів конструкцій і мінімізує помилки в проектуванні та обчисленнях.

**Ключові слова:** висяче покриття, напружено-деформований стан, обчислювальні методи, показники надійності, руйнування покриття.

УДК 624:014

*Прядко Ю.М., Руднєва І.М., Рибаков Ю., Бартоло Х.* **Новий підхід до проектування систем покриттів висячого типу** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 191-220.

*Розглядаються різні підходи до обчислення, проектування та оцінки надійності конструкцій висячих покриттів. Запропоновано новий алгоритм оцінки надійності висячого покриття на стадії проектування та новий метод розрахунку висячого покриття.*

Іл. 15. Бібліогр. 106 назв.

UDC 624:014
*Priadko I.N., Rudnieva I.N., Ribakov Y., Bartolo H.* **A new approach to the design of suspension roof systems** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 191-220. – Eng.
*Different approaches in computation, design and reliability assessment of roof structures are considered. A new algorithm to assess the reliability of suspension roofs at the design stage is proposed and a novel method for computational design and reliability evaluation of suspension roofs is presented.*
Fig. 15. Ref. 106.

УДК 624:014
*Прядко Ю.Н., Руднева И.Н., Рыбаков Ю., Бартоло Х.* **Новый подход к проектированию систем покрытий висячего типа** // Сопротивление материалов и теория сооружений: науч.-техн. сборник. – К.: КНУСА, 2020. – Вып. 104. – С. 191-220.
*Рассматриваются различные подходы к вычислению, проектированию и оценке надежности конструкций висячих покрытий. Предложен новый алгоритм оценки надежности висячего покрытия на стадии проектирования и новый метод расчета висячего покрытия.*
Ил. 15. Библиогр.106 назв.

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UDC 514.18

# SYSTEM OF MODELING OF STRUCTURAL ELEMENTS OF VENTILATION SYSTEMS BY POLYCOORDINATE TRANSFORMATIONS

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DOI: 10.32347/2410-2547.2020.104. 221-228

**Abstract.** In this work, a computer system for modeling geometric objects is constructed. This system is instrumental in solving various problems that occur in construction, in particular in the design of ventilation systems. Our approach is based on a method of the polypoint transformations, namely on deformation modeling. Deformations of geometric objects could be described based on the given parameters of a dynamic deformation rather than on analytical equations. An object's form is changing due to a deformation of a space in which an object is located. Using the machinery of the polypoint transformations, a computer system for modeling geometric objects has been created. The system provides tools that simplify the constriction of surfaces with various types of sections.

**Key words:** deformation modeling, polypoint transformations, computer modeling, ventilation system, gearbox.

**1. Introduction.** There are various ways to supply and remove indoor air. The choice of ventilation system must take into account technological requirements for working conditions and living space, as well as economic factors. When designing ventilation, it is necessary to apply appropriate design and planning solutions using modern information technologies.

The composition of the system depends on its type. One of the most commonly used is mechanical systems. They include the following components: ducts; grates; diffusers; fans; heaters; filters, gearboxes and more. In any ventilation, ducts are an important structural element for supplying fresh air and removing polluted air. When designing ducts in construction, it is necessary to take into account the various forms of cross sections when connecting ducts, which is difficult in practice. Similar problems also arise when designing gearboxes, various ventilation systems for residential and domestic industrial premises.

The difficulty of this task is that the transitional structures listed may have cross-sections of different shapes at the ends that need to be joined: for example, round on one side and square on the other. This problem will be solved by creating a system of modeling by means of polycoordinate transformations.

**2. Literature Review and the Problem Statement.** In [1] formulas of polypoint transformations are given, the concept of poly-point coordinates is introduced and the method of transformation of a straight line in a point cascade is described.

[2, 3] provides examples of how poly-point transforms are used to control the shape of an object. The analysis of different ways of polypoint transformations is carried out.

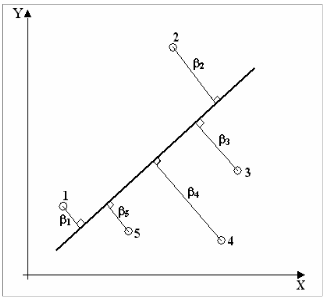
[4] provides an example of the use of different types of polypoint transformations to solve the extrapolation problem.

The analysis of these studies indicates the need to create a computer system for modeling geometric objects, which would allow solving a number of problems that arise in the design and design in construction.

**3. Formulating the goals of the article.** The purpose of this study is a computer-aided system for modeling geometric objects, which is based on the theory of polypoint transformations, created with the help of modern information technologies, which would greatly simplify the existing processes of constructing surfaces with different types of sections.

**4. Main Materials of the Study.** Polycoordinate transformations [1] can be used in various fields of production at the stage of modeling of investigated processes. Polycoordinate transformations are divided into polytissues and polypoint.

Let's take a closer look at polypoint transformations using a multipoint framework (Figure 1).



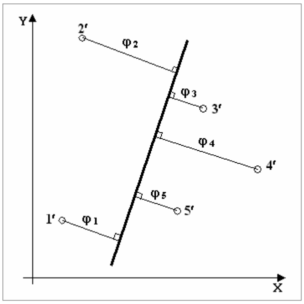



Fig. 1. Polypoint transformations on a plane

Polypoint transformations allow you to change the position of a straight line (object of transformation) by manipulating the points of the transformation base. As can be seen from Figure 1, the initial base (points 1,2,3,4,5) was changed to 1', 2', 3', 4', 5'. In this case, the position of the line changed according to the change in the basis of points.

This is achieved by decoupling the system, which establishes a functional relationship between the polycoordinate coefficients of the direct before and after transformations ($\beta_i$ and $\varphi_i$).

Since polypoint transformations allow one line to be transferred from the initial basis to another, two and several lines can be "transferred" in the same way. This, in turn, means that you can transform objects in this way. Figure 2 shows the polypoint circle transformation. As you move the points, the circle turns into a closed curve.

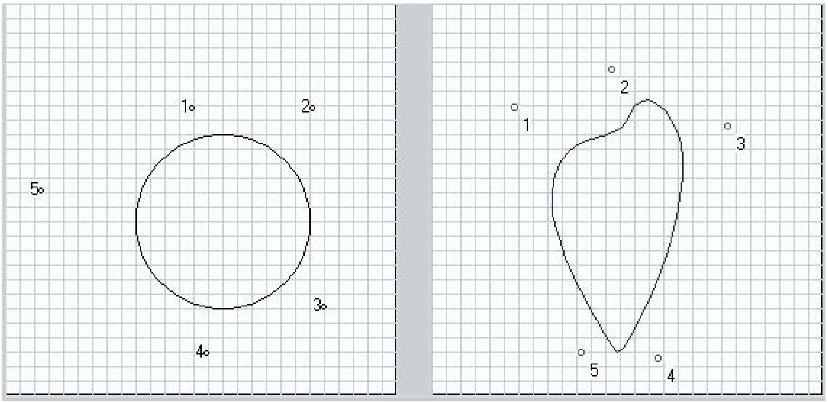



Fig. 2. Conversion of a circle at the five-point base

Thus, polypoint transformations allow you to track the deformation of a geometric object, affecting only the space that limits that object. There are different ways of influencing the basis [2, 3]. For example, you can move the basis points in the plane, and you can enter and manipulate the weights by increasing or decreasing the weight at a particular point of the basis, or at all points in the same way. Figure 3 shows an example of a circle (object of transformation) with a change of weight is converted into a square (with increasing weight), or a curvilinear rhombus (with decreasing weight).

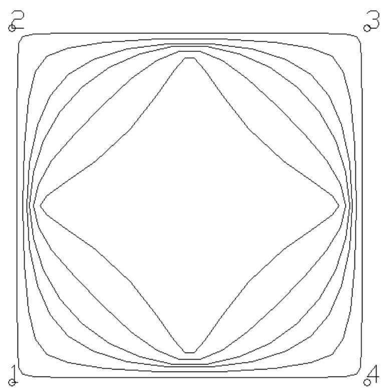



Fig. 3. Weight transformations of a circle at a four point basis

In three-dimensional polypoint transformations, the images will be not planes but straight. Two point bases are introduced: initial and converted. As the weights of the basis points change, the shape of the three-dimensional objects changes. These transformations also have different modifications depending on the appearance of the scales.

Let's take a closer look at polypoint transforms using a multipoint frame.

The plane in the initial basis can be described by a system of equations:

$$\gamma_i = ax_i + by_i + cz_i + dh_i, i = 1...n .$$

The area after conversion will be determined by such a system:

$$\varphi_i = AX_i + BY_i + CZ_i + DH_i,\ i = 1...n,$$

where $\varphi_i$ – the distance from the plane (the prototype) to the points in the initial base.

The system determines *A*,*B*,*C*,*D* – coefficients of the transformed plane.

The formula of multipoint transformation is written as:

$$\varphi_i = \omega_i \gamma_i,\ i = 1...n.$$

So,

$$\omega_i \gamma_i = Ax_i + By_i + Cz_i + Dh_i,\ i = 1...n.$$

Functional for unambiguously solving the problem:

$$\sum (\varphi_i - \gamma_i)^2 \to 0,\ i = 1...n.$$

Therefore, it is necessary to find partial derivatives for all four variables.

$$\frac{\partial \sum (\varphi_i - \gamma_i)^2}{\partial A} = 2\sum X_i (AX_i + BY_i + CZ_i + DH_i - \gamma_i) = 0,$$

$$\frac{\partial \sum (\varphi_i - \gamma_i)^2}{\partial B} = 2\sum Y_i (AX_i + BY_i + CZ_i + DH_i - \gamma_i) = 0,$$

$$\frac{\partial \sum (\varphi_i - \gamma_i)^2}{\partial C} = 2\sum Z_i (AX_i + BY_i + CZ_i + DH_i - \gamma_i) = 0,$$

$$\frac{\partial \sum (\varphi_i - \gamma_i)^2}{\partial D} = 2\sum H_i (AX_i + BY_i + CZ_i + DH_i - \gamma_i) = 0.$$

These four equations form a linear system of equations.

$$A \cdot V = B,$$

where

$$A = \begin{bmatrix} \sum X_i^2 & \sum X_i Y_i & \sum X_i Z_i & \sum X_i H_i \\ \sum Y_i X_i & \sum Y_i^2 & \sum Y_i Z_i & \sum Y_i H_i \\ \sum Z_i X_i & \sum Z_i Y_i & \sum Z_i^2 & \sum Z_i H_i \\ \sum H_i X_i & \sum H_i Y_i & \sum H_i Z_i & \sum H_i^2 \end{bmatrix},$$

$$V = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}, B = \begin{bmatrix} \sum X_i \gamma_i \\ \sum Y_i \gamma_i \\ \sum Z_i \gamma_i \\ \sum H_i \gamma_i \end{bmatrix}.$$

Unleashing the system, we get the values *A*, *B*, *C*, *D* - the coefficients of the plane after conversion.

The algorithm for converting a three-dimensional body can be described as follows:

• the 3D object is immersed in a point base. This is done by the user by selecting the basis points based on the conditions imposed on the task, such as the final shape of the object;

• the body to be deformed is represented by a set of planes (triangles). This can be done, for example, by triangulation;

• consecutive polypoint transformations of each plane are performed and their intersection is determined;

• using the existing methods of spatial interpolation (or the capabilities of modern graphics packages) smoothies the resulting surface.

Figure 4 shows an example of a deformation of a sphere as the basis weights change.

The polypoint transformations considered allow modeling of deformations of different geometric elements. Exterior, modeling gearboxes (adapters) with different breaks in front of different limbs, which allow to connect with each other the ducts of different system ventilations of internal living and industrial premises. Examples of some reducers are presented in Figure 5.

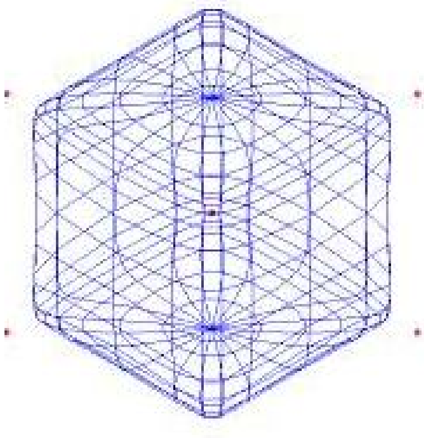


Fig. 4. Three-dimensional poly-point transformations

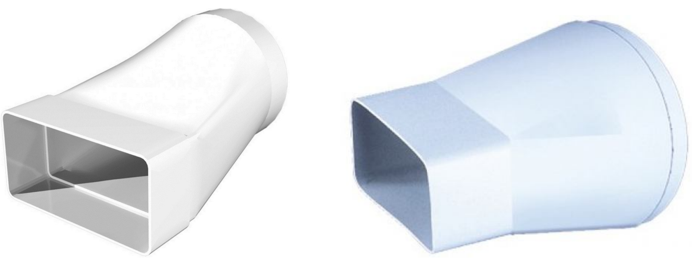


Fig. 5. Gearboxes with rectangular and circular sections

To visualize the processes of deformation of an object on the basis of polypoint transformations, a system has been developed that allows to control the positions of the basis points, ie their coordinates, and also to set the weights of these points. The system lets you view the animation. The object rotates around three axes in real time. This allows the user to view in real time the shape change of the object and to analyze the deformations that have taken place.

An example of how the system works is shown in Figure 6.

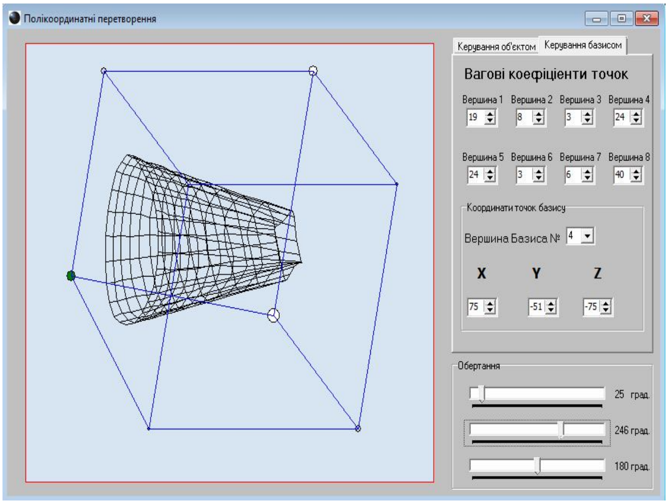



Fig. 6. Deformation of an object after changing the coordinates of the basis points

As can be seen from Figure 6, the user can deform the object and get the shape he needs, by simply manipulating the controls.

**5. Conclusions.** Studies have shown that polycoordinate transformations can not only model the shape of deformed objects with the ability to visually track these processes, but also construct technical objects such as reducers - adapters with different cross-sections at the ends. In the process, a computer system was created whereby the user could deform the object and get the shape it needed by simply manipulating the controls, which can be useful in the process of producing gearboxes.

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*Стаття надійшла до редакції 20.12.2019 р.*

*Сидоренко Ю.В., Кривда О.В., Лещинська І.В.*

**СИСТЕМА МОДЕЛЮВАННЯ КОНСТРУКТИВНИХ ЕЛЕМЕНТІВ ВЕНТИЛЯЦІЙНИХ СИСТЕМ ЗАСОБАМИ ПОЛІКООРДИНАТНИХ ПЕРЕТВОРЕНЬ**

Дослідження присвячене необхідності створення системи моделювання геометричних об’єктів, яка дозволила б вирішити ряд завдань, що виникають при конструюванні та проектуванні в будівництві, а саме у вентиляційних системах. Завдання вирішується з використанням політочкових перетворень. За допомогою деформаційного моделювання можна відображати процеси зміни форми геометричних об’єктів без певного виду аналітичного представлення, користуючись тільки параметрами динамічної деформації. Зміну форми об’єкта викликає деформація простору, в якому знаходиться об’єкт. Представником даного класу моделей є політочкові перетворення. На основі апарату політочкових перетворень була створена комп’ютерна система моделювання геометричних об’єктів, яка дозволяє спростити процеси конструювання поверхонь з різними видами перерізів.

**Ключові слова:** деформаційне моделювання; політочкові перетворення; комп’ютерне моделювання, вентіляційна система; редуктор.

*Сидоренко Ю.В., Кривда Е.В., Лещинская И.В.*

**СИСТЕМА МОДЕЛИРОВАНИЯ КОНСТРУКТИВНЫХ ЭЛЕМЕНТОВ ВЕНТИЛЯЦИОННЫХ СИСТЕМ СРЕДСТВАМИ ПОЛИКООРДИНАТНЫХ ПРЕОБРАЗОВАНИЙ**

Исследование посвящено необходимости создания системы моделирования геометрических объектов, которая позволила бы решить ряд задач, возникающих при конструировании и проектировании в строительстве, а именно в вентиляционных системах. Задача решается с использованием политочечных преобразований. С помощью деформационного моделирования можно отображать процессы изменения формы геометрических объектов без определенного вида аналитического представления, пользуясь только параметрами динамической деформации. Изменение формы объекта вызывает деформацию пространства, в котором находится объект. Представителем данного класса моделей являются политочечные преобразования. На основе аппарата политочечных преобразований была создана компьютерная система моделирования геометрических объектов, которая позволяет упростить процессы конструирования поверхностей с различными видами сечений.

**Ключевые слова:** деформационное моделирование; политочечные преобразования; компьютерное моделирование, вентиляционные системы; редуктор.

УДК 514.18

*Сидоренко Ю.В., Кривда О.В., Лещинська І.В.* **Система моделювання конструктивних елементів вентиляційних систем засобами полікоординатних перетворень** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 221-228.

*Дослідження присвячене необхідності створення системи моделювання геометричних об’єктів, яка дозволила б вирішити ряд завдань, що виникають при конструюванні та проектуванні в будівництві, а саме у вентиляційних системах. Завдання вирішується з використанням політочкових перетворень.*

Табл. 0. Іл. 5. Бібліогр. 4 назв.

UDC 514.18

*Sydorenko Iu.V., Kryvda O.V., Leshchynska I.V.* **System of modeling of structural elements of ventilation systems by polycoordinate transformations** // Strength of Materials and Theory of Structures: Scientific and technical collected articles. - Kyiv: KNUBA, 2020. – Issue. 104. – P. 221-228.

*The research is devoted to the need to create a system of modeling geometric objects that would solve a number of problems that arise in the design and design in construction, namely in ventilation systems. The problem is solved using polypoint transformations.*

Tab. 0. Fig. 5. Ref. 4.

УДК 514.18
*Сидоренко Ю.В., Кривда Е.В., Лещинская И.В.* **Система моделирования конструктивных элементов вентиляционных систем средствами поликоординатных преобразований** // Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2020. – Вып. 104. – С. 221-228. – Англ.

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UDC 539.3;624.1

# SCIENTIFIC SUBSTANTIATION OF ENGINEERING PREPARATION MEASURES DUE TO THE INFLUENCE OF CONSTRUCTION IN THE DENSE BUILDING CONDITIONS

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DOI: 10.32347/2410-2547.2020.104.229-241

The method of studying the stress-strain state of reinforcement structures of combined soil massifs has been proposed with the geometric and physical nonlinearity in the formulation of the problem based on the nonlinear theory of elasticity and plasticity of the soil. The study of the stress-deformed state of the computational domain from the standpoint of the mechanics of the deformed solid body had been carried out, using algorithms for solving the problems of the theory of elasticity and plasticity, with the construction of universal computational models of the combined half-space, that allows to determine more reasonably the magnitude of the stress-strain state of complex soil bases in interaction with the reinforcement structures, the surrounding buildings foundations and the whole complex of the surrounding buildings structures. The influence of new construction on the condition of soil bases and foundations of adjacent buildings had been evaluated by determining the change of pressure on the reinforcement structure and determining the stress-strain state change of this structure and the foundation of the existing house. The change in the deformation of the foundation of the existing building, ie the oscillation of the soil foundation and the maximum amplitude of uneven subsidence of the foundations of the existing building have been determined. Each formulation of the problem had to include its own reliability analysis and a specific approach that requires numerical modeling and development of appropriate measures to scientifically substantiate engineering preparation measures in dense building. Further design of protective reinforcement structures for new construction under the conditions of the building reconstruction has carried out considering the impact on the existing buildings and structures and the adjacent soil mass of different stages of construction, begining with the arrangement of protective reinforcement structures, the development of a excavation due to the effect of unloading the foundation, and the sequence of erection of engineering structures.

**Keywords:** new construction, scientific substantiation, stress-strain state, reinforcement structures, engineering preparation.

**Introduction.** In the process of reconstruction of urban territories by carrying out new construction in the conditions of dense construction, in particular, large-scale use of underground space in difficult engineering-geological conditions, the activation of additional processes in the soil bases of territories and foundations of existing buildings and structures had been revealed. Research of the interaction of soil bases with the objects of existing construction in the process of reconstruction of districts are associated with the determination of the stress-deformed state and stability of the soil massif, and the determination of the deformability and strength of the structures of its strengthening. Estimation of the stress state of the half-space involves comparing the results of the calculation with the maximum permissible deformations and displacements, ie, possible local areas of loss of stability and development of plastic deformations, that is, reaching the boundary condition of the soil bases in the continuous development of shear deformations.

**Problems of designing enclosing structures of deep ditches in the conditions of dense building**. Tasks for the study of enclosure constructions should be solved with the simultaneous combination of such scientific directions as nonlinear theory of elasticity and plasticity, nonlinear soil mechanics, structural mechanics of combined structures, engineering geology, calculation of foundations and foundations of deep formation, also studies of the behavior of structures of existing buildings and structures under changed conditions.

Development of theoretical bases for the study of large elastic and elastic-plastic deformations of a continuous medium, mathematical modeling of its joint work with structures of buildings and structures, the study of the tense state and the development of calculation methods, the influence of engineering-geological conditions on the processes of reconstruction, issues of engineering and engineering a lot of work is devoted to the development of the theory of reliability, however, the application of any single theory or methodology does not allow to study all aspects of engineering ovky, the whole complex of factors influence the reconstruction of existing urban areas typically associated with the construction of new facilities, insufficiently studied soil deformation processes in the fundamentals surrounding dense housing structures in complex geological conditions. All this requires the development and improvement of special theoretical, constructive and planning decisions on the scientific substantiation of the reconstruction, the creation and development of effective methods of mathematical modeling and scientific research of engineering preparation objects, which most fully take into account the specifics of the interaction of structures with soil semi-space scientific analysis of the possible volume of reconstruction, as well as the creation of appropriate concepts for consideration the impact of new construction in dense building and predicting its consequences, which generally determine the priorities of modern civil engineering.

Thus, in order to solve similar problems and scientific and technical substantiation of urban reconstruction decisions under the above conditions, it is necessary to combine the mentioned sciences of the construction industry within the framework of continuous environment mechanics in the general formulation, that is, considering geometric and physical nonlinearities, states at different stages of deformation of inhomogeneous materials, interaction of multimodal materials, solids with a continuous elastic-plastic environment, etc. Therefore, the complex problem of boundary equilibrium of the soil massif in a flat formulation with wall-to-soil enclosing structures, structures of reinforcement of inhomogeneous multilayer soil masses, shallow foundations, pile foundations, and foundations has been considered.

**Methodology for the condition research of the enclosure structures in interaction with the soil semi-space**. Therefore, to carry out a reliable analysis of the degree of new construction impact on the state of the foundations of adjacent existing buildings, it is necessary to solve a complex scientific problem related to the methods of continuum mechanics in the most general approach. The solution to this complex problem tasks associated with the development of a research methodology combined space and the interaction of solid deformable bodies from the soil mass, based on the laws of the nonlinear theory of elasticity and plasticity, nonlinear soil mechanics, variational methods, apparatus connecting nonlinear programming, efficient numerical methods - finite element method – attached to discretization in space, and finite difference method – attached to discretization in time [3].

Determination of the stress-strain state of a ditch fence, foundation soil and structures located near the ditch, is carried out on the basis of solving the problems of soil mechanics, in which real soil is replaced by a specific model [7]. To carry out researches the impact on the stress-strain state of the foundations and foundations of adjacent buildings the methodology for modeling the interaction of enclosure structures with inhomogeneous soil semi-space has been developed based on a nonlinear theory of elasticity and plasticity, advanced mechanical model of soil semi-space stability, considering the extended fluidity criterion, developed basic relationships of the finite element method, based on an efficient scheme with geometric and physical nonlinearity in the formulation of the problem [1, 2].

A flat problem of nonlinear elasticity theory about interaction of a fence structure with soil space is considered. The formulation of the problem assumes discrete modeling of substantially inhomogeneous soil layers, as well as the presence of solid inclusions (which are several orders of magnitude greater than the stiffness of the soil mass layers), modeling elements of fence structures, foundation soils, and foundation structures of surrounding buildings, as well as anchors, which causes the presence of stress concentrations and the development of plastic deformations at the boundaries of elements inclusions in the soil in the first limiting state in accordance with nonlinear soil mechanics. For the calculation of local stability losses in the presence of significant displacements and continuous development in local zones of plastic deformation shifts, the problem of flat deformation of an inhomogeneous anisotropic half-space is considered, taking into account geometric and physical nonlinearity in the formulation of the problem.

In the initial ratios of the proposed methodology, the equilibrium of a continuous medium is described by the first principle of virtual work, in the absence of restrictions on the nature of external influences, ie can be taken into account both conservative and non-conservative forces, for example, tracing forces in the conditions of geometric nonlinearity and in the presence in the presence of unilateral connections:

$$\int_{v}\left(\sigma'^{ij}+c^{ijkl}{}_{(e,p)}\right)\delta\gamma_{ij}dv-\int_{v}p^{i'}\delta u_{i'}dv-\int_{s}q^{i'}\delta u_{i'}ds=0\,, \tag{1}$$

where $\sigma'^{ij}$ there are components of the initial stress tensor; $c^{ijkl}{}_{(e,p)}$ there are components of the elastic tensor in the elastic-plastic state of the material; $\delta\gamma_{ij}$ there is a variation in the increment of the Cauchy-Green finite deformation tensor; $p^{i'}$, $q^{i'}$ there are components of generalized vectors of volume and surface forces in a global Cartesian coordinate system; $\delta u_{i'}$ there are variations of the vector components of the increments of displacements in the global coordinate system.

Variational equation (1) is described in increments of displacements, strains and stresses when the initial state is natural, that is, the stresses are zero, and describes the equilibrium elementary volume (finite element) of an arbitrary continuous medium in accordance with the energy methods of the problem solving approach, regardless of its physical properties, it is adequate to such a stress state, when a small additional influence can disturb the equilibrium. This stress state is also characterized by the fact that the shear resistance in the elementary region (finite element) is determined in the limit state for this type of soil. This condition relates to the second phase of the boundary states of the soil with a significant development of shear deformations in the soil massif [8].

In the proposed method of solving the problems of soil mass stability, the criterion of soil stability or fluidity in a separate homogeneous isotropic elementary region (finite element) is described in a universal form (in the form of stress invariants) based on the Mises extended fluidity criterion, which also includes the Coulomb-Mohr principle, when the intensity of stress is described by hydrostatic pressure, considering the angle of internal friction (tg$\varphi$) and soil adhesion values ($c$), and taking into account not only the second but also the third invariant of the tensor-deviator of the stress functions through the Lode-Nadai invariant:

$$f\left(\tilde{\sigma},\tilde{S},\tilde{\varepsilon}^{(P)},\alpha,\varphi,c\right)=\frac{3}{2}I_1\left(\tilde{S}^2\right)\left(\cos\alpha-\frac{1}{\sqrt{3}}\sin\alpha\cdot\sin\varphi\right)^2-$$

$$-\left[\frac{1}{\sqrt{3}}I_1\left(\tilde{\sigma}\right)\sin\varphi-\sqrt{3}\,c\cdot\cos\varphi\right]^2=0\,;$$

$$\alpha=\frac{1}{3}\arcsin\left\{-\sqrt{6}\,\frac{I_1\left(\tilde{S}^3\right)}{\left[I_1\left(\tilde{S}^2\right)\right]^{\frac{3}{2}}}\right\}, \tag{2}$$

where $\tilde{\sigma},\tilde{S},\tilde{\varepsilon}^{(P)}$ there are tensors of general (total) stresses, stresses of the deviatorial part and plastic deformations, respectively; $I_1(\tilde{S}^2)$, $I_1(\tilde{S}^3)$ there are the first invariants of the square and the cube of the stress tensor; $\varphi$, $c$ there are angle of internal friction and specific soil adhesion respectively.

Thus, the theory of plastic flow, which is based on the construction of differential relations between stresses and strains or in the form of increments, allows to perform modelling the materials elastic-plastic behavior more accurately. The plasticity phenomenon, which depends on the load history, needs to calculate the derivatives and increments of plastic deformation over the entire load history, with subsequent integration of the accumulated stresses. The surface of plasticity is a surface of fluidity that for hardening materials can change as the stress state changes. Negative values of the load function correspond to the elastic region. During unloading the plastic deformation increment and the Odquist plasticity parameter are zero and the incomplete differential of the loading function is less than zero. Under neutral loading, the stress state is on the elasticity border and no change in the plasticity surface occurs. For numerical researches of the soil massif stability, the variational equilibrium equations (1) and the load surface equation in the six-dimensional stress space (2) can be used as the initial ratios in the proposed methodology.

Thus, the proposed theory is a development of the theory of the boundary stress state of the soil half-space based on the introduction of an extended yield criterion for a flat problem of nonlinear theory of elasticity and plasticity, which involves determining the magnitude of the second critical load, in which continuous stretches of boundary stress state occur in the soil half-space.

The theory developed is based on the ideas of generalizing the dependencies of soil mechanics and consists in constructing the relations of the stress-strain state of the computational domain from the positions of the mechanics of the deformed solid, using algorithms for solving the problems of the theory of elasticity and plasticity, with the construction of universal computational models of combined half-space, which makes it possible to more accurately determine the magnitude of the stress-strain state of complex soil bases, which interact with the enclosure structures of the fortifications, the foundations of the adjoining buildings and the whole complex of structures of the surrounding building [4, 5]. Based on these provisions, a methodology for investigating the interaction of protective protective structures with soil half-space in the transboundary state has been developed, considering the geometrical and physical nonlinearities in the problem formulation, in the implementation of the evolution of complex loading taking into account the active and passive components of the load and the effect of unloading the combined half-space, providing a more accurate account of both elastic and plastic deformations, and allows to determine more reasonably the magnitude of stress-strain state of soil bases and foundations, constructions of buildings and structures of surrounding development, which are in interaction with complex soil bases, that is, it allows to study the processes that occur in soil bases during the reconstruction of urban territory, which essentially constitute an assessment of the impact of new construction on adjacent buildings, especially in difficult engineering-geological conditions.

ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104

**Scientific substantiation of measures for engineering preparation of construction in the conditions of dense building**. For the purpose of to determine the factors of influence on the existing buildings during the reconstruction of a section of urban territory in the conditions of dense housing within the framework of scientific and technical support for the design and construction of an apartment building with built-in office premises and underground parking has been performed design and calculation researches with the implementation of numerical calculations of half-space to a depth of 41.0 m in the central section of the study site with a length of 150.0 m.

Numerical researches have applied the methodology of solving the problem of nonlinear soil mechanics by the boundary equilibrium of soil arrays when interacting with the enclosure structures of deep ditches and elements of foundations of new and existing buildings. The results of the combined half-space study using a new model of the multilayer soil massif equations state have been obtained for a specific problem in the variant of the pit enclosing structure interaction with the ground half-space, and the foundations of the existing residential five-storeyed building, when the active pressure on the retaining wall in the soil exceeds the passive pressure of the soil resistance at the base of the pit under the building being designed.

The purpose of the research is to determine the change in pressure on the enclosure structure over time and to determine the change in the stress-strain state of the enclosure structure itself and the basis of the existing dwelling house. The result of determining the change in the deformation of the foundation of a 5-storey building is the fluctuation of the soil foundation and the maximum amplitude of uneven subsidence of the foundations of an existing building, which is a criterion for determining the influence of adjacent construction in conditions of dense urban development on the condition of foundations and foundations of adjacent buildings.

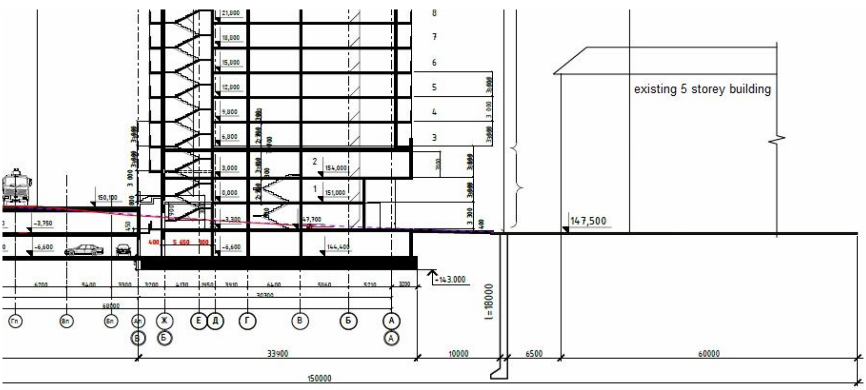



Fig. 1. The section of the projected multi-storey residential building and the existing 5-storey residential building

The initial data are determined physical and mechanical characteristics of the layers of soil half-space according to the data of engineering and geological surveys, considering correction of soil multilayer half-space properties in depth, as well as the physical and mechanical characteristics of inclusions in the half-space of concrete and reinforced concrete elements, among which are piles of enclosing structures such as solid "wall in soil", grate and solid reinforced concrete foundation slab of concrete, slabs of foundations of existing buildings and concrete wall blocks.

The estimated load per 1 sq.m of the grid surface area has been determined, which is 82.32 kN/m$^2$. The given load on the cutting edge of the foundations of a five-storey existing residential building is 14.22 kN per 0.5 m of the length of the outer wall. A discrete model and calculation scheme of interaction of enclosing structures of the pit, pile foundations of a new residential building and the foundations of an existing dwelling house with soil half-space have been constructed.

The initial variation of the motion equation in accordance with energy methods describes the equilibrium of the elementary volume of an arbitrary continuous medium, regardless of its physical and mechanical properties. The proposed methodology implements the applied approach of variational principles and the theory of boundary stress deformed body, when the resulting solutions are related to the first distribution of elastic regions into elastic and non-elastic regions with developed zones of elastic-plastic (soil-shear) deformations. The original finite element model in the deformation process is transformed in accordance with the criterion of fluidity (destruction) of the soil array and is divided into two zones of stress-strain state determination: elastic and elastic-plastic using a Coulomb – Mohr criterion loading surface considering the tensor-deviator invariant of the stress function through the Lode – Nadai invariant [8]. The proposed methodology also uses Mises' extended modified fluidity criterion, which enables more accurate solutions to the stability problems of the combined multilayer half-space.

The discrete model and the calculation scheme of the multilayered soil half-space were constructed taking into account the presence of inclusions of the structures of the ditches, the foundations of the new building, the existing structures and cavities.

The grid area of the discrete model $S_1$, $S_2$, $S_3$, is regular and placed within $S_1 = 1, M1$, $S_2 = 1, M2$, $S_3 = 1, M3$. Dimensions values of grid area are equal to:

$$S_1 = M1 = 2\,;\; S_2 = M2 = 25\,;\; S_3 = M3 = 104\,.$$

Thus, the number of nodes in the grid two-layer area is equal to:

$$N_{UX} = M1 \times M2 \times M3 = 2 \times 25 \times 104 = 5200\,.$$

Accordingly, the number of nonlinear equations is the system of:

$$K = 3 \times N_{UX} = 3 \times 5200 = 15600$$

equations without consideration the imposed boundary conditions.

The grid area describes a discrete finite element model of 24 x 103 = 2472 finite element elements, including cavities with boundaries, which is determined by the grid coordinates $S_1$, $S_2$, initial and final nodes of the regular area - respectively 1, 63; 2, 64; i 1, 64; 3, 104. These coordinates are determined the permanent cavity for both variants of the calculation scheme.

The transformed cavity, which models the pit development of the building underground part, in the discrete model is determined by the corresponding nodal grid coordinates; within the regular domain boundary nodes of inclusions of soil layers of multilayered half-space are described, as well as the inclusions elements of the enclosures "walls in soil" and foundation structures.

The geometric dimensions of the estimated half-space are 41x150 m, the thickness of the half-space is equal 50 cm.

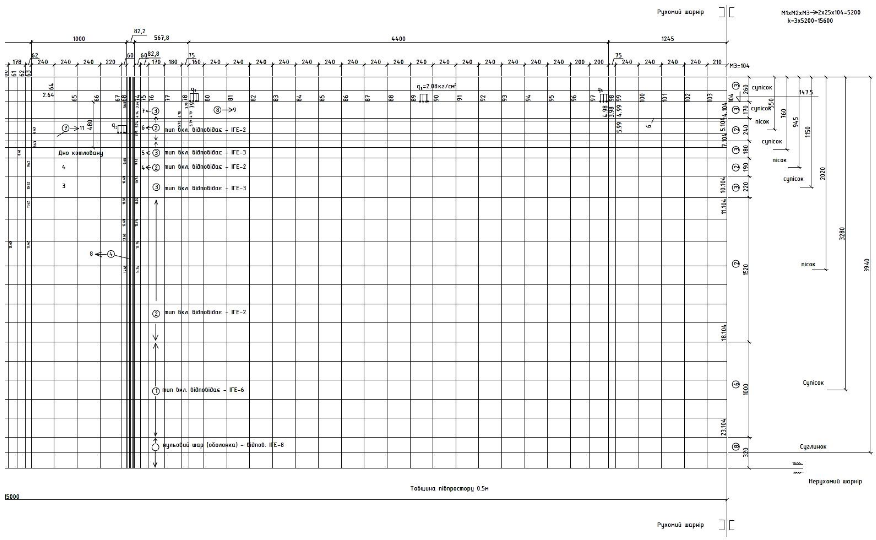


Fig. 2. Discrete model and calculation scheme of half-space:
influence on the stress-strain state of half-space from a 5-storey building with a free pit

**Results of numerical research of a discrete model of combined half-space.**

The accepted calculation variant of interaction the fencing structures and the existing building foundations with soil half-space at a free pit is a classic for solving this problematic task, and, as a rule, the most dangerous in terms of the stability of the pit slope. But in this version of the problem statement, this option was quite safe, because the pit depth is due to the difference planning mark on 2,5 m is only 4,5 m and the active pressure from such a slope prism was insignificant.

The calculation is implemented at three values of the depth of the "wall in the soil" – 17.2 m, 12.6 m i 10.3 m. Based on these results, the corresponding appropriate plots of, settlements, and internal efforts in the wall-in-soil structure elements have been constructed, that are shown on Fig. 3, 4, 5.

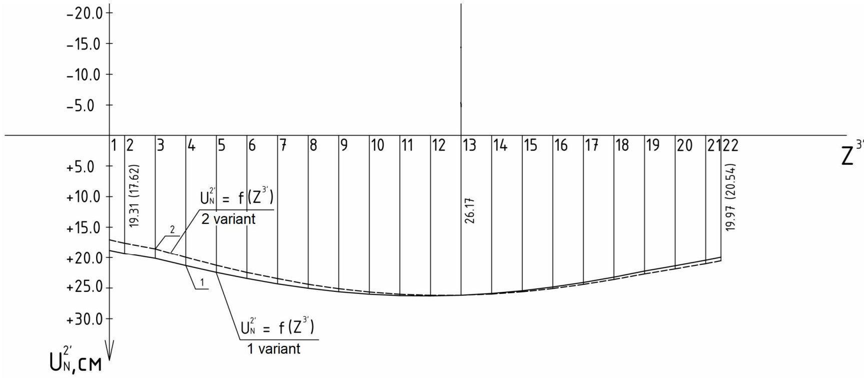
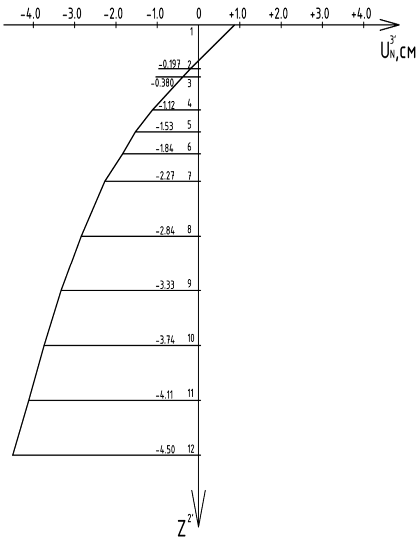



Fig. 3. Plot of nodal displacements (vertical sediments) placed on the sole of the foundation from additional external loads under different conditions:
1 variant – before developing of the pit; 2 variant – after development of the pit to the mark of the free bottom (before arranging the pile foundation under a new tall building)



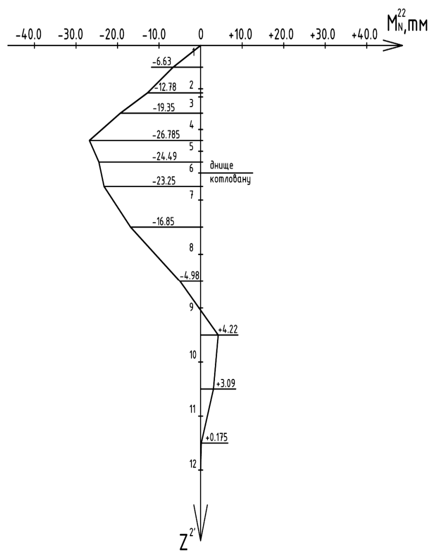




Fig. 4. Plot of transverse deflections "wall-in-soil"

Fig. 5. Plot of bending moments in cross section "wall-in-soil"

According the character graphics displacements of the free pit bottom there soil bulging, but insignificant – in the center of the pit

$$U_{1165}^{2} = -10{,}15 \text{ cm}.$$

At the width of the pit 81.23 m such output is negligible. According to the plot of vertical movements of the sole of the foundation of the existing 5-storey building it can be seen that the average uneven subsidence in the half-span of the foundation length is:

$$\frac{4,56}{2200} \cong 0,0021 < 0,0024,$$

that is less than the standard non-uniform clockwise rotation, how the sliding prism moves under active pressure.

The stress-strain state of the wall-in-soil structure is illustrated by a plot of transverse deflections (Fig. 4) and plot of bending moments in cross section "wall-in-soil" (Fig. 5).

From the analysis of the nodes displacements located on the vertical face of the wall-in-soil (retaining wall) construction, it can be concluded, that the transverse displacements of the retaining wall occur as a significant whole by rotating it substantially as a whole clockwise. In such a turn, the internal forces are insignificant within 50% margin of safety of the retaining wall strength.

**Conclusions and recommendations**

The task of pressure changing on the enclosure structure in time and determining the change in the stress-strain state of the enclosure structure and the existing dwelling house basis has been solved.

The change in the 5-storey building foundation deformation has been determined, that is, the oscillation of the soil foundation and the maximum amplitude of uneven subsidence of the existing building foundations, that is a criterion for the influence determining of adjacent construction in an urban area on the soil bases condition and foundations of adjacent buildings.

Results of scientific researches have shown that according to the results of numerical calculations it is enough to design a protective screen of enclosing structures to a depth of 10-12 m, but according to the results of the second variant, in the presence of fluctuations of active and passive pressure on the enclosure "wall in soil" it is necessary to design the screen to a depth of 17.0 m, with the maximum amplitude of non-uniform sediments almost approaching the limit relative normative value.

The impact of the new building on the existing nearby five-story apartment building was significant enough. There is a change in pressure on the protective enclosure, which causes the maximum permissible subsidence of the foundations bottom and the heeling change.

The amount of sedimentation will not cause the development of cracks, but a protective screen – the enclosure structure in conjunction with this building should be arranged at a depth of at least 17.0 m, and a diameter of at least 600 mm, with appropriate seams of settlement.

The recommendations made can be taken into account when making design decisions considering the possible dangers from the impact of new multi-storey building for scientific substantiation of the necessary measures of engineering preparation of new construction [6]. In the future proposed methodology of scientific substantiation of measures for engineering preparation of territories and forecasting of their possible consequences, on the basis of the analysis of numerical studies of the stress state and the assessment of the stability of the real objects basics by generalized design parameters, will allow to build principles of reconstruction of urban territories with dense development and difficult geological conditions and provide appropriate guidance to determine the impact of the reconstruction and the extent of the necessary engineering preparation to protect the site and preserve existing construction.

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*Стаття надійшла до редакції 01.01.2020*

*Prusov D.E.*

**SCIENTIFIC SUBSTANTIATION OF ENGINEERING PREPARATION MEASURES DUE TO THE INFLUENCE OF CONSTRUCTION IN THE DENSE BUILDING CONDITIONS**

The method of studying the stress-strain state of reinforcement structures of combined soil massifs has been proposed with the geometric and physical nonlinearity in the formulation of the problem based on the nonlinear theory of elasticity and plasticity of the soil. The study of the stress-deformed state of the computational domain from the standpoint of the mechanics of the deformed solid body had been carried out, using algorithms for solving the problems of the theory of elasticity and plasticity, with the construction of universal computational models of the combined half-space, that allows to determine more reasonably the magnitude of the stress-strain state of complex soil bases in interaction with the reinforcement structures, the surrounding buildings foundations and the whole complex of the surrounding buildings structures. The influence of new construction on the condition of soil bases and foundations of adjacent buildings had been evaluated by determining the change of pressure on the reinforcement structure and determining the stress-strain state change of this structure and the foundation of the existing house. The change in the deformation of the foundation of the existing building, ie the oscillation of the soil foundation and the maximum amplitude of uneven subsidence of the foundations of the existing building have been determined. Each formulation of the problem had to include its own reliability analysis and a specific approach that requires numerical modeling and development of appropriate measures to scientifically substantiate engineering preparation measures in dense building. Further design of protective reinforcement structures for new construction under the conditions of the building reconstruction has carried out considering the impact on the existing buildings and structures and the adjacent soil mass of different stages of construction, begining with the arrangement of protective reinforcement structures, the development of a excavation due to the effect of unloading the foundation, and the sequence of erection of engineering structures.

**Keywords:** new construction, scientific substantiation, stress-strain state, reinforcement structures, engineering preparation.

*Прусов Д.Е.*

**НАУКОВЕ ОБҐРУНТУВАННЯ ЗАХОДІВ З ІНЖЕНЕРНОЇ ПІДГОТОВКИ З УРАХУВАННЯМ ВПЛИВУ БУДІВНИЦТВА В УМОВАХ ЩІЛЬНОЇ ЗАБУДОВИ**

Запропонована методика дослідження напружено-деформованого стану конструкцій укріплень ґрунтових масивів з урахуванням геометричної і фізичної нелінійності в постановці задачі на основі нелінійної теорії пружності і пластичності ґрунтів. Виконано дослідження напружено-деформованого стану розрахункової області з позицій механіки деформованого твердого тіла, із застосуванням алгоритмів розв'язання задач теорії пружності і пластичності, з побудовою універсальних розрахункових моделей комбінованого півпростору, що дозволяє більш обґрунтовано визначати величину напружено-деформованого стану складних ґрунтових основ, які перебувають у взаємодії з огороджувальними конструкціями укріплень, фундаментів прилеглих будинків і всього комплексу споруд навколишньої забудови. Проведено оцінку впливу нового будівництва на стан основ і фундаментів суміжних будівель шляхом визначення зміни тисків на огороджувальну конструкцію у часі та визначенні зміна напружено-деформованого стану самої огороджувальної конструкції і основи існуючого житлового будинку. Визначена зміна деформування основи існуючого будинку, тобто коливання ґрунтової основи та максимальна амплітуда нерівномірного осідання фундаментів існуючої будівлі. Кожна постановка задачі повинна включати власний аналіз на достовірність і особливий підхід, який вимагає проведення чисельного моделювання та розробки відповідних заходів для наукового обґрунтування заходів з інженерної підготовки будівництва в умовах щільної забудови. Подальше проектування захисних огороджувальних конструкцій під нове будівництво в умовах реконструкції забудови проводиться з урахуванням впливу на існуючі будівлі та споруди і прилеглий ґрунтової масив різних етапів будівництва, починаючи з улаштування захисних огороджувальних конструкцій, розробки котловану з урахуванням ефекту розвантаження основи, та послідовності зведення інженерних конструкцій.

**Ключові слова**: нове будівництво, наукове обґрунтування, напружено-деформований стан, конструкції укріплення, інженерна підготовка.

УДК 539.3;624.1
*Прусов Д.Е.* **Наукове обґрунтування заходів з інженерної підготовки з урахуванням впливу будівництва в умовах щільної забудови** / Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 229-241.

*Запропоновано методику дослідження напружено-деформованого стану конструкцій укріплень ґрунтових масивів та проведено оцінку впливу нового будівництва для наукового обґрунтування необхідних заходів інженерної підготовки нового будівництва та рекомендацій щодо прийняття проектних рішень з урахуванням можливих небезпек.*

Іл. 5. Бібліогр. 8 назв.

UDC 539.3;624.1
*Prusov D.E.* **Scientific substantiation of engineering preparation measures due to the influence of construction in the dense building conditions** / Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 229-241.

*The method of studying the stress-strain state of strengthening structures of combined soil massifs has been proposed and the impact of new construction has been evaluated for scientific substantiation of necessary engineering preparation measures and recommendations for making design decisions due to possible dangers.*
Fig. 5. References 8 items.

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UDC 539.3

# APPLICATION OF STIFFNESS RINGS FOR IMPROVEMENT OF OPERATING RELIABILITY OF THE TANK WITH SHAPE IMPERFECTIONS

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DOI: 10.32347/2410-2547.2020.104.242-254

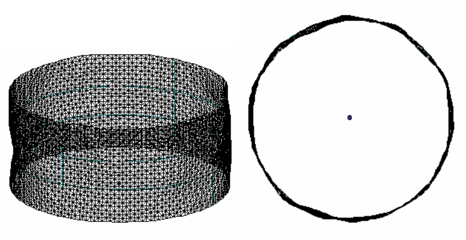
An efficiency of using of two stiffness rings for improvement of operating reliability of the tank with real shape imperfection at the action of combination load was evaluated. The computer model of the tank was constructed in the form of the thin cylindrical shell by of the program complex of finite element analysis. The tank stability problem under separate and joint action of surface pressure and axial compression was solved by the Lancosh method in linear formulation and as a nonlinear static problem by the Newton-Raphson method. The region of the tank failure-free work, which has the graphical presentation, confirmed the improvement of the tank wall stability due to the use of stiffness rings, especially in the area of surface pressure action.

**Keywords:** finite element method, operating reliability, stability, tank, failure-free region, thin-walled shell, shape imperfection, combined load, stiffness ring.

**Introduction.** Many works were aimed at investigating the thin-walled shell [1-15], their stability reliability [1, 2, 8], especially the influence of initial imperfections. One of the approaches is V.T. Coiter’s one [6], which proposed an asymptotic analysis based on the general theory of supercritical behavior. Another approach consists in a direct analysis of the nonlinear deformation of a shell with a curved shape of the middle surface based on one of the grid methods of discretization of the resolving equations. In present time the apparatus of nonlinear differential equations are used for full description of the general laws of the stress-strain state of shells with shape imperfections. The mathematical methods, which are realized in the program complexes, are used for solving the shells stability problem and make it possible to investigate complicated nonlinear systems with multivariant parameters.

In this article the numerical technique for studying the stability of the oil tank under combined load with application of the program complex of finite element analysis NASTRAN is presented [7, 10, 12]. The presence of shape imperfections of the tank wall significantly reduced its stability. Therefore the strengthening of the wall with the stiffness rings to improve the operating reliability of the imperfect tank is proposed in this article. The tank stability problem is solved by the Lancosh method in linear formulation and as a nonlinear static problem by the Newton-Raphson method. The influence of stiffness rings on the critical values of the combined load and the stress-strain state of the tank at different loading steps are investigated. The region of the tank failure-free work is presented.

**1. Finite element model of the tank with real shape imperfection.** The oil tank is a cylindrical shell with a radius of $R$ =19,963 m, height $H$ =17,88 m. The shell thickness is variable in height every 1,49 m and is: 15,24; 14,22; 13,0; 11,56; 10,43; 9,46; 8,60; 7,70; 7,53; 7,40; 7,46; 7,16 (mm). The wall of the tank is made of steel with mechanical characteristics: $E$ =2,06·10$^{11}$ Pa, $\mu$ =0,3, $\rho$ =7800 kg/m$^3$. The stiffness ring shows a folded cross section in the form of the brand 100x300x8 mm. The rings are made of steel with the same mechanical characteristics.



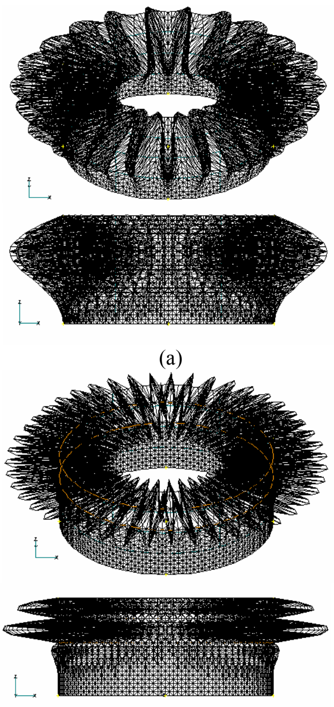
(a) (b)

Fig. 1. Finite element model of an imperfect tank: (a) side view; (b) top view

The tank calculation model is constructed in a finite element program complex [10] in a cylindrical coordinate system. The shape imperfections in the reservoir wall as a result of the theodolite measurements were obtained. The shell wall model with imperfect geometry is represented as a triangular finite element grid. To visualize the real imperfections on a certain scale a special program has been created. Fig. 1 (a), (b) shows the finite element model of the tank in different planes in a 1:20 scale.

The simulation of the combined load is carried out in accordance with a numerical technique [8, 12], which requires solving stability problems of the tank under surface pressure and axial compression separately for each load.

**2. The tank stability under surface pressure.** First, the stability problem of the perfect tank without and taking into account the stiffness rings under surface pressure in linear formulation is considered. To determine the critical value of surface pressure, a linear stability loss problem (Buckling) is solved by Lancosh method. Fig. 2 presents the first buckling form of the perfect shell without and with stiffness rings.



(a)

(b)

Fig. 2. The first buckling form of the perfect tank under surface pressure: (a) without stiffness rings; (b) with stiffness rings

The critical surface pressure value is

$q_{cr}^{0}$ =1179,61 N/m$^2$ for the unsupported perfect shell and $q_{cr}^{0}$ = 4226,44 N/m$^2$ – for the perfect shell with two stiffness rings. The stiffness rings increased the critical surface pressure by 3,58 times and changed the shell buckling form: increasing the number of waves both in the circumferential and longitudinal directions.

The stability of the imperfect tank with stiffness rings under surface pressure in non-linear formulation is considered. The procedure of solving the non-linear static problem (Nonlinear Static) by the modified Newton-Raphson method is applied. Surface pressure is supplied in the form $q = \beta q_{cr}^{0}$, where $\beta$ – loading coefficient, $q_{cr}^{0}$ = 4226,44 N/m$^2$. Fig. 3 shows the loading curves of the shell surface pressure for two nodes, in which maximum displacements (m) were observed at different loading stages.

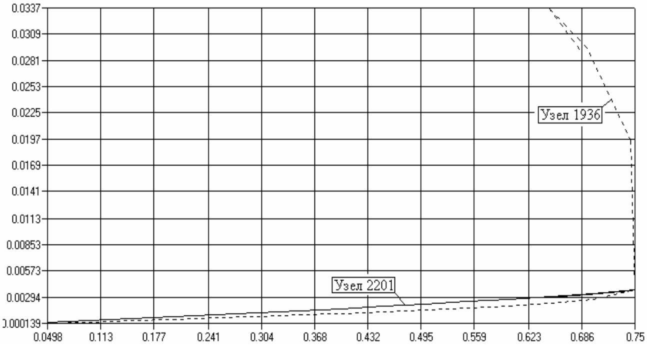



Fig. 3. Loading curves of the imperfect tank with stiffness rings by surface pressure

The imperfect tank with stiffness rings in the postcritical state lost of the stability at the critical loading coefficient $\beta_{cr}$ =0,684. The critical (limited) value of the surface pressure is $q_{cr}^{imp}$ =4226,44 · 0,684=2890,88 N/m$^2$. The stress-strain states of an imperfect tank at the different loading stages are shown in Fig. 4.

The maximum equivalent stresses in the wall elements from the outside of the imperfect shell (Plate Top VonMises Stress) under surface pressure $q = [0,05;0,45;0,744;0,684]\, q_{cr}^{0}$ are $\sigma = [0,685;6,531;40,894;65,658]$ MPa, which are lower than the design resistance of steel $R_y = 240$ MPa.

**3. The tank stability under axial compression.** The tank calculation models as a shell and a shell sector are constructed in a finite element program complex [10] in a cylindrical coordinate system. First, the stability problem of the perfect tank without and with stiffness rings under axial compression in linear formulation (Bucling) is solved by Lancosh method. The first buckling forms of the perfect shell sector without and with stiffness rings are shown in Fig. 5.

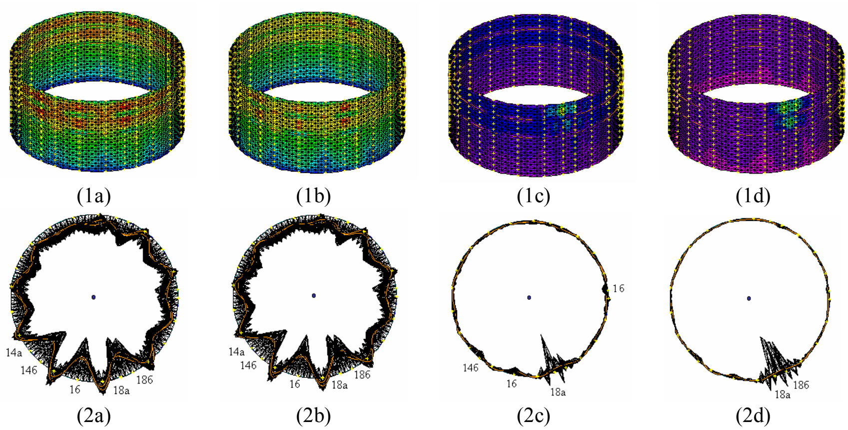



Fig. 4. The stress states (1) and deformated forms (2) of the imperfect tank with stiffness rings under surface pressure: (a) $0,05q_{cr}^{0}$ ; (b) $0,45q_{cr}^{0}$ ; (c) $0,744q_{cr}^{0}$ ; (d) $0,684q_{cr}^{0}$

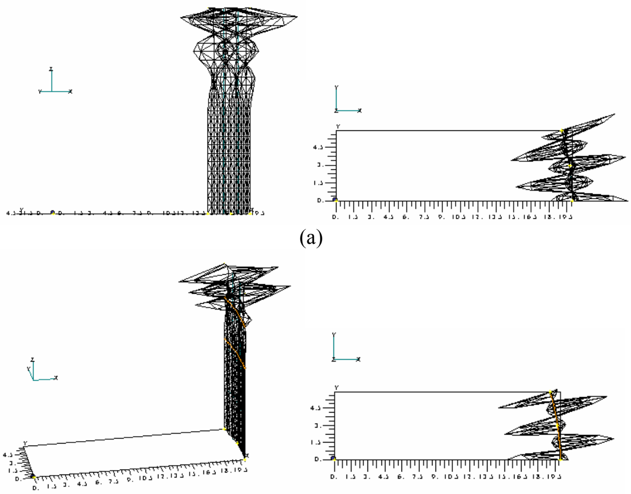



Fig. 5. The first buckling forms of the perfect tank sector under axial compression:
(a) - without stiffness rings; (b) with stiffness rings

The critical value of axial compression is $P_{cr}^{0}$=384957,1 N/m for the unsupported perfect shell and $P_{cr}^{0}$=389612,8 N/m − for the perfect shell with two stiffness rings. The stiffness rings increased the critical axial compression on 1,2 %.

The stability of the imperfect shell with stiffness rings under axial compression in non-linear formulation (Nonlinear Static) is considered using the modified Newton-Raphson method. Axial compression is supplied in the form $P = \beta P_{cr}^{0}$, where $\beta$ – loading coefficient, $P_{cr}^{0}$ = 389612,8 N/m. Fig. 6 presents the loading curves of axial compression for three tank nodes, in which maximum displacements (m) were observed at different loading stages.

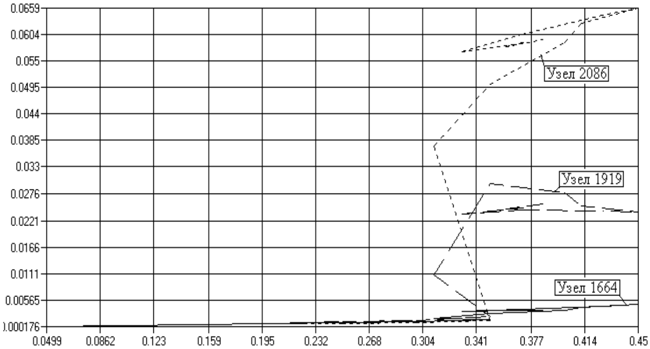
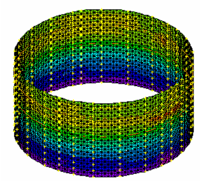
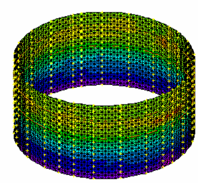
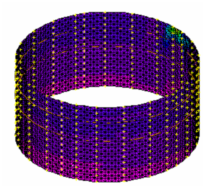



Fig. 6. Loading curves of the imperfect tank with stiffness rings by axial compression

Imperfect tank with stiffness rings in the postcritical state lost of the stability at the critical loading coefficient $\beta_{cr}$ =0,386. The critical (limited) axial compression value is $P_{cr}^{imp}$ =389612,8 · 0,386=150390,54 N/m. Fig. 7 shows the stress-strain states of an imperfect tank at the different stages of axial compression loading.


(1a)


(1b)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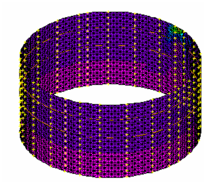
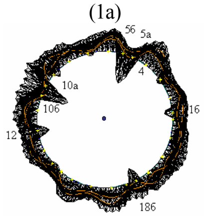
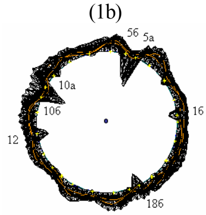

(1d)



(2a)



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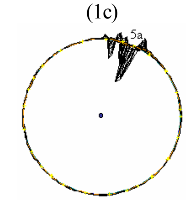

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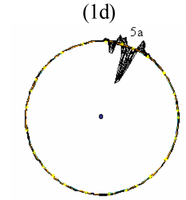


(2d)

Fig. 7. The stress states (1) and deformated forms (2) of the imperfect tank with stiffness rings under axial compression: (a) $0,05q_{cr}^{0}$ ; (b) $0,35q_{cr}^{0}$ ; (c) $0,426q_{cr}^{0}$ ; (d) $0,386q_{cr}^{0}$

The maximum equivalent stresses on the outside of the shell (Plate Top VonMises Stress) at different loading stages by axial compression $P = [0{,}05; 0{,}35; 0{,}426; 0{,}386] P_{cr}^0$ are $\sigma = [3{,}144; 22{,}384; 151{,}647; 139{,}736]$ MPa and less than the design resistance of steel $R_y = 240$ MPa.

**4. The tank stability under combined load.** The first step of the research is determining of the critical combination of axial compression and surface pressure which act on the perfect tank without and with the stiffness rings. The perfect tank stability problem in a linear formulation is solved by Lancosh method. The critical combinations of axial compression and surface pressure are determined by the formulas:

$$\left[\tilde{P}_{cr}^*; \tilde{q}_{cr}^*\right] = \left[\tilde{\mu}\,\alpha P_{cr}^0; \tilde{\mu}(1-\alpha)\, q_{cr}^0\right],\ \left[P_{cr}^*; q_{cr}^*\right] = \left[\mu\,\alpha P_{cr}^0; \mu(1-\alpha) q_{cr}^0\right],$$

where $P_{cr}^0 = 384957{,}1$ N/m, $q_{cr}^0 = 4226{,}44$ N/m$^2$ – the critical values, respectively, of axial compression and surface pressure at their separate action on the perfect tank as defined above; $\alpha$ – a dimensionless combination factor with values from 0 to 1 in 0,1.

Tables 1 and Table 2 present the combined load values for the various axial compression and surface pressure combinations $\left[\alpha P_{cr}^0; (1-\alpha) q_{cr}^0\right]$, the critical combined load coefficients for the perfect shell without $\tilde{\mu}$ and taking into account $\mu$ the stiffness rings and the corresponding values of critical combined load $\left[\tilde{P}_{cr}^* / P_{cr}^0; \tilde{q}_{cr}^* / q_{cr}^0\right]$, $\left[P_{cr}^* / P_{cr}^0; q_{cr}^* / q_{cr}^0\right]$.

Table 1

Critical combined load values for perfect shell without stiffness rings

| $\alpha$ | $\left[\alpha P_{cr}^0; (1-\alpha) q_{cr}^0\right]$ [N/m; N/m$^2$] | $\tilde{\mu}$ | $\left[\tilde{P}_{cr}^*; \tilde{q}_{cr}^*\right]$ [N/m; N/m$^2$] | $\left[\tilde{P}_{cr}^* / P_{cr}^0; \tilde{q}_{cr}^* / q_{cr}^0\right]$ |
|---|---|---|---|---|
| 0 | [0; 4226,44] | 0,279 | [0; 1179,18] | [0; 0,279] |
| 0,1 | [38961,28; 3803,80] | 0,30429 | [9848,07; 1265,76] | [0,02528; 0,29949] |
| 0,2 | [77922,56; 3381,15] | 0,33439 | [27354,56; 1186,95] | [0,07021; 0,28084] |
| 0,3 | [116883,84; 2958,58] | 0,37098 | [46389,55; 1174,19] | [0,11907; 0,27782] |
| 0,4 | [155845,12; 2535,86] | 0,41634 | [70506,03; 1147,25] | [0,18096; 0,27145] |
| 0,5 | [194806,4; 2113,22] | 0,47391 | [101386,2; 1099,82] | [0,26022; 0,26022] |
| 0,6 | [233767,68; 1690,5] | 0,54899 | [127346,4; 920,95] | [0,32685; 0,21790] |
| 0,7 | [272728,96; 1267,93] | 0,64997 | [181138,2; 842,12] | [0,46492; 0,19925] |
| 0,8 | [311690,24; 845,29] | 0,78943 | [257255,7; 697,67] | [0,66029; 0,16507] |
| 0,9 | [350651,52; 422,64] | 0,97323 | [373700,1; 450,42] | [0,95916; 0,10657] |
| 1 | [389612,8; 0] | 0,988 | [384937,4; 0] | [0,988;0 ] |

Table 2

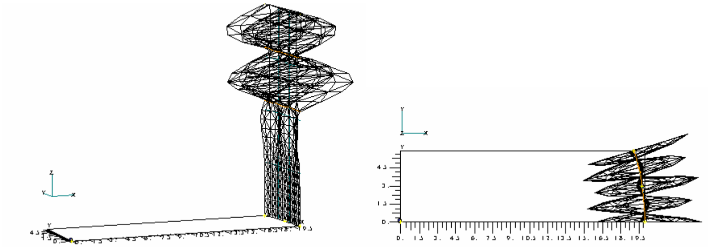
Critical combined load values for perfect shell with stiffness rings

| $\alpha$ | $[\alpha P_{cr}^0; (1-\alpha)q_{cr}^0]$ [N/m; N/m$^2$] | $\mu$ | $[P_{cr}^*; q_{cr}^*]$ [N/m; N/m$^2$] | $[P_{cr}^* / P_{cr}^0; q_{cr}^* / q_{cr}^0]$ |
|---|---|---|---|---|
| 0 | [0; 4226,44] | 1 | [0; 4226,44] | [0; 1] |
| 0,1 | [38961,28; 3803,80] | 0,83067 | [32364,08; 3159,71] | [0,08307; 0,74761] |
| 0,2 | [77922,56; 3381,15] | 1,04982 | [81804,35; 3549,59] | [0,20996; 0,83985] |
| 0,3 | [116883,84; 2958,58] | 1,06983 | [125045,96;3165,10] | [0,32095; 0,74888] |
| 0,4 | [155845,12; 2535,86] | 1,08664 | [169347,23;2755,57] | [0,43466; 0,65198] |
| 0,5 | [194806,4; 2113,22] | 1,09820 | [213935,61;2320,73] | [0,54910; 0,54910] |
| 0,6 | [233767,68; 1690,5] | 0,99229 | [231964,86;1677,54] | [0,59537; 0,39692] |
| 0,7 | [272728,96; 1267,93] | 1,02185 | [278687; 1295,63] | [0,71529; 0,30655] |
| 0,8 | [311690,24; 845,29] | 1,04551 | [325875,26; 883,76] | [0,83641; 0,20910] |
| 0,9 | [350651,52; 422,64] | 1,09505 | [383979,19; 462,81] | [0,98554; 0,10950] |
| 1 | [389612,8; 0] | 1 | [389612,8; 0] | [1; 0] |

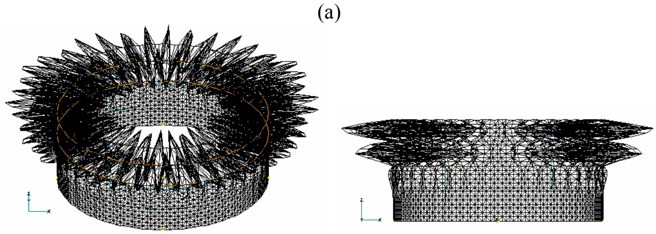
In the case of considering the tank with the stiffness rings (Table 2) at the action of combined load with $\alpha = [0,1;\ 0,6;\ 0,7]$ the critical load coefficients $\mu$ were defined for the shell sector. If we compare the critical combined load coefficients for the shell without stiffness rings (Table 1) and taking them into account (Table 2), we see that using of stiffness rings increases the overall stability of the tank with ideal wall shape. Fig. 8 shows the buckling forms of the perfect tank with stiffness rings at the different combination factor.

Let's consider the shell stability, taking into account the stiffness rings and imperfections of wall shape. The stability problem in non-linear formulation (Nonlinear Static) is considered using the modified Newton-Raphson method. The critical combined load coefficient $\beta_{cr}$ are determined for the load with a combination factor values $\alpha = [0;\ 0,3;\ 0,5;\ 0,8;\ 1]$. The critical combinations of axial compression and surface pressure in the action on the imperfect tank with stiffness rings are determined by the formula: $\left[P_{cr}^{imp}; q_{cr}^{imp}\right] = \beta_{cr}\left[P_{cr}^*; q_{cr}^*\right]$ and are presented in Table 3 (the values $\left[P_{cr}^*; q_{cr}^*\right]$ are shown in Table 2).

As an example, the behavior of the imperfect shell with stiffness rings under combinated load with a combination factor $\alpha = 0,3$ is showed. Fig. 9 presents a loading curves for two model nodes with maximum displacements at different stages loading. A dimensionless loading coefficient $\beta$ is deposited along the abscissa axis and maximum nodal displacements (m) along the ordinate axis.



(a)



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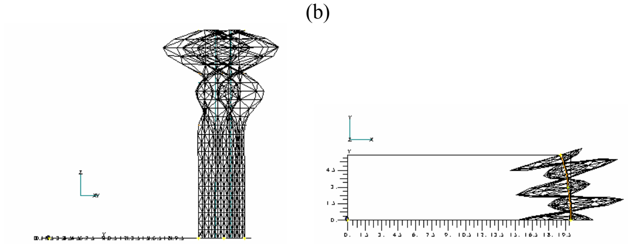

(c)

Fig. 8. Buckling form of the perfect tank with stiffness rings under combained load:
(a) α =0,1; (b) α =0,5; (c) α =0,7

Table 3

Critical combined load values for imperfect shell with stiffness rings

| α | $[P_{cr}^{*}; q_{cr}^{*}]$ [N/m; /m$^2$] | $\beta_{cr}$ | $[P_{cr}^{imp}; q_{cr}^{imp}]$ [N/m; N/m$^2$] | $\left[P_{cr}^{imp} / P_{cr}^{0}; q_{cr}^{imp} / q_{cr}^{0}\right]$ |
|---|---|---|---|---|
| 0 | [0; 4226,44] | 0,684 | [0; 2890,88] | [0; 0,684] |
| 0,3 | [125045,96; 3165,10] | 0,559 | [69900,69; 1769,29] | [0,17941; 0,41862] |
| 0,5 | [213935,61; 2320,73] | 0,43 | [91992,31; 997,91] | [0,23611; 0,23611] |
| 0,8 | [325875,26; 883,76] | 0,3 | [97762,58; 265,13] | [0,25092; 0,06273] |
| 1 | [389612,8; 0] | 0,386 | [150390,54; 0] | [0,386; 0] |

The imperfect tank with stiffness rings in the postcritical state under combined load with a combination factor $\alpha = 0,3$ lost of the stability at $\beta_{cr}$ =0,559. The critical (limited) combined load values are presented in Table 3. Fig. 10 shows the corresponding stress-strain states of the imperfect tank at the different stages of combined loading.

The values of the maximum equivalent stresses and maximum nodal displacements for all values of the combination factor α are given in Table 4. All values of the maximum equivalent stresses in the shell wall elements are less than the design resistance of steel $R_y = 240$ MPa.

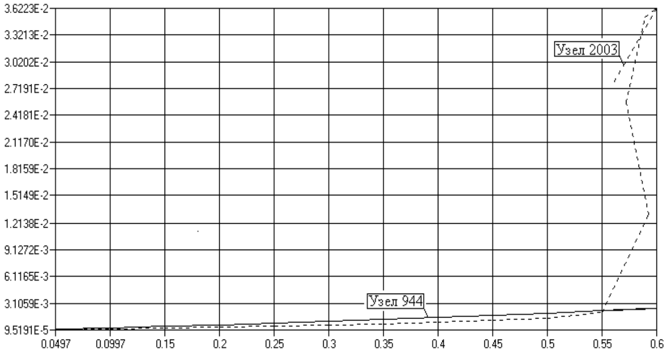


Fig. 9. Loading curves of the imperfect tank with stiffness rings by combined load (α = 0, 3)

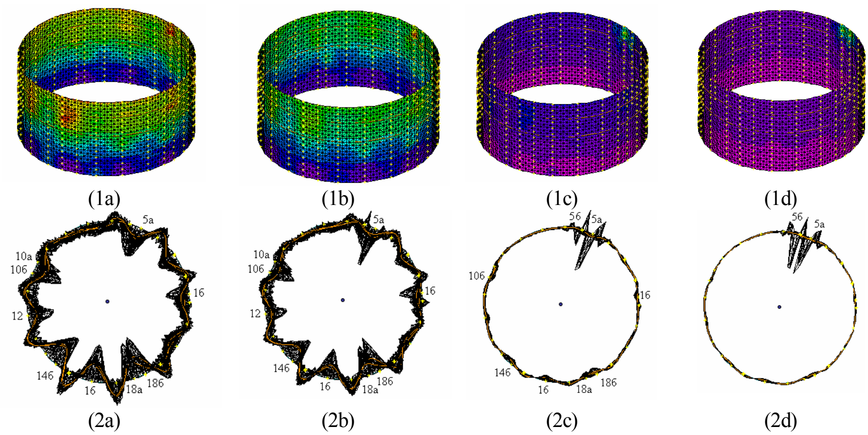


Fig. 10. The stress states (1) and deformated forms (2) of the imperfect tank with stiffness rings under combined load (α = 0, 3) : (a) β = 0, 05 ; (b) β = 0, 55 ; (c) β = 0, 592 ; (d) β = 0, 559

Table 4

| α | $[P_{cr}^{imp}; q_{cr}^{imp}]$ [N/m; N/m²] | Maximum equivalent stresses, МПа | Maximum nodal displacements, m |
|---|---|---|---|
| 0 | [0; 2890,88] | 65,658 | 0,0337 |
| 0,3 | [69900,69; 1769,29] | 68,834 | 0,0362 |
| 0,5 | [91992,31; 997,91] | 102,019 | 0,0464 |
| 0,8 | [97762,58; 265,13] | 112,706 | 0,0492 |
| 1 | [150390,54; 0] | 151,644 | 0,0659 |

**5. The influent of the stiffness rings on the operating reliability of the tank with real wall imperfections.**

The theory of constructure reliability has one of the basic concepts as a concept of failure [1, 2, 8]. The failure of the shell in stability is considered, because this type of failure for thin-walled shell structures is more dangerous.

In our case, the operating reliability of the imperfect tank $R$ is defined as probability of the shell reaction vector $S(\tau)$ in the stability region $\Omega^{imp}$ during the time interval $\left[0 \le \tau \le t\right]$: $R = P_{suc} = \Pr ob\left[S(\tau) \in \Omega^{imp}\right]$. The probability of failure is an addition to the reliability function: $P_{fail}(t) = 1 - P_{suc}$.

An addition the operating reliability of the imperfect tank can be estimated by constructing the region of the design combined load and the stability regions of the perfect tank without and with two stiffness rings.

The design combined load from wind pressure, snow and the weight of the shell coating spacer ring is determined according to DBN B.1.2-2-2006 "Loads and Impacts" [54]. Its value is $[P_r; q_r]$ = [16601,25 N/m; 532 N/m$^2$]. If we take into account consider the critical values of surface pressure and axial compression, which separately acting on the perfect tank with the stiffness rings $P_{cr}^0 = 384957{,}1$ N/m and $q_{cr}^0 = 4226{,}44$ N/m$^2$, then the design combined load value is $[P_r; q_r]$=[0,0415; 0,126].

The stability regions of the perfect tank without $\tilde{\Omega}_0$ and with stiffness rings $\Omega_0$ under combined load are shown in Fig. 11(a). Fig. 11(b) presents the stability regions of the imperfect tank with stiffness rings $\Omega^{imp}$ and the design combined load as an area 1.

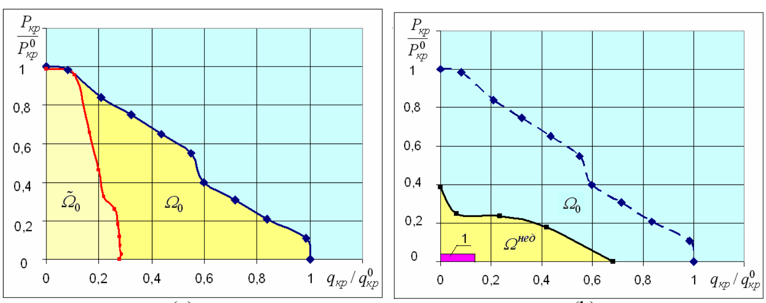


Fig. 11. The stability regions: (a) the perfect tank without $\tilde{\Omega}_0$ and with stiffness rings $\Omega_0$;
(b) the imperfect tank with stiffness rings $\Omega^{imp}$ and the design combined load $\Omega_r$

Design reliability of success work of shell for limit states is $P_{suc} = 99{,}9$ % [1]. Let's consider the reaction vectors $S(\tau)$ of the perfect tank without and with the stiffness rings during time interval $[0 \le \tau \le t]$. They located in the according stability regions $\tilde{\Omega}_0$ and $\Omega_0$ in Fig. 11(a). The stability region of perfect tank with stiffness rings $\Omega_0$, which is bigger than stability region $\tilde{\Omega}_0$ on 65,3%. It means, the addition of the stiffness rings into the construction raise the tank general stability, especially, in an area of surface pressure. We see in Fig. 11(b) the imperfects of tank wall reduced stability region of perfect tank with the stiffness rings $\Omega_0$ on 66,6%. The stiffness rings secure an operating reliability in general stability of the imperfect tank: $P_{suc}^{imp}$ = 99,9+65,3-66,6=98,6%. However, the stability reserve factor in the area of axial compression is less than the same one in the area of surface pressure.

**Conclusion**. Operating reliability, which was presented as a failure-free in stability of the oil tank with shape imperfections under combined action of axial compression and surface pressure, was investigated. The efficiency of the use of two stiffness rings for improving of the tank stability was confirmed. The developed numerical technique with application of the program complex of finite element analysis NASTRAN was effective. The critical combined load values at the solving of the tank stability problem in nonlinear formulation by Newton-Raphson method were more accurate than the values, which were got by Lancosh method. The graphically presentation of the tank stability regions comfirmed the improvement of the tank wall stability as result of application of the stiffness rings, especially in the area of surface pressure action.

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*Стаття надійшла до редакції 01.01.2020*

*Lukianchenko O.O.*

**APPLICATION OF STIFFNESS RINGS FOR IMPROVEMENT OF OPERATING RELIABILITY OF THE TANK WITH SHAPE IMPERFECTION**

Strengthening of the tank thin wall taking into account real shape imperfections at the joint action of axial compression and surface pressure was offered with the stiffness rings. An efficiency of the use of two stiffness rings for improvement of operating reliability in the tank stability was evaluated. The numerical technique for studying the stability of thin imperfect shells with application of the program complex of finite element analysis procedures was presented. The computer model of the tank with wall real imperfections were constructed in the form of a thin cylindrical shell using spline-curves in a cylindrical coordinate system.The tank stability problem under separate and joint action of surface pressure and axial compression was solved by the Lancosh method in linear formulation and as a nonlinear static problem by the Newton-Raphson method. Precritical and postcritical behavior of the shell was considered. The influence of stiffness rings on the critical values of the combined load and the stress-strain state of the tank at different loading steps were investigated. The region of the tank failure-free work, which has the graphical presentation, confirmed the improvement of the tank wall stability due to the use of stiffness rings, especially in the area of surface pressure action.

**Keywords:** finite element method, operating reliability, stability, tank, failure-free region, thin-walled shell, shape imperfection, combined load, stiffness ring.

*Лук'янченко О.О.*

**ЗАСТОСУВАННЯ КІЛЕЦЬ ЖОРСТКОСТІ ДЛЯ ПОКРАЩЕННЯ ЕКСПЛУАТАЦІЙНОЇ НАДІЙНОСТІ РЕЗЕРВУАРА З НЕДОСКОНАЛОСТЯМИ ФОРМИ**

Зміцнення тонкої стінки резервуара з реальною недосконалістю форми при сумісній дії осьового стиснення та поверхневого тиску запропонована кільцями жорсткості. Оцінена ефективність використання двох кілець жорсткості для підвищення експлуатаційної надійності резервуара. Представлена чисельна методика дослідження стійкості тонких недосконалих оболонок із застосуванням процедур програмного комплексу скінченно-елементного аналізу. Комп'ютерна модель резервуара з реальними недосконалостями стінки побудована у вигляді тонкої циліндричної оболонки із застосування сплайн-кривих в циліндричній системі координат. Задача стійкості резервуара при окремій та сумісній дії поверхневого тиску та осьового стиснення досліджувалась методом Ланцоша в лінійній постановці та як нелінійна задача статики методом Ньютона-Рафсона. Розглянута докритична та закритична поведінка оболонки. Досліджено вплив кілець жорсткості на критичні значення комбінованого навантаження та напружено-деформований стан резервуара на різних кроках навантаження. Область безвідмовної роботи резервуара, яке має графічне представлення, підтвердило підвищення стійкості стінки резервуара внаслідок застосування кілець жорсткості, особливо, в області дії поверхневого тиску.

**Ключові слова:** метод скінченних елементів, надійність експлуатації, стійкість, резервуар, область безвідмовної роботи, тонкостінна оболонка, недосконалість форми, комбіноване навантаження, кільце жорсткості.

UDC 539.3
*Lukianchenko O.O.* **Application of stiffness rings for improvement of operating reliability of the tank with shape imperfection** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 242-254.

*An efficiency of using of two stiffness rings for improvement of operating reliability of the tank with real shape imperfection at the action of combination load was evaluated. The computer model of the tank was constructed in the form of the thin cylindrical shell by of the program complex of finite element analysis. The tank stability problem under separate and joint action of surface pressure and axial compression was solved by the Lancosh method in linear formulation and as a nonlinear static problem by the Newton-Raphson method. The region of the tank failure-free work, which has the graphical presentation, confirmed the improvement of the tank wall stability due to the use of stiffness rings, especially in the area of surface pressure action.*
Tab. 4. Fig. 11. References 15 items.

УДК 539.3
*Лук'янченко О.О.* **Застосування кілець жорсткості для покращення експлуатаційної надійності резервуара з недосконалостями форми** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 242-254.

*Оцінена ефективність використання двох кілець жорсткості для підвищення експлуатаційної надійності резервуара з реальною недосконалістю форми при дії комбінованого навантаження. Комп'ютерна модель резервуара побудована у вигляді тонкої циліндричної оболонки за допомогою програмного комплексу скінченно-елементного аналізу. Задача стійкості резервуара при окремій та сумісній дії поверхневого тиску та осьового стиснення вирішена методом Ланцоша в лінійній постановці і як нелінійна задача статики методом Ньютона-Рафсона. Область безвідмовної роботи резервуара, яка має графічне представлення, підтвердила підвищення стійкості стінки резервуара за рахунок використання кілець жорсткості, особливо, в області дії поверхневого тиску.*
Табл. 4. Іл. 11. Бібліогр. 15 назв.

УДК 539.3
*Лукьянченко О.А.* **Применение колец жесткости для улучшения эксплуатационной надежности резервуара с несовершенствами формы //** Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2020. – Вып. 104. – С. 242-254.

*Оценена эффективность применения двух колец жесткости для повышения эксплуатационной надежности резервуара с реальным несовершенством формы при действии комбинированной нагрузки. Компьютерная модель резервуара построена в виде тонкой цилиндрической оболочки с помощью программного комплекса конечно-элементного анализа. Задача устойчивости резервуара при отдельном и совместном действии поверхностного давления и осевого сжатия была решена методом Ланцоша в линейной постановке и как нелинейная задача статики методом Ньютона-Рафсона. Область безотказной работы резервуара, которая имеет графическое представление, подтвердила повышение устойчивости стенки резервуара за счет использования колец жесткости, особенно в области действия поверхностного давления.*
Табл. 4. Ил. 11. Библиогр. 15 назв.

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UDC 539.375

# BASIC RELATIONSHIPS FOR PHYSICALLY AND GEOMETRICALLY NONLINEAR PROBLEMS OF DEFORMATION OF PRIMATIC BODIES

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DOI: 10.32347/2410-2547.2020.104.255-264

The initial relations of thermo elastic-plastic deformation of prismatic bodies are given in the paper. The basic concepts, indifference of deformation tensors, with the condition of energy conjunction in description of the shaping process are laid out on the basis of classical works.

**Keywords:** prismatic bodies, physical and geometric nonlinearity, thermo elasticplastic deformation, shaping process, Finger measure, Aldroid derivative.

**Introduction.** A number of responsible structures elements, which are prismatic bodies, are undergoing a significant shaping in the process of manufacturing and operation, which often take place at high temperatures, which leads to changes in the physical and mechanical characteristics of the material and the development of various types of deformations. Due to the possibility of simultaneous occurrence of plasticity and creep deformations caused both of the presence of force load and external temperature influences, determining the bearing capacity of these objects requires the solution of the problems of thermo elastoplasticity. The solution authenticity of such problems of the deformable body mechanics depends essentially on the adequacy of the physical relations used to the considered processes of the material deformation, in particular taking into account the presence of large deformations.

The purpose of this work is to select adequately the basic relations of geometrically nonlinear problems of thermo elasto-plasticity for prismatic bodies.

**Initial relations for the problems of the theory of elasticity, plasticity and creep.** Consider a curvilinear prismatic body of complex shape (Fig. 1) with variable geometric and physical characteristics in the basic coordinate system $z^{i'}$. It is used to describe boundary conditions, external influences, and object configuration. Fig. 1 shows also a local curvilinear coordinate system $x^i$ that is related to its geometry.

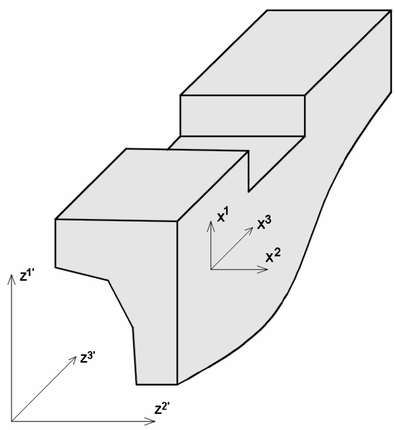

Fig. 1. Curvilinear prismatic body of complex shape

The transformation tensor that determines the relationship between the local and basic coordinate systems is known at each point in the body:

$$z^{i'}_{,j} = \frac{\partial z^{i'}}{\partial x^j}. \tag{1}$$

The indexes indices by Latin letters taking values 1, 2, 3, and taking the values 1, 2 when indices in Greek letters hereinafter.

The covariant components of the metric tensor of the local coordinate system are represented by the covariant components of the metric tensor of the basic coordinate system according to formula:

$$g_{ij} = z^{m'}_{,i} z^{n'}_{,j} g_{mn}. \tag{2}$$

It is most advisable to use a Cartesian coordinate system as a basis for the study of prismatic bodies. Three components of the metric tensor are non-zero in this case:

$$g_{1'1'} = 1, \quad g_{2'2'} = 1, \quad g_{3'3'} = 1. \tag{3}$$

Then the covariance components of the metric tensor of the local coordinate system are determined by the formula:

$$g_{ij} = z^{m'}_{,i} z^{n'}_{,j}. \tag{4}$$

We find the covariance components of the metric tensor of the local coordinate system using the following relation:

$$g^{ij} = \frac{A(g^{ij})}{g}. \tag{5}$$

where $A(g^{ij})$ is the algebraic complement of the each element in a matrix composed of the covariance components of the metric tensor, $g = \det(g_{ij})$ - the determinant of that matrix.

The relation for determining the deformation components due to the displacements in the local coordinate system have the form [20]:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i}\right) - u_k \Gamma^k_{ij}, \tag{6}$$

where $\Gamma^k_{ij}$ - the second kind Christoffel symbols.

In the basis Cartesian coordinate system all the Christoffel symbols are equal to zero and the displacements in the local and base coordinate systems are related by the ratios:

$$u_k = u_{m'} z^{m'}_{,k} . \qquad (7)$$

On the basis of formulas (6) and (7) we obtain the expression of the components of the strain tensor in the local coordinate system by displacements in the basic one:

$$\varepsilon_{ij} = \frac{1}{2}\left( u_{m',i} z^{m'}_{,j} + u_{m',j} z^{m'}_{,i} \right) . \qquad (8)$$

In problems of thermoelasticity the components of the complete deformation tensor are equal to amount of elastic and temperature components:

$$d\varepsilon_{ij} = d\varepsilon^{e}_{ij} + d\varepsilon^{T}_{ij} , \qquad (9)$$

where $\varepsilon^{T}_{ij} = \alpha_T T g_{ij}$ , $\alpha_T$ - coefficient of linear expansion of material, $T$ – an increase of temperature in the investigated point of the body relative to its original state.

Components of the stress tensor under elastic loading connected through the components of the strain tensor in accordance with Hooke's law:

$$\sigma^{ij} = C^{ijmn} \varepsilon^{e}_{mn} , \qquad (10)$$

or subject to (7)

$$\sigma^{ij} = C^{ijmn} (\varepsilon_{mn} - \varepsilon^{T}_{mn}) . \qquad (11)$$

The components of the elasticity tensor constant for isotropic bodies are found from the relations:

$$C^{ijmn} = \lambda g^{mn} g^{ij} + \mu (g^{mi} g^{nj} + g^{mj} g^{ni}) , \qquad (12)$$

where are the Lame coefficients $\lambda$ and $\mu$ are determined by the Poisson's ratio $\nu = \nu(z^{i'}, T)$ and material elasticity modulus (Young's modulus) $E = E(z^{i'}, T)$ , that depend on the temperature *T*:

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}, \quad \mu = \frac{E}{2(1+\nu)} . \qquad (13)$$

To describe the process of deformation beyond the elasticity of a material whose physical properties depend on temperature, we use the theory of plastic flow [1].

It is supposed that the material is homogeneous and isotropic in the initial state, plastic non-compressed and change of material’s volume is linear-elastic:

$$d\varepsilon^{p}_{ij} = 0, \quad d\varepsilon_{ij} = d\varepsilon^{e}_{ij} . \qquad (14)$$

The increment of complete deformation $d\varepsilon_{ij}$ is equal to amount of elastic deformation $d\varepsilon^{e}_{ij}$, temperature deformation $d\varepsilon^{T}_{ij}$ and deformation of plasticity $d\varepsilon^{p}_{ij}$ :

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} + d\varepsilon_{ij}^{T} . \qquad (15)$$

Elastic deformations are related to the stress of the Hooke law (10). The area of elastic deformation is limited in the space of stresses by the yield surface:

$$f_p(\sigma^{ij}, \chi, T) = 0 . \qquad (16)$$

In accordance with the hypothesis of isotropic hardening under the conditions of Mises' fluidity, the equations of the yield surface are as follows:

$$f_p = \frac{1}{2} s_{ij} s^{ij} - \tau_s^2(\chi, T) = 0 , \qquad (17)$$

where $\tau_s(\chi,T)$ - yield limit under pure shear, $\chi$ - Odquist's strengthening parameter:

$$\chi = \int_{\varepsilon_{ij}^{p}} \sqrt{\frac{2}{3} d\varepsilon_{ij}^{p} d\varepsilon_{p}^{ij}} . \qquad (18)$$

The components of the stress deviator included in expression (17) are determined by the formula:

$$s^{ij} = \sigma^{ij} - \frac{1}{3} \delta_{mn} \sigma^{mn} g^{ij} . \qquad (19)$$

Stress deviator is associated with an increase in plastic deformation in accordance with the associated law of plastic yield:

$$d\varepsilon_{ij}^{p} = \lambda_p \frac{\partial f_p}{\partial s^{ij}} = \lambda_p s_{ij} . \qquad (20)$$

In case of creep deformations presence the equations of state are adopted in accordance with the theory of strengthening [8]. It is assumed that the complete increments of deformation are defined as the sum of four components:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{T} + d\varepsilon_{ij}^{p} + d\varepsilon_{ij}^{c} . \qquad (21)$$

The creep surface equation looks like:

$$f_c = \frac{3}{2} s_{ij} s^{ij} - \tau_c^2(\psi, T, \varepsilon_i) = 0 . \qquad (22)$$

The creep limit is determined by the formula:

$$\tau_c = \left[ \frac{\varepsilon_i^c}{\alpha} (\psi)^{\beta} \right]^{\frac{1}{\gamma}} , \qquad (23)$$

where $\alpha$, $\beta$, $\gamma$ are temperature dependent constants which characterized a creep properties of material; $\psi$ - strengthening parameter:

$$\psi = \int_{\varepsilon_{ij}^{c}} \sqrt{\frac{2}{3} d\varepsilon_{ij}^{c} d\varepsilon_{c}^{ij}} . \qquad (24)$$

The increase of creep deformations is found by the components of the stress deviator:

$$d\varepsilon_{ij}^{c} = \lambda_c \frac{\partial f_c}{\partial s^{ij}} = \lambda_c s_{ij} . \qquad (25)$$

**Determination of deformations in geometrically nonlinear problems.** We will still use [4, 5, 6] the basic Cartesian coordinate system $Z^{i'}$ when considering spatial objects in geometrically nonlinear formulation and the local coordinate system $x^i$, provided that it is "frozen" into the medium and deformed with it. The positions of each particle of body at any time are determined by the radius vector:

$$\bar{r} = \bar{r}(Z^i, t) . \qquad (26)$$

We suppose that the reference initial configuration is formed by vectors $\bar{r}_0$ at time $t_0$, topical – vector $\bar{r}_t = R$ at time $t$. We also introduce the reference variable configuration that corresponds to the time $\tilde{t}$ which is close enough to $t$:

$$t = \tilde{t} + \Delta t . \qquad (27)$$

We denote the metric tensors of these states $\overset{€}{g}$, $\overset{€}{\tilde{g}}$, $\overset{€}{G}$ respectively (Fig. 2).

The increase of time $\Delta t$ chosen in a such way that during the transition from the reference variable configuration to the actual metric tensor components were corresponded to the ratio:

$$\Delta\overset{€}{G} = \overset{€}{G} - \overset{€}{\tilde{g}}, \quad \Delta G_{ij} \langle\langle G_{ij} . \qquad (28)$$

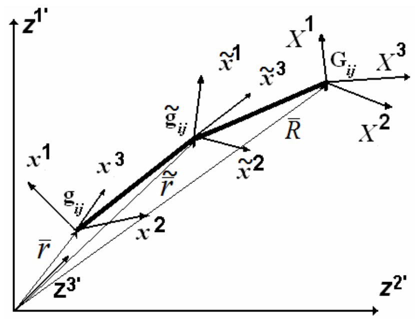


Fig. 2. Three configurations of coordinate system

The covariance components of metric tensors of configurations being entered into consideration are calculated similarly (4) through the transformation components tensor of the respective configurations.

To identify the components $\Delta\overset{€}{G}$ we will write an expression for the radius vector of a point in the current configuration $\bar{R}$, as the sum of the vector $\bar{r}_{\tilde{t}} = \bar{\tilde{r}}$ in the variable reference configuration and displacement vector $\bar{u}$ (Fig. 3):

$$\bar{R} = \bar{\tilde{r}} + \bar{u} , \qquad (29)$$

or, using of index notation:

$$Z^{m'} = \tilde{Z}^{m'} + u^{m'} . \qquad (30)$$

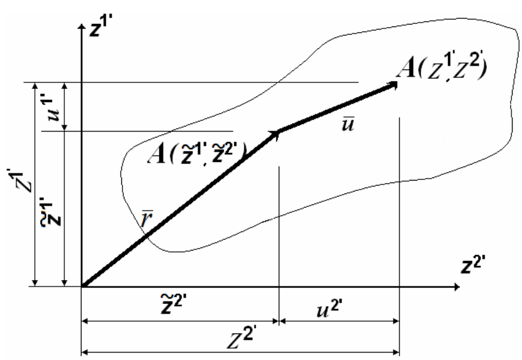


Fig. 3. Changing the position of a point according to the entered reference variable configuration

The components of the transformation tensor that determine the relationship between the local and basic coordinate systems in the current configuration are determined by the formula:

$$Z^{m'}_{,i} = \tilde{Z}^{m'}_{,i} + u^{m'}_{,i}. \quad (31)$$

The covariant components of the metric tensor of the actual configuration are represented using of formula (4):

$$G_{ij} = Z^{m'}_{,i} Z^{m'}_{,j}. \quad (32)$$

Turning (32) and taking into account (31), we obtain:

$$G_{ij} = \tilde{Z}^{m'}_{,i}\tilde{Z}^{m'}_{,j} + \tilde{Z}^{m'}_{,i}u^{m'}_{,j} + u^{m'}_{,i}\tilde{Z}^{m'}_{,j} + u^{m'}_{,i}u^{m'}_{,j} = \tilde{g}_{ij} + \Delta G_{ij}, \quad (33)$$

where

$$\Delta G_{ij} = \tilde{Z}^{m'}_{,i}u^{m'}_{,j}u^{m'}_{,i} + u^{m'}_{,i}u^{m'}_{,j}. \quad (34)$$

Counter-variant components $\Delta G_{ij}$ are determined by the condition:

$$G^{ij}G_{jl} = \delta^i_l \quad (35)$$

or

$$(\tilde{g}^{ij} + \Delta G^{ij})(g_{jl} + \Delta G_{jl}) - \delta^i_l = 0. \quad (36)$$

Neglecting small increments of $\Delta G^{ij}\Delta G_{jl}$ value, we get:

$$\Delta G^{ij} g_{jl} + \tilde{g}^{ij}\Delta G_{jl} = 0, \quad (37)$$

where

$$\Delta G^{ik} = -\tilde{g}^{ij}\Delta G_{jl}\tilde{g}^{lk}. \quad (38)$$

We write the expressions for the strain tensor in the current configuration using the Finger measure $\overset{\frown}{F}$ [2, 3]:

$$\overset{\frown}{\varepsilon} = \frac{1}{2}(F - \overset{\frown}{G}). \quad (39)$$

Counter-variant components of the Finger measure $F^{ij}$ is equal to the corresponding components of the metric tensor $g^{ij}$ of reference initial configuration.

We present the counter-variant components of the deformation tensor in the current configuration as follows:

$$\varepsilon^{ij} = \frac{1}{2}(F^{ij} - G^{ij}) = \frac{1}{2}(g^{ij} - G^{ij}). \quad (40)$$

Using a variable reference configuration, we represent (40) as amount of:

$$\varepsilon^{ij} = \frac{1}{2}(g^{ij} - \tilde{g}^{ij} + \tilde{g}^{ij} - G^{ij}) = \tilde{\varepsilon}^{ij} + \Delta\varepsilon^{ij}. \quad (41)$$

The components of the strain tensor $\overset{\frown}{\varepsilon}$ in the variable reference configuration relative to the initial reference one are indicated there as $\tilde{\varepsilon}^{ij}$:

$$\tilde{\varepsilon}^{ij} = \frac{1}{2}(g^{ij} - \tilde{g}^{ij}), \quad (42)$$

and components of the strain tensor in the transition from the variable reference to the actual configuration are indicated through $\Delta\varepsilon^{ij}$:

$$\Delta\varepsilon^{ij} = \frac{1}{2}(\tilde{g}^{ij} - G^{ij})\,. \tag{43}$$

Counter-variant components of the deformation increment during transition from the reference variable to the actual configuration, taking into account (38), represented by the relations:

$$\Delta\varepsilon^{ij} = \frac{1}{2}(\tilde{g}^{ij} - G^{ij}) = \frac{1}{2}(\tilde{g}^{ij} - \tilde{g}^{ij} - \Delta G^{ij}) = -\frac{1}{2}\tilde{g}^{im}\Delta G_{mn}\tilde{g}^{jn}\,, \tag{44}$$

and the covariance components are:

$$\Delta\varepsilon_{kl} = \Delta\varepsilon^{ij}G_{ik}G_{jl} \approx \Delta\varepsilon^{ij}\tilde{g}_{jl} = \frac{1}{2}\Delta G_{kl}\,. \tag{45}$$

Using expression (34), we write the covariance components of the strain tensor in the current configuration through displacements:

$$\Delta\varepsilon_{ij} = \frac{1}{2}(\tilde{Z}^{m'}_{,i}u^{m'}_{,j} + u^{m'}_{,i}\tilde{Z}^{m'}_{,j} + u^{m'}_{,i}u^{m'}_{,j})\,. \tag{46}$$

On the other hand, the increment of the strain tensor $\Delta\overset{\text{€}}{\mathcal{E}}$ can be expressed as the product of the strain rate tensor at $\Delta t$.

$$\Delta\overset{\text{€}}{\mathcal{E}} = \overset{\text{€}}{\mathcal{E}}^{ol}\cdot\Delta t\,. \tag{47}$$

The Aldroid derivative of the tensor $\overset{\text{€}}{\mathcal{E}}$ we represent with the relation [7]:

$$\overset{\text{€}}{\mathcal{E}}^{ol} = \dot{\overset{\text{€}}{\mathcal{E}}} - \nabla\overline{\vartheta}^{T}\overset{\text{€}}{\mathcal{E}}\,. \tag{48}$$

Taking into account (39) and equivalence to zero of the operator $\nabla\overset{\text{€}}{G} = 0$, we get:

$$\overset{\text{€}}{\mathcal{E}}^{ol} = \frac{1}{2}[(\dot{\overset{\text{€}}{F}} - \dot{\overset{\text{€}}{G}}) - \nabla\overline{\vartheta}^{T}(\overset{\text{€}}{F} - \overset{\text{€}}{G}) - (F - \overset{\text{€}}{G})\nabla\overline{\vartheta}] = \frac{1}{2}\Big[\nabla\overline{\vartheta}^{T}\overset{\text{€}}{F} + \overset{\text{€}}{F}\nabla\overline{\vartheta} -$$

$$-\dot{\overset{\text{€}}{G}} - \nabla\overline{\vartheta}^{T}\overset{\text{€}}{F} + \nabla\overline{\vartheta}^{T}\overset{\text{€}}{G} - \overset{\text{€}}{F}\nabla\overline{\vartheta} + \overset{\text{€}}{G}\nabla\overline{\vartheta}\Big] = -\frac{1}{2}(\dot{\overset{\text{€}}{G}} + \nabla\overset{\text{€}}{G}) = \frac{1}{2}\frac{\partial\overset{\text{€}}{G}}{\partial t}. \tag{49}$$

Then at $\Delta t \to 0$:

$$\Delta\overset{\text{€}}{\mathcal{E}} = -\frac{1}{2}\frac{\partial\overset{\text{€}}{G}}{\partial t}\Delta t = -\frac{1}{2}\Delta\overset{\text{€}}{G}\,, \tag{50}$$

which is equivalent to component form (45).

**Conclusion.** The initial relations for physically and geometrically nonlinear problems of deformation process for space prismatic bodies being formulated above. It will allow to create new types of finite elements and to obtain corresponding ratios for calculating the coefficients of stiffness matrices and nodal reactions for a new class of problems.

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*Стаття надійшла до редакції 03.02.2020 р.*

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**MAIN RELATIONSHIPS FOR PHYSICALLY AND GEOMETRICALLY NONLINEAR PROBLEMS OF DEFORMATION OF PRIMATIC BODIES**

A number of responsible structures elements, which are prismatic bodies, are undergoing a significant shaping in the process of manufacturing and operation, which often take place at high temperatures, which leads to changes in the physical and mechanical characteristics of the material and the development of various types of deformations. The solution authenticity of such problems of the deformable body mechanics depends essentially on the adequacy of the physical relations used to the considered processes of the material deformation, in particular taking into account the presence of large deformations.

The initial relations of thermo elastic-plastic deformation of prismatic bodies are given in the paper. A Cartesian coordinate system used as a basis for the study of prismatic bodies. The relation for determining the deformation components through displacement values in the local coordinate system are formulated. The components of the complete thermo elastic-plastic and creep deformation tensor are taken as amount of appropriate deformation components. The plastic deformation described with associated law of plastic yield, a creep deformation – in accordance with the theory of strengthening The basic concepts, indifference of deformation tensors, with the condition of energy conjunction in description of the shaping process are laid out on the basis of classical work.

**Keywords:** prismatic bodies, physical and geometric nonlinearity, thermo elasticplastic deformation, shaping process, Finger measure, Aldroid derivative.

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**ОСНОВНЫЕ СООТНОШЕНИЯ ДЛЯ ФИЗИЧЕСКИХ И ГЕОМЕТРИЧЕСКИХ НЕЛИНЕЙНЫХ ЗАДАЧ ДЕФОРМИРОВАНИЯ ПРИЗМАТИЧЕСКИХ ТЕЛ**

В работе приведены исходные соотношения термовязкоупругопластического деформирования призматических тел. На основе классических работ изложены основные понятия, индифферентность тензоров деформаций при условии энергетической сопряженности для описания процесса формоизменения.

**Ключевые слова:** призматические тела, физическая и геометрическая нелинейность, термовязкоупругопластическое деформирование, формоизменение, мера Фингера, производная Олдроида.

УДК 539.375
*Максим'юк Ю.В., Пискунов С.О., Шкриль О.О., Максим'юк О.В.* **Основні співвідношення для фізично і геометрично нелінійних задач деформування призматичних тіл** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – Київ: КНУБА, 2020. – Вип. 104. – С. 255-264.

*В роботі наведені вихідні співвідношення термов'язкопружнопластичного деформування призматичних тіл. На основі класичних робіт викладені основні поняття, індиферентність тензорів деформацій при умові енергетичної сполученості для опису процесу формозмінення.*
Табл. 0. Іл. 3. Бібліогр. 8 назв.

UDC 539.375
*Maksimyuk Yu.V., Pyskunov S.O., Shkril' A.A., Maksimyuk O.V.,* **Basic relations for physically and geometrically nonlinear problems of deformation of prismatic bodies** // Strength of Materials and Theory of Structures: Scientific-&-Technical collected articles – Kyiv: KNUBA, 2020. – Issue 104. – P. 255-264. – Engl.

*The initial relations of thermo elastic-plastic deformation of prismatic bodies are given in the paper. The basic concepts, indifference of deformation tensors, with the condition of energy conjunction in description of the shaping process are laid out on the basis of classical works.*
Tabl. 0. Fig. 3. Ref. 8.

УДК 539.375
*Максимюк Ю.В., Пискунов С.О., Шкрыль А.А., Максимюк О.В.* Основные соотношения для физических и геометрических нелинейных задач деформирования призматических тел // Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2020. – Вып. 104. – С. 255-264. – Англ.

*В работе приведены исходные соотношения термовязкоупругопластического деформирования призматических тел. На основе классических работ изложены основные понятия, индифферентность тензоров деформаций при условии энергетической сопряженности для описания процесса формоизменения.*
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UDC 519.853, 624.04

# AN IMPROVED GRADIENT-BASED METHOD TO SOLVE PARAMETRIC OPTIMISATION PROBLEMS OF THE BAR STRUCTURES

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DOI: 10.32347/2410-2547.2020.104.265-288

The paper considers parametric optimisation problems for the bar structures formulated as non-linear programming tasks. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimisation problem. Equivalent Householder transformations of the resolving equations of the method have been proposed. They increase numerical efficiency of the algorithm developed based on the method under consideration. Additionally, proposed improvement for the gradient-based method also consists of equivalent Givens transformations of the resolving equations. They ensure acceleration of the iterative searching process in the specified cases described by the paper due to decreasing the amount of calculations. The comparison of the optimisation results of truss structures presented by the paper confirms the validity of the optimum solutions obtained using proposed improvement of the gradient-based method. The efficiency of the propoced improvement of the gradient-based method has been also confirmed taking into account the number of iterations and absolute value of the maximum violation in the constraints.

**Keywords:** parametric optimisation, non-linear programming task, gradient-based method, bar system, finite-element method

**Introduction.** Over the past 50 years, numerical optimisation and finite element method [7] have individually made significant advances and have together been developed to make possible the emergence of structural optimisation as a potential design tool. In recent years, great efforts have been also devoted to integrate optimisation procedures into the CAD facilities. With these new developments, lots of computer packages are now able to solve relatively complicated industrial design problems using different structural optimisation techniques.

Applied optimum design problems for the bar structures in some cases are formulated as parametric optimisation problems, namely as searching problems for unknown structural parameters, whose provide an extreme value of the specified purpose function in the feasible region defined by the specified constraints. In this case structural optimisation performs by variation of the structural parameters when the structural topology, cross-section types and node type connections of the bars, the support conditions of the bar system, as well as loading patterns and load design values are prescribed and constants. Besides, mathematical model of the parametric optimisation problem of the structures includes the set of design variables, the objective function, as well as constraints, whose reflect in general case non-linear interdependences between them [10].

In cases if the purpose function and constraints of the mathematical model are continuously differentiable functions, as well as the search space is smooth, then the parametric optimization problems are successfully solved using gradient-based non-linear methods [11]. The gradient-based methods operate with the first derivatives or gradients only both of the objective function and constraints. The methods are based on the iterative construction such sequence of the approximations of the design variables that provides the convergence to the optimum solution (optimum values of the structural parameters) [17].

Additionally, a sensitivity analysis is a useful optional feature that could be used in scope of the numerical algorithms developed based on the gradients methods [8].

Although many papers are published on the parametric optimization of the structures, the development of a general computer program for the design and optimisation of building structures according to specified design codes remains an actual task. Therefore, in this paper, a gradient-based method is considered as investigated object. The main research question is the development of mathematical support and numerical algorithm to solve parametric optimisation problems of the building structures with orientation on software implementation in a computer-aided design system.

**1. Parametric optimisation problem formulation.** Let us consider a parametric optimisation problem of a structure consists of the bar members, which can be formulated as presented below: to find optimum values for geometrical parameters of the structure, bar's cross-section sizes and initial pre-stressing forces introduced into the redundant members of the bar system, whose provide the extreme value of the determined optimality criterion and satisfy all load-bearing capacities and stiffness requirements. We assume, that the structural topology, cross-section types and node type connections of the bars, the support conditions of the bar system, as well as loading patterns and load design values are prescribed and constants.

The formulated parametric optimisation problem can be stated as a non-linear programming task in the following mathematical terms: to find unknown structural parameters $\vec{X} = \{X_\iota\}^T$, $\iota = \overline{1, N_X}$, providing the least value of the determined objective function:

$$f^* = f(\vec{X}^*) = \min_{\vec{X} \in \Im} f(\vec{X}), \qquad (1.1)$$

in feasible region (search space) $\Im$ defined by the following system of constraints:

$$\boldsymbol{\psi}(\vec{X}) = \left\{ \psi_\kappa(\vec{X}) = 0 \mid \kappa = \overline{1, N_{EC}} \right\}; \qquad (1.2)$$

$$\boldsymbol{\varphi}(\vec{X}) = \left\{ \varphi_\eta(\vec{X}) \le 0 \mid \eta = \overline{N_{EC} + 1, N_{IC}} \right\}; \qquad (1.3)$$

where $\vec{X}$ is the vector of the design variables (unknown structural parameters); $f$, $\psi_\kappa$, $\varphi_\eta$ are the continuous functions of the the vector argument; $\vec{X}^*$ is the optimum solution or optimum point (the vector of optimum values of the structural parameters); $f^*$ is the optimum value of the optimum criterion (objective function); $N_{EC}$ is the number of constraints-equalities $\psi_\kappa(\vec{X})$, whose define hyperplanes of the feasible solutions; $N_{IC}$ is the number of constraints-inequalities $\varphi_\eta(\vec{X})$, whose define a feasible region in the design space $\Im$.

The vector of the design variables Eq. (1.1) can include as components unknown geometrical parameters of the structure, unknown cross-sectional sizes of the structural members, as well as unknown initial pre-stressing forces introduced into the specified redundant members of the structure.

The specific technical-and-economic index (material weight, material cost, construction cost etc.) or another determined indicator can be considered as the objective function Eq. (1.1) taking into account ability to formulate it analytical expression as a function of design variables $\vec{X}$.

Load-bearing capacities constraints (strength and stability inequalities) for all design sections of the structural members subjected to all design load combinations at the ultimate limit state as well as displacements constraints (stiffness inequalities) for the specified nodes of the bar system subjected to all design load combinations at the serviceability limit state should be included into the system of constraints Eqs. (1.2) – (1.3). Additional requirements, whose describe structural, technological and serviceability particularities of the building structure under consideration, as well as constraints on the building functional volume can be also included into the system Eqs. (1.2) – (1.3).

**2. An improved gradient-based method to solve the parametric optimisation problem.** The parametric optimisation problem stated as non-linear programming task by Eqs. (1.1) – (1.4) can be solved using a gradient-based method. The method of *objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations* ensures effective searching for solution of the non-linear programming tasks occurred when optimum designing of the building structures [5, 9].

The gradient-based method operates with the first derivatives or gradients only both of the objective function Eq. (1.1) and constraints Eqs. (1.2) – (1.3). The method is based on the iterative construction such sequence Eq. (2.1) of the approximations of the design variables Eq. (1.4) that provides the convergence to the optimum solution (optimum values of the structural parameters):

$$\vec{X}_{t+1} = \vec{X}_t + \Delta\vec{X}_t, \tag{2.1}$$

where $\vec{X}_t = \{X_\iota\}^T$, $\iota = \overline{1, N_X}$ is the current approximation to the optimum solution $\vec{X}^*$ that satisfies both constraints-equalities Eq. (1.2) and constraints-inequalities Eq. (1.3) with the extreme value of the objective function Eq. (1.1); $\Delta\vec{X}_t = \{\Delta X_\iota\}^T$, $\iota = \overline{1, N_X}$, is the increment vector for the current values of the design variables $\vec{X}_t$ (see Fig. 2.1); $t$ is the iteration's index. Start point of the iterative searching process $\vec{X}_{t=0}$ can be assigned as engineering's estimation of the admissible design of the structure.

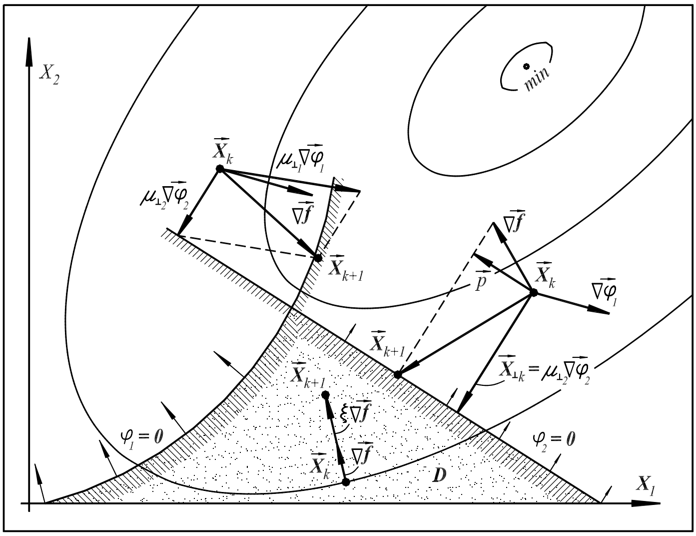


Fig. 2.1. Step to the optimum point depending on location of the current approximation in the N-dimension search space: graphical illustration

The active constraints only of constraints system Eqs. (1.2) – (1.3) should be considered at each iteration. Set of active constraints numbers **A** calculated for the current approximation $\vec{X}_t$ to the optimum solution (current design of the structure) is determined as:

$$\mathbf{A} = \boldsymbol{\kappa} \cup \boldsymbol{\eta},\ \boldsymbol{\kappa} = \left\{\kappa \,\middle|\, \left|\psi_\kappa(\vec{X}_t)\right| \ge -\varepsilon\right\},\ \boldsymbol{\eta} = \left\{N_{EC} + \eta \,\middle|\, \varphi_\eta(\vec{X}_t) \ge -\varepsilon\right\}.$$

where $\varepsilon$ is small positive number introduced here in order to diminish the oscillations on movement alongside of the active constraints surface.

Increment vector $\Delta\vec{X}_t$ for the current values of the design variables $\vec{X}_t$ can be determined by the following equation:

$$\Delta\vec{X}_t = \Delta\vec{X}_\perp^t + \Delta\vec{X}_\parallel^t, \tag{2.2}$$

where $\Delta\vec{X}_\perp^t$ is the vector calculated subject to the condition of liquidation the constraint's violations; $\Delta\vec{X}_\parallel^t$ is the vector determined taking into consideration the improvement of the objective function value. Vectors $\Delta\vec{X}_\parallel^t$ and $\Delta\vec{X}_\perp^t$ are directed parallel and perpendicularly accordingly to the subspace with the vectors basis of the linear-independent constraint's gradients, such that:

$$\left(\Delta\vec{X}_\perp^t\right)^T \Delta\vec{X}_\parallel^t = 0 . \tag{2.3}$$

The values of the constraint's violations for the current approximation $\vec{X}_t$ of the design variables are accumulated into the following vector:

$$\mathbf{V} = \left(\psi_\kappa(\vec{X})\forall\kappa\in\boldsymbol{\kappa};\ \varphi_\eta(\vec{X})\forall\eta\in\boldsymbol{\eta}\right).$$

Let introduce into further consideration set $\mathbf{L}$, $\mathbf{L}\subseteq\mathbf{A}$, of the constraint's numbers, such that the gradients of the constraints at the current approximation $\vec{X}_t$ to the optimum solution are linear-independent.

Component $\Delta\vec{X}_\perp^t$ is calculated from the equation presented below:

$$\Delta\vec{X}_\perp^t = [\nabla\varphi]\vec{\mu}_\perp . \tag{2.4}$$

where $[\nabla\varphi]$ is the matrix that consists of components $\frac{\partial\psi_\kappa}{\partial X_\iota}$ and $\frac{\partial\varphi_\eta}{\partial X_\iota}$, here $\iota=\overline{1,N_X}$, $\kappa\in\mathbf{L}$, $\eta\in\mathbf{L}$; $\vec{\mu}_\perp$ is the column-vector that defines the design variables increment subject to the condition of liquidation the constraint's violations. Vector $\vec{\mu}_\perp$ can be calculated as presented below.

In order to correct constraint's violations $\mathbf{V}$, vector $\Delta\vec{X}_\perp^t$ to a first approximation should also satisfy Taylor's theorem for the continuously differentiable multivariable function in the vicinity of point $\vec{X}_t$ for each constraint from set $\mathbf{L}$, namely:

$$-\mathbf{V} = [\nabla\varphi]^T \Delta\vec{X}_\perp^t . \tag{2.5}$$

With substitution of Eq. (2.4) into the Eq. (2.5) we obtain the system of equations to determine column-vector $\vec{\mu}_\perp$:

$$[\nabla\varphi]^T [\nabla\varphi]\vec{\mu}_\perp = -\mathbf{V} . \tag{2.6}$$

Component $\Delta\vec{X}_\parallel^t$ is determined using the following equation:

$$\Delta\vec{X}_\parallel^t = \xi\times\vec{p}_{\nabla f} = \xi\left(\nabla\vec{f} - [\nabla\varphi]\vec{\mu}_\parallel\right), \tag{2.7}$$

where $\nabla\vec{f}$ is the vector of the objective function gradient in the current point (current approximation of the design variables) $\vec{X}_t$; $\vec{p}_{\nabla f}$ is the projection of the objective function gradient vector onto the active constraints surface in the current point $\vec{X}_t$; $\vec{\mu}_{\parallel}$ is the column-vector that defines the design variable's increment subject to the improvement of the objective function value. Column-vector $\vec{\mu}_{\parallel}$ can be calculated approximately using the least-square method by the following equation:

$$[\nabla\varphi]\vec{\mu}_{\parallel} \approx \nabla\vec{f}\,, \tag{2.8}$$

or from the equation presented below:

$$[\nabla\varphi]^T[\nabla\varphi]\vec{\mu}_{\parallel} = [\nabla\varphi]^T\nabla\vec{f}\,; \tag{2.9}$$

where $\xi$ is the step parameter, which can be calculated subject to the desired increment $\Delta f$ of the purpose function on movement along the direction of the purpose function anti-gradient. The increment $\Delta f$ can be assign as 5...25% from the current value of the objective function $f(\vec{X}_t)$:

$$\Delta f = \xi\left(\nabla\vec{f}\right)^T\nabla\vec{f}\,,\ \xi = \frac{\Delta f}{\left(\nabla\vec{f}\right)^T\nabla\vec{f}}\,, \tag{2.10}$$

where in case of minimisation Eq. (1.1) $\Delta f$ and $\xi$ accordingly have negative values. The parameter $\xi$ can be also calculated using the dependency presented below:

$$\xi = \frac{\Delta f}{\left(\vec{p}_{\nabla f}\right)^T\nabla\vec{f}}\,, \tag{2.11}$$

that follows from the condition of attainment the desired increment of the objective function $\Delta f$ on movement along the direction of the objective function anti-gradient projection onto the active constraints surface. Step parameter $\xi$ can be also selected as a result of numerical experiments performed for each type of the structure individually [6, 13].

Using Eqs. (2.4) and (2.7), Eq. (2.2) can be rewritten as presented below:

$$\Delta\vec{X}_t = [\nabla\varphi]\,\vec{\mu}_{\perp} + \xi\left(\nabla\vec{f} - [\nabla\varphi]\,\vec{\mu}_{\parallel}\right), \tag{2.12}$$

or

$$\Delta\vec{X}_t = \xi\,\nabla\vec{f} + [\nabla\varphi]\left(\vec{\mu}_{\perp} - \xi\,\vec{\mu}_{\parallel}\right), \tag{2.13}$$

where column-vectors $\vec{\mu}_{\perp}$ and $\vec{\mu}_{\parallel}$ are calculated using Eq. (2.6) and Eq. (2.8) or Eq. (2.9).

The linear-independent constraints of the system Eqs. (1.2) – (1.3) should be detected when constructing the matrix of the active constraints gradients $[\nabla\varphi]$ used by Eq. (2.6) and Eq. (2.8) or Eq. (2.9). Selection of the linear-independent constraints can be performed based on the equivalent transformations of the resolving equations of the gradient-based method using the non-degenerate transformation matrix $\mathbf{H}$, such that the sub-diagonal elements of the matrix $\mathbf{H}[\nabla\varphi]$ equal to zero. Besides,

$$\mathbf{H}^T\mathbf{H} = \mathbf{I}; \quad (2.14)$$

$$\mathbf{H} = \mathbf{H}_t \times \ldots \times \mathbf{H}_i \times \ldots \times \mathbf{H}_2 \times \mathbf{H}_1; \quad (2.15)$$

where $\mathbf{I}$ is the unit matrix; $t$ is the total number of the linear-independent gradients of the active constraints, $\mathbf{H}_i$ is the transformation matrix, such that $\mathbf{H}_i^T\mathbf{H}_i = \mathbf{I}$, at the same time the sub-diagonal element are equal to zero in matrix $\mathbf{H}_i \times \mathbf{H}_{i-1} \times \ldots \times \mathbf{H}_2 \times \mathbf{H}_1 \times [\nabla\varphi]$ for column's numbers $\overline{1, i}$. Described conditions are satisfied by the orthogonal matrix of the elementary mapping (Householder's transformation) [18].

Let us present here the following algorithm to form set $\mathbf{L}$ and to construct matrix $\mathbf{H}[\nabla\varphi]$.

1. $i = 0$, $\mathbf{L} = \varnothing$ and $[\nabla\mathbf{\Phi}]_0 = [\nabla\varphi]$ should be assumed, where $[\nabla\varphi]$ is the matrix that comprises from the column-gradients of all active constraints. All columns of matrix $[\nabla\mathbf{\Phi}]_0$ should be marked as 'not used' (or linear-independent).

2. $i = i + 1$.

3. Among all 'not used' columns of matrix $[\nabla\mathbf{\Phi}]_{i-1}$, whose correspond to the constraints-equalities Eq. (1.2), one $j^{\text{th}}$ column with extreme value of the specified criterion should be selected (for example, the following criterion $\ell_j^2 = \sum_{k=i}^{N_X} g_{kj}^2$ can be considered as such criterion, where $g_{kj}$ are the $j^{\text{th}}$ column's components of matrix $[\nabla\mathbf{\Phi}]_{i-1}$). At the same time all $k^{\text{th}}$ columns of matrix $[\nabla\mathbf{\Phi}]_{i-1}$, for whose the following inequality $\ell_k^2 \le \varepsilon_1$ met, should be marked as 'used', here $\varepsilon_1$ is the small positive number. In case when no constraints-equalities exist or all constraints-equalities Eq. (1.2) are marked as 'used', the selection of $j^{\text{th}}$ column should be performed among all 'not used' columns of matrix $[\nabla\mathbf{\Phi}]_{i-1}$, whose correspond to the constraints-inequalities Eq. (1.3). If $\ell_j^2 \le \varepsilon_1$, then generation of set $\mathbf{L}$ and matrix $\mathbf{H}[\nabla\varphi]$ is finished. $\mathbf{H}[\nabla\varphi] = [\nabla\varphi]_{i-1}$. In case of $\ell_j^2 \le \varepsilon_1$ and $i = 1$ (i. e. $\mathbf{L} = \varnothing$), there is a contradiction in the system of constraints Eq. (1.2) – (1.3). In other case, moving to the next step performs.

4. $k^{\text{th}}$ number of the constraint, that corresponds to the $j^{\text{th}}$ column number,

should be included into set $\mathbf{L}$ , $\mathbf{L} \leftarrow \mathbf{L} + \{k\}$ .

5. Calculate $[\nabla\mathbf{\Phi}]_i = \mathbf{H}_i [\nabla\mathbf{\Phi}]_{i-1}$. It is reasonable to execute the multiplication only for 'not used' columns. It should be noted, when using Householder's transformation matrix $\mathbf{H}_i$ is not constructed evidently [18]. At the same time, matrix $[\nabla\mathbf{\Phi}]_i$ may be constructed within the ranges of matrix $[\nabla\mathbf{\Phi}]_{i-1}$ when no additional memory is needed.

6. If $i = 1$, then $[\nabla\varphi]_i = \vec{q}_j$, where $\vec{q}_j$ is $j^{\text{th}}$ column-vector of matrix $[\nabla\mathbf{\Phi}]_i$. When $i > 1$ $[\nabla\varphi]_i$ is constructed using extension of the matrix $[\nabla\varphi]_{i-1}$ by the column-vector $\vec{q}_j$. $j^{\text{th}}$ column of matrix $[\nabla\mathbf{\Phi}]_i$ is selected as 'used', then moving to the step 2 performs.

Using Householder's transformations described above triangular structure of the nonzero elements of matrix $\mathbf{H}[\nabla\varphi]$ is formed step-by-step. Besides, Eq. (2.6) and Eq. (2.8) can be rewritten as follow:

$$\left([\nabla\varphi]^T \mathbf{H}^T\right)\left(\mathbf{H}[\nabla\varphi]\right)\vec{\mu}_\perp = -\mathbf{V}\,; \qquad (2.16)$$

$$\mathbf{H}[\nabla\varphi]\vec{\mu}_\parallel \approx \mathbf{H}\nabla\vec{f}\ . \qquad (2.17)$$

In order to calculate column-vectors $\vec{\mu}_\perp$ and $\vec{\mu}_\parallel$ it takes only to perform forward and backward substitutions in Eq. (2.16) and Eq. (2.17).

To accelerate the convergence of the minimisation algorithm presented above, $h^{\text{th}}$ columns should be excluded from matrix $\mathbf{H}[\nabla\varphi]$. These columns correspond to those constraints from Eq. (1.3), for whose the following inequality satisfies:

$$\mu_{\perp h} - \xi \times \mu_{\parallel h} > 0\,. \qquad (2.18)$$

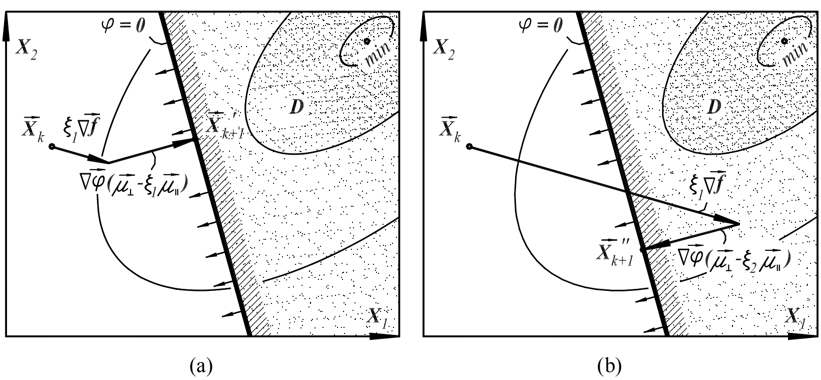


Fig. 2.2. Graphical illustration for the selection of the constraints-inequalities: graphical illustration: $a -$ $\mu_{\perp h} - \xi_1\mu_{\parallel h} < 0$ ; $b -$ $\mu_{\perp h} - \xi_2\mu_{\parallel h} > 0$

Actually, when $\mu_{\perp h} - \xi_2 \mu_{\parallel h} > 0$ return onto the active constraints surface from the feasible region $\Im$ with simultaneous degradation of the objective function value perform (see Fig. 2.2, *b*). At the same time, in case of:

$$\mu_{\perp h} - \xi_1 \mu_{\parallel h} < 0 , \tag{2.19}$$

both improvement of the objective function value and return from the inadmissible region onto the active constraints surface perform (see Fig. 2.2, *a*).

When excluding $h^{\text{th}}$ columns from matrix $\mathbf{H}[\nabla\varphi]$ corresponded to those constraints for whose Eq. (2.12) satisfies, matrix $(\mathbf{H}[\nabla\varphi])_{red}$ with broken (non-triangular) structure of the non-zero elements is obtained. The set $\mathbf{L}$ of the linear-independent active constraints numbers transforms into the set $\mathbf{L}_{red}$ respectively. At the same time, the vector of the constraint's violations $\mathbf{V}$ reduced into the vector $\mathbf{V}_{red}$ accordingly.

In order to restore triangular structure of the matrix $(\mathbf{H}[\nabla\varphi])_{red}$ with zero sub-diagonal elements Givens transformations (Givens rotations) [1, 18] can be used. Givens transformations for the matrix $(\mathbf{H}[\nabla\varphi])_{red}$ consist of construction such square matrix $\mathbf{G}_{wz}$, for which corresponded $wz^{\text{th}}$ element of matrix $\mathbf{G}_{wz}(\mathbf{H}[\nabla\varphi])_{red}$ returns into zero (see Fig. 2.3) [12]. Since $c^2 + s^2 = 1$ by definition, so it follows:

$$(\mathbf{G}_{wz})^T \mathbf{G}_{wz} = \mathbf{I} . \tag{2.20}$$

Obvious method to calculate $c$ and $s$ for $d^{\text{th}}$ non-zero sub-diagonal element and for $a^{\text{th}}$ diagonal element is:

$$c = \frac{a}{r}, \quad s = \frac{d}{r}; \tag{2.21}$$

where

$$r = \sqrt{a^2 + d^2} . \tag{2.22}$$

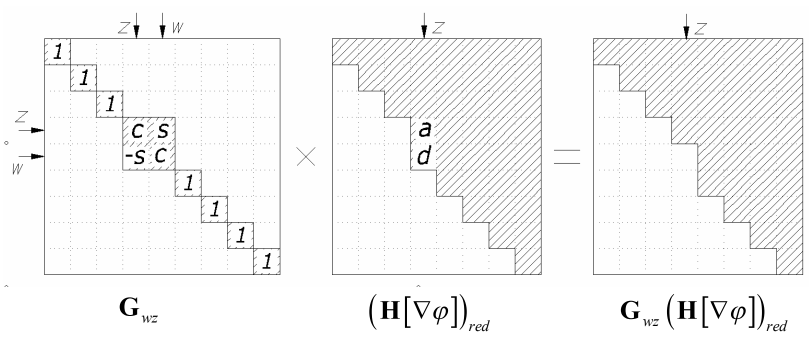


Fig. 2.3. Scheme for Givens rotations (non-zero elements of the matrixes are dashed)

The Givens matrix $\mathbf{G}$ may be calculated similarly to the matrix $\mathbf{H}$ using the following equation:

$$\mathbf{G} = \mathbf{G}_{\gamma} \times \ldots \times \mathbf{G}_{i} \times \ldots \times \mathbf{G}_{2} \times \mathbf{G}_{1} . \qquad (2.23)$$

where $\gamma$ is the number of the Givens transformations. So, Givens transformations should be executed several times (with different values $z$ and $w$), while the matrix $\mathbf{G}_{wz}\left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}$ has no all zero sub-diagonal elements (for example presented by Fig. 2.3, $\gamma = 5$).

Taking into account Givens transformations Eq. (2.16) and Eq. (2.17) to calculate column-vectors $\left(\vec{\mu}_{\perp}\right)_{red}$ and $\left(\vec{\mu}_{\parallel}\right)_{red}$ can be rewritten as:

$$\left(\left[\nabla\varphi\right]^{T}\mathbf{H}^{T}\right)_{red}\mathbf{G}^{T}\mathbf{G}\left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\perp}\right)_{red} = -\mathbf{V}_{red} ; \qquad (2.24)$$

$$\mathbf{G}\left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\parallel}\right)_{red} \approx \mathbf{GH}\nabla\vec{f} ; \qquad (2.25)$$

and the main resolving equation of the gradient-based method Eq. (2.12) and Eq. (2.13) can be rewritten as presented below:

$$\Delta\vec{X}_{t} = \left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\perp}\right)_{red} + \xi\left(\nabla\vec{f} - \left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\parallel}\right)_{red}\right), \qquad (2.26)$$

or

$$\Delta\vec{X}_{t} = \xi\,\nabla\vec{f} + \left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\left(\vec{\mu}_{\perp}\right)_{red} - \xi\left(\vec{\mu}_{\parallel}\right)_{red}\right). \qquad (2.27)$$

Proposed improvement for the method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations consists of equivalent transformations of the resolving equations using Householder transformations. The transformations with matrix $\mathbf{H}$ presented by Eq. (2.24) and Eq. (2.25) of the resolving equations of the gradient-based method Eq. (2.6) and Eq. (2.8) increase numerical efficiency of the algorithm developed based on the gradient-based method described above.

Additionally, proposed improvement for the gradient-based method includes equivalent transformations of the resolving equations using Givens rotations. The transformations with matrix $\mathbf{G}$ presented by Eq. (2.24) and Eq. (2.25) ensure acceleration of the iterative searching process Eq. (2.1) in case when Eq. (2.18) takes into account due to decreasing the amount of calculations.

It should be noted, that lengths of the gradient vectors for objective function Eq. (1.1) as well as for constraints Eqs. (1.2) – (1.3) remain as they were in scope of the proposed equivalent transformations ensuring the dependability of the optimisation algorithm.

Determination the convergence criterion is the final question when using the iterative searching for optimum point Eq. (2.1) described above. Taking into consideration the geometrical content of the gradient steepest descent method, we can assume that, at the permissible point $\vec{X}_{t}$ the component of the increment vector $\Delta\vec{X}_{\|}^{t}$ for the design variables should be vanish, $\Delta\vec{X}_{\|}^{t} \to 0$, in case of approximation to the optimum solution of the non-linear programming task presented by Eqs. (1.1) – (1.4). So, the following convergence criterion of the iterative procedure Eq. (2.1) can be assign:

$$\left\|\Delta\vec{X}_{\|}^{t}\right\| = \sqrt{\sum_{\iota=1}^{N_X}\left(\Delta X_{\|,\iota}^{t}\right)^2} < \varepsilon_1, \tag{2.28}$$

where $\varepsilon_1$ is the small positive number.

Taking into consideration Eq. (2.28) let formulate the following stop criteria in the iterative searching procedure Eq. (2.1).

*Stop criterion 1*: in case of the objective function gradient in the current approximation $\vec{X}_t$ of the design variables is close to zero value indicating on extreme character of the current approximation, as well as violated constraints are absent:

$$\left[\begin{array}{l}\mathbf{\Sigma} = \varnothing, \\ -\varepsilon \geq \nabla\vec{f} \geq +\varepsilon;\end{array}\right. \tag{2.29}$$

where $\mathbf{\Sigma}$ is the set of the violated constraints numbers, $\mathbf{\Sigma} = \left\{ s \middle| \left|\psi_s(\vec{X}_t)\right| > \varepsilon;\ \varphi_s(\vec{X}_t) > \varepsilon \right\}$;

*Stop criterion 2*: in case of the projection of the objective function gradient in the current approximation $\vec{X}_t$ onto the active constraints surface is close to zero value or objective function gradient is perpendicularly to the active constraints surface indicating impossible further improvement of the objective function value, as well as violated constraints are absent:

$$\left[\begin{array}{l}\mathbf{\Sigma} = \varnothing; \\ -\varepsilon \geq \vec{p} \geq +\varepsilon;\end{array}\right. \tag{2.30}$$

*Stop criterion 3*: when in the current approximation $\vec{X}_t$ of the iterative searching procedure (2.2) the total number of the active constraints $t$ equals to the number of design variables $N_X$, as well as all active constraints are $\varepsilon$-active (both not violated constraints and those ones for whose inequality Eq. (2.12) met):

$$\left[\begin{array}{l}\mathbf{\Sigma} = \varnothing; \\ t = N_X; \\ \mu_{\perp f} - \xi \times \mu_{\| f} < 0, \forall f \in \mathbf{L}.\end{array}\right. \tag{2.31}$$

This stop criterion for the iteration process Eq. (2.1) corresponds to the case when the current approximation $\vec{X}_t = \left(X_\iota^t\right)^T$, $\iota = \overline{1, N_X}$, to the optimum solution locates at the intersection point of the constraints (so called, vertex). In this case, no correction of the constraints violations is needed and further improvement of the purpose function value is not possible.

*Stop criterion 4*: when the purpose function values within two consecutive iterations are the same with acceptable accuracy subject to the absence of the violated constraints:

$$\left[\begin{array}{l} \boldsymbol{\Sigma} = \varnothing; \\ f\left(\vec{X}_{t-1}\right) \approx f\left(\vec{X}_t\right). \end{array}\right. \tag{2.32}$$

**3. Results and discussion.** In order to estimate an efficiency of the new methods or algorithms, we should perform a comparison with alternative methods or algorithms presented by other authors using different optimisation techniques. Criteria to implement such comparison are described, i.e. by the papers [2, 6]. Many of them, such as robustness, amount of functions calculations, requirements to the CPU memory, numbers of iterations etc. cannot be used due to lack of corresponded information in the technical literature. Therefore, an efficiency estimation of the method of objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations presented above will be based on comparison of the optimisation results obtained using proposed improvement of the gradient-based method, as well as of the results presented by the literature and widely used for testing. Initial data and mathematical models of the parametric optimisation problems considered below were assumed as the same as described in the literature.

***3.1. Parametric optimisation of a three-bar truss.*** Optimisation of a three-bar truss (see Fig. 3.1) has been firstly solved by Schmit L. A. [15] using a non-linear programming method. Besides, the task has been also considered by the authors of the paper [6].

A parametric optimisation problem was formulated as searching for optimum cross-sectional areas $b_1$, $b_2$ and $b_3$ of the truss bars providing the least value of the truss weight subject to normal stresses and flexural stability constraints, as well as displacements and eigenvalue constraints. Load cases for truss under consideration are presented by Table 3.1.

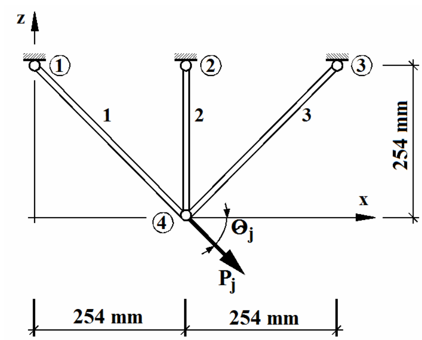

Fig. 3.1. Three-bar truss

Table 3.1

Load cases for considered truss

| Load case $j$ | 1 | 2 | 3 |
|---|---|---|---|
| $\theta_j$, ° | 45 | 90 | 135 |
| $P_j \times 10^3$, pound-force | 40 | 30 | 20 |
| $P_j$, ton-force | 18.144 | 13.608 | 9.072 |

Initial data for optimisation of the truss are as follows: unit weight of the truss material is $\rho g = 0.1$ pound/inch$^3$ = $2.768 \cdot 10^{-6}$ t/cm$^3$; modulus of elasticity is $E = 10^7$ pound/inch$^2$ = 703.066 t/cm$^2$; allowable stresses value for the 1st and 3rd truss member is $\sigma_1^a = \sigma_3^a = 5000$ pound/inch$^2$ = 0.3515 t/cm$^2$; for the 2nd truss member is $\sigma_2^a = 2000$ pound/inch$^2$ = 1.4061 t/cm$^2$; non-dimensional factor used to calculate second moment area of inertia for each truss member is $\beta = 1$, $I_i = \beta b_i$; ultimate vertical $z^a$ and horizontal $x^a$ displacements of the truss nodes are $x^a = z^a = 0.005$ inch = 0.127 mm; lower limit value for eigenvalue is $\zeta_0 = 1.872 \cdot 10^8$.

The objective function can be written as presented below:

$$\psi_0 = \rho g l \left( b_1\sqrt{2} + b_2 + b_3\sqrt{2} \right) \to \min\,; \tag{3.1}$$

where $l$ is the truss height, $l = 25.4$ cm (see Fig. 3.1). Let formulate strength constraints for each truss members for all load cases as follows:

$$\psi_{3(i-1)+j} = \frac{\left| N_i^j \right|}{b_i \sigma_i^a} - 1 \le 0\,; \tag{3.2}$$

where $N_i^j$ is the axial force for $i$ th truss member subjected to $j$ th load case, $i = \overline{1,3}$, $j = \overline{1,3}$. Besides, let include to the system of constraints the inequalities describing that the design variables should have positive values:

$$\psi_{9+i} = -b_i \le 0\,; \tag{3.3}$$

Flexural buckling constraints for all truss members can be written using Hooke law as presented below:

$$\psi_{12+3(i-1)+j} = -\frac{\left( x_4^j + z_4^j \right) l}{\pi^2 \beta b_i} - 1 \le 0\,; \tag{3.4}$$

where $x_4^j$, $z_4^j$ are linear displacements for 4th node of the truss subjected to $j$ th load case along the directions of $0x$ and $0z$ axes respectively. Constraints on the minimum values of the eigenvalues can be written analytically using calculation results of the eigenvalues stability problem for truss under consideration:

$$\psi_{22} = \frac{2\sqrt{2}\rho\, l^2 \zeta_0 \left( \frac{b_1 + b_3}{b_2}\sqrt{2} + 1 \right)}{3E\left( \frac{b_1 + b_3}{b_2} + \sqrt{2} - \sqrt{\left( \frac{b_1 - b_3}{b_2} \right)^2 + 2} \right)} - 1 \le 0 . \quad (3.5)$$

Let also formulate displacements constraints for 4th truss node in the plane $x0z$:

$$\psi_{22+j} = -1 - \frac{x_4^j}{x^a} \le 0 ; \quad (3.6)$$

$$\psi_{25+j} = \frac{x_4^j}{x^a} - 1 \le 0 ; \quad (3.7)$$

$$\psi_{28+j} = -1 - \frac{z_4^j}{z^a} \le 0 ; \quad (3.8)$$

$$\psi_{31+j} = \frac{z_4^j}{z^a} - 1 \le 0 . \quad (3.9)$$

Starting from start values of the design variables $\vec{b}^0 = (64.5160,\ 32.2580,\ 32.258)^T$ cm$^2$ with truss weight $G^0 = 116.602$ N optimum solution $\vec{b}^* = (57.4878,\ 12.4482,\ 27.4299)^T$ cm$^2$ with optimum weight $G^* = 91.383$ N has been obtained. Comparison of the optimisation results for three-bar truss under consideration obtained by authors of the paper [6] and in this article is presented by Table 3.2. Step-by-step characteristics of the iterative searching for optimum design of the three-bar truss are presented by Table 3.3.

Table 3.2

Comparison of the optimisation results for three-bar truss

| Truss member number, $i$ | Start values of the design variables | Optimum cross-section areas for $i$ th truss member, cm$^2$ | |
|---|---|---|---|
| | | Paper [6] | This paper |
| 1 | 64.5160 | 59.225688 | 57.487781 |
| 2 | 32.2580 | 13.935456 | 12.448249 |
| 3 | 32.2580 | 24.838660 | 27.429940 |
| **Truss weight, N** | **116.602** | **91.588438** | **91.382689** |

11 iterations have been performed. Iterative searching process for the optimum point was stopped due to the following stop criterion: increment of the design variables within two consecutive iterations was less than 0.0001, as well as there were no violated constraints.

Table 3.3
Step-by-step characteristics of the iterative searching for optimum design of the three-bar truss

| Iteration number | Current values of the design variables, $cm^2$ | | | Objective function value, ton-force | Numbers of the active constraints | Maximum violation of the constraints |
|---|---|---|---|---|---|---|
| | $b_1$ | $b_2$ | $b_3$ | | | |
| 0 | 64.5160 | 32.2580 | 32.2580 | 0.01189005130 | – | – |
| 1 | 44.5160 | 22.2580 | 22.2580 | 0.00820412802 | 15 | 0.358346583 |
| 2 | 60.78468 | 12.2580 | 12.2580 | 0.00812434854 | 10, 15, 18, 22, 24, 33, 34 | 0.462764185 |
| 3 | 40.78469 | 14.85774 | 22.2580 | 0.00731284150 | 15, 18, 22, 26 | 0.422373593 |
| 4 | 55.57631 | 15.86132 | 21.5254 | 0.00878127279 | 15, 22 | 0.100763871 |
| 5 | 57.39339 | 13.01145 | 26.33674 | 0.00923995830 | 15, 22 | 0.011887395 |
| 6 | 57.58708 | 12.55355 | 27.23679 | 0.00931651373 | 15, 22 | 0.000269880 |
| 7 | 57.49673 | 12.45692 | 27.41397 | 0.00931835396 | 15, 22 | $9.66743 \cdot 10^{-6}$ |
| 8 | 57.48847 | 12.44889 | 27.42879 | 0.00931844122 | 15, 22 | $7.55468 \cdot 10^{-8}$ |
| 9 | 57.48783 | 12.44830 | 27.42985 | 0.00931844180 | 15, 22 | $1.42876 \cdot 10^{-9}$ |
| 10 | 57.48778 | 12.44825 | 27.42993 | 0.00931844180 | 15, 22 | $8.73750 \cdot 10^{-11}$ |
| 11 | 57.48778 | 12.44825 | 27.42994 | 0.00931844180 | 15, 22 | $6.55025 \cdot 10^{-12}$ |

***3.2. Optimisation of a ten-bar cantilever truss.*** A parametric optimization problem of a ten-bar cantilever truss (see Fig. 3.2) is widely used in the literature [3, 6, 14, 16] in order to compare different methods for solving optimisation problems. The parametric optimisation problem is formulated as follows: to find unknown cross-sectional areas for each truss member $\vec{b} = (b_i)^T$, $i = \overline{1,10}$, with weight minimisation of the truss subjected to stresses constraints in all truss bars, node displacements constraints, as well as constraints on the minimal cross-section areas.

The truss under consideration is undergone for two load cases (see Fig. 3.2 together with Table 3.4). Initial data for optimisation of the truss are as follows: unit weight of the truss material is $\rho g = 0.1$ pound/inch$^3$ = $2.768 \cdot 10^{-6}$ t/cm$^3$; modulus of elasticity is $E = 10^7$ pound/inch$^2$ = 703.066 t/cm$^2$; non-dimensional factor used to calculate second moment area of inertia for each truss member is $\beta = 1.0$ $\left(I_i = \beta b_i^2\right)$; lower limit value for cross-sectional areas for all truss bars is $b^L = 0.10$ inch$^2$ = 0.64516 cm$^2$; allowable stresses value for the all truss member is $\sigma^a = 25 \cdot 10^3$ pound/inch$^2$ = 1.758 t/cm$^2$; ultimate vertical $z^a$ and horizontal $x^a$ displacements of the truss nodes are $x^a = z^a = 2$ inch = 50.8 mm. Start value $b_0 = 1.0$ inch$^2$ = 6.4516 cm$^2$ was used as start approximation for variable cross-sections areas for all bars of the truss under consideration.

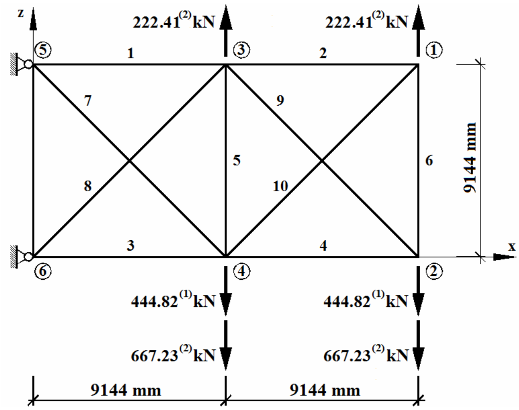


Fig. 3.2. Ten-bar cantilever truss

Table 3.4

Load cases for ten-bar cantilever truss

| Load case number | Node number (see Fig. 3.2) | Concentrated load along axis $0z$, $\times 10^3$ pound | Concentrated load along axis $0z$, t |
|---|---|---|---|
| 1 | 2 | –100.0 | -45.35901659 |
| | 4 | –100.0 | -45.35901659 |
| 2 | 1 | 50.0 | 22.67950830 |
| | 2 | –150.0 | -68.03852489 |
| | 3 | 50.0 | 22.67950830 |
| | 4 | –150.0 | -68.03852489 |

Variable cross-section areas for each truss member $\vec{b} = (b_i)^T$, $i = \overline{1,\ 10}$, were considered as design variables. The objective function can be written as presented below:

$$\psi_0 = \rho g l \left( b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + \sqrt{2}\left(b_7 + b_8 + b_9 + b_{10}\right)\right) \to \min ; \quad (3.10)$$

where $l$ is the truss height, $l = 914.4$ cm (see Fig. 3.2). Constraints on lower limit value for variable cross-sectional areas for all truss bars are written as follows:

$$\psi_i = 1 - \frac{b_i}{b_i^L} \le 0 . \quad (3.11)$$

Stresses constraints can be formulated as presented below:

$$\psi_{10+i} = \frac{|N_i|}{b_i \sigma_i^a} - 1 \le 0 . \quad (3.12)$$

where $N_i$ is the axial force in the $i^{\text{th}}$ truss member. Displacement constraints for the truss nodes are written as follows:

$$\psi_{20+j} = -1 - x_j / x^a \le 0 ; \qquad (3.13)$$

$$\psi_{24+j} = x_j / x^a - 1 \le 0 ; \qquad (3.14)$$

$$\psi_{28+j} = -1 - z_j / z^a \le 0 ; \qquad (3.15)$$

$$\psi_{32+j} = z_j / z^a - 1 \le 0 ; \qquad (3.16)$$

where $x_j$, $z_j$ are linear displacements of $j^{\text{th}}$ truss node, $j = \overline{1,4}$.

Table 3.5

Comparison of the optimisation results for the 10-bar cantilever truss

| Bur number, $i$ | Start values for design variables | Optimal cross-section area for $i^{\text{th}}$ truss member, cm$^2$ | | | |
|---|---|---|---|---|---|
| | | for the first load case | | for the second load case | |
| | | Paper [6] | This paper | Paper [6] | This paper |
| 1 | 6.4516 | 193.7479996 | 197.0312484 | 152.0255024 | 151.8842240 |
| 2 | 6.4516 | 0.645160000 | 0.645160000 | 0.645160000 | 0.645160000 |
| 3 | 6.4516 | 150.1545384 | 149.6078266 | 163.0770932 | 163.1232003 |
| 4 | 6.4516 | 98.61915760 | 98.22918330 | 92.5352988 | 92.75779895 |
| 5 | 6.4516 | 0.645160000 | 0.645160000 | 0.645160000 | 0.645160000 |
| 6 | 6.4516 | 3.590315400 | 3.559530885 | 12.70836168 | 12.70787948 |
| 7 | 6.4516 | 48.18248428 | 48.02706763 | 80.0062916 | 79.87214746 |
| 8 | 6.4516 | 136.7610168 | 135.7825567 | 82.9030600 | 82.79629923 |
| 9 | 6.4516 | 139.4706888 | 138.9183389 | 130.870706 | 131.1797334 |
| 10 | 6.4516 | 0.645160000 | 0.645160000 | 0.645160000 | 0.645160000 |
| **Truss weight, kN** | **1.8666727** | **22.51500912** | **22.51356469** | **20.80022802** | **20.80595725** |
| Number of active constraints | | 4 | 5 | 4 | 6 |
| Numbers of active constraints | | – | 2, 5, 10, 13, 31 | – | 2, 5, 10, 17, 31, 32 |
| Modulus of the maximum violation in the constraints | | $0.27 \cdot 10^{-4}$ | $\lvert\delta\psi_{13}\rvert = 2.041 \times 10^{-13}$ | $0.17 \cdot 10^{-3}$ | $\lvert\delta\psi_{17}\rvert = 2.824 \times 10^{-12}$ |

Starting from the initial truss design with start weight $G^0 = 1.867$ kN optimal solution with optimum weight $G^* = 22.514$ kN has been obtained for the truss subjected to the first load case. Additionally, starting from the initial truss design with start weight $G^0 = 1.867$ kN optimal solution with optimum weight $G^* = 20.806$ kN has been obtained for the truss subjected to the first load case. Comparison of the optimisation results for three-bar truss under consideration obtained by authors of the paper [6] and in this article is presented by Table 3.5.

For both loaded cases iterative searching process for the optimum point was stopped due to the following stop criterion: increment of the design variables within two consecutive iterations was less than 0.0001, as well as there were no violated constraints.

Comparison of the optimisation results for the ten-bar cantilever truss obtained using the proposed improved method of objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations with optimisation results presented by the literature [3, 6, 14, 16] are shown in Table 3.6.

Table 3.6

Comparison of the optimisation results for the 10-bar cantilever truss

| Weight, kN | Load case 1 | | Load case 2 | |
|---|---|---|---|---|
| | Stresses constraints only | All constraints | Stresses constraints only | All constraints |
| This paper | 7.086663425 | 22.51356469 | 7.404064841 | 20.8059573 |
| The paper [6] | 7.086783276 | 22.51500912 | 7.408610546 | 20.8003614 |
| The paper [16] | 7.087005686 | 22.58284417 | 7.404251310 | 20.8039200 |
| The paper [14] | 7.086783276 | 22.58199901 | 7.404162346 | 20.8038755 |
| The paper [3] | 7.2149804 | 22.59685600 | – | 22.5065575 |

***3.3. Optimisation of a 24-bar translational tower.*** Parametric optimization problem for a translational tower (see Fig. 3.3) has been considered by the paper [6]. The translation tower is subjected to 2 load cases (see Table 3.7). Taking into account the symmetry of the structural form, the vector of the design variables has been reduced to 7 variable cross-section areas for 25 structural members of the tower under consideration (see Table 3.8). The parametric optimization problem is formulated as searching for optimum cross-sectional areas $\vec{X} = (X_i)^T$, $i = \overline{1,7}$, of the tower structural members, whose provide the least weight of the tower subjected to stresses constraints, node displacements constraints, as well as constraints on the minimal cross-section areas.

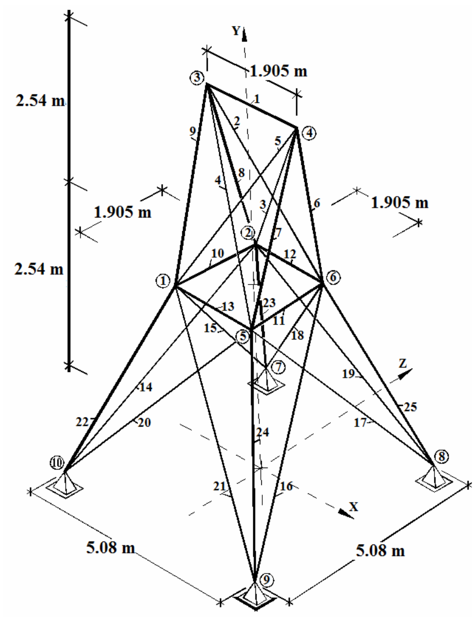


Fig. 3.3. Design model of the translational tower

Initial data for optimisation of the tower are as follows: unit weight of the tower material is $\rho g = 0.1$ pound/inch$^3$= 2.768 t/m$^3$; modulus of elasticity is $E = 10^7$ pound/inch$^2$ = 703.074 t/cm$^2$; non-dimensional factor used to calculate second moment area of inertia for each tower structural member is $\beta = 1.0$ $\left(I_i = \beta b_i^2\right)$; lower limit value for cross-sectional areas for all tower members is $A^L = 0.01$ inch$^2$ = 0.0645 cm$^2$; ultimate node displacements of the tower are $x^a = y^a = z^a = 0.35$ inch = 8.89 mm; allowable stresses value for the all tower member is $\sigma^a = \pm 40 \cdot 10^3$ pound/inch$^2$ = ±2.8122 t/cm$^2$.

Start value $A_0 = 1.0$ inch$^2$ = 6.4516 cm$^2$ was used as start approximation for variable cross-sections areas for all members of the tower under consideration. Dimensions of the optimisation problem were 7 design variables and 129 constraints.

Comparison of the optimisation results for the translational tower is presented by Table 3.8. At the continuum optimum point there were 5 active constraints: 3rd node displacement constraint of the tower along axis $0x$ for 1st and 2nd load cases, 3rd node displacement constraint along axis $0z$ for 1st load case, 4th node displacement constraint along axis $0x$ for 2nd load case, as well as 4th node displacement constraint along axis $0z$ for 1st load case. Internal axial forces at the optimum design of the translational tower are shown by Table 3.9.

Table 3.7

Load cases for translational tower

| Load case number | Node number (see Fig. 3.3) | Direction of the node load application | | |
|---|---|---|---|---|
| | | $0x$ | $0y$ | $0z$ |
| 1 | 1 | 0.2268 | – | – |
| | 2 | 0.2268 | – | – |
| | 3 | 0.4536 | –2.2680 | 4.5359 |
| | 4 | – | –2.2680 | 4.5359 |
| 2 | 3 | – | –2.2680 | 9.0718 |
| | 4 | – | –2.2680 | –9.0718 |

Table 3.8

Comparison of the optimisation results for the translational tower

| Design variable | Tower structural members (see Fig. 3.3) | Optimal cross-section areas for tower members, cm$^2$ | |
|---|---|---|---|
| | | Paper [6] | This paper |
| $A_1$ | 1 | 0.0645 | 0.0939 |
| $A_2$ | 2, 3, 4, 5 | 13.2103 | 0.2444 |
| $A_3$ | 6, 7, 8, 9 | 19.3322 | 23.8915 |
| $A_4$ | 10, 11, 12, 13 | 0.0645 | 8.6632 |
| $A_5$ | 14, 15, 16, 17 | 4.4213 | 5.0950 |
| $A_6$ | 18, 19, 20, 21 | 10.4626 | 1.8024 |
| $A_7$ | 22, 23, 24, 25 | 17.2335 | 25.2070 |
| Tower weight, t | | 0.2472 | 0.2207 |

Table 3.9

Internal axial forces at the optimum design of the translational tower, ton-force

| Bar No.* | Load case | | Bar No.* | Load case | | Bar No.* | Load case | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | | 1 | 2 | | 1 | 2 |
| 1 | –0.1433 | 0.00 | 10 | –0.0821 | –0.5387 | 18 | –0.4685 | 2.5114 |
| 2 | –0.3292 | 0.00 | 11 | –0.1304 | 0.5387 | 19 | –0.5370 | –2.5114 |
| 3 | 0.0654 | 0.00 | 12 | –3.9711 | 0.00 | 20 | 0.3682 | 2.5114 |
| 4 | –0.2108 | 0.00 | 13 | 2.5968 | 0.00 | 21 | 0.2997 | –2.5114 |
| 5 | 0.1839 | 0.00 | 14 | –1.4834 | –0.4803 | 22 | 5.8160 | –3.7169 |
| 6 | –7.7237 | 4.8443 | 15 | 1.2554 | –0.4803 | 23 | –7.6930 | –3.7170 |
| 7 | 5.0976 | 4.8443 | 16 | –1.5765 | 0.4803 | 24 | 5.0270 | 3.7169 |
| 8 | –7.4008 | –4.8443 | 17 | 1.1623 | 0.4803 | 25 | –8.4820 | 3.7169 |
| 9 | 5.4205 | –4.8443 | | | | | | |

Iterative searching process for the optimum point was stopped due to the following stop criterion: increment of the design variables within two consecutive iterations was less than $1\times10^{-6}$, as well as there were no violated constraints (maximum value among constraint violations was $0.049\times10^{-10}$).

**Conslusion.** The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been considered by the paper. Equivalent Householder transformations of the resolving equations of the method have been proposed. They increase numerical efficiency of the algorithm developed based on the method under consideration.

Additionally, proposed improvement for the gradient-based method also includes equivalent transformations (Givens rotations) of the resolving equations. They ensure acceleration of the iterative searching process in specified cases described by the paper due to decreasing the amount of calculations.

Lengths of the gradient vectors for objective function, as well as for constraints remain as they were in scope of the proposed equivalent transformations ensuring the reliability of the optimisation algorithm.

The comparison of the optimisation results presented by the paper confirms the validity of the optimum solutions obtained using proposed improvement of the gradient-based method. Start values of the design variables have no influence on the optimum solution of the non-linear problem confirming in such way accuracy and validity of the optimum solutions obtained using the algorithm developed based on the presented improved gradient-based method. The efficiency of the propoced improvement of the gradient-based method has been also confirmed taking into account the number of iterations and absolute value of the maximum violation in the constraints. The deviations availabled in some presented results can be explained, on the one hand, by using a numerical approach to the iterative searching with specified accuracy (as in the optimisation of the ten-bar cantilever truss), on the other hand, by possible existence of several local optimum points (as in the optimisation of the translation tower).

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*Стаття надійшла 05.03.2020*

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**МОДИФІКАЦІЯ ГРАДІЄНТНОГО МЕТОДУ ДЛЯ РОЗВ'ЯЗКУ ЗАДАЧ ПАРАМЕТРИЧНОЇ ОПТИМІЗАЦІЇ СТЕРЖНЕВИХ КОНСТРУКЦІЙ**

У статті розглядаються задачі параметричної оптимізації стержневих конструкцій, які формулюються в термінах задачі нелінійного програмування. Об'єктом дослідження виступає метод, що грунтуються на обчисленні градієнтів функції мети та обмежень, а задачею дослідження – розробка математичного та алгоритмічного забезпечення для розв'язку задач параметричної оптимізації конструкцій при орієнтації на програмну реалізацію в системі автоматизованого проектування.

Для розв'язку задач параметричної оптимізації використовується метод проекції градієнта функції мети на поверхню активних обмежень з одночасною ліквідацією нев'язок в обмеженнях. У статті запропоновані еквівалентні перетворення Хаусхолдера для розв'язувальних рівнянь розглядуваного методу оптимізації, які підвищують обчислювальну ефективність алгоритму, розробленого на основі градієнтного методу. Окрім того запропоновані еквівалентні перетворення Гівенса для розв'язувальних рівнянь розглядуваного методу, які для визначених випадків, обумовлених у статті, пришвидшують ітераційний процес пошуку оптимального розв'язку внаслідок скорочення обсягу обчислень. Довжини векторів градієнтів функції мети та обмежень математичної моделі залишаються незмінними при запропонованих еквівалентних перетвореннях, що забезпечує надійність алгоритму оптимізації.

Порівняння результатів оптимізаціних розрахунків стержневих систем, представлене у статті, підтверджує достовірність оптимальних розв'язків, отриманих з використанням запропонованої модифікації градієнтного методу. Ефективність запропонованої модифікації градієнтного методу оптимізації, що розглядається, також підтверджується кількістю ітерацій та абсолютним значенням максимальної нев'язки в обмеженнях.

**Ключові слова:** параметрична оптимізація, задача нелінійного програмування, градієнтний метод, стержнева система, метод скінчених елементів

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**AN IMPROVED GRADIENT-BASED METHOD TO SOLVE PARAMETRIC OPTIMISATION PROBLEMS OF THE BAR STRUCTURES**

The paper considers parametric optimisation problems for the bar structures formulated as non-linear programming tasks. In the paper a gradient-based method is considered as investigated object. The main research question is the development of mathematical support and numerical algorithm to solve parametric optimisation problems of the building structures with orientation on software implementation in a computer-aided design system.

The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimisation problem. Equivalent Householder transformations of the resolving equations of the method have been proposed by the paper. They increase numerical efficiency of the algorithm developed based on the method under consideration. Additionally, proposed improvement for the gradient-based method also consists of equivalent Givens transformations of the resolving equations. They ensure acceleration of the iterative searching process in the specified cases described by the paper due to decreasing the amount of calculations. Lengths of the gradient vectors for objective function, as well as for constraints remain as they were in scope of the proposed equivalent transformations ensuring the reliability of the optimisation algorithm.

The comparison of the optimisation results of truss structures presented by the paper confirms the validity of the optimum solutions obtained using proposed improvement of the gradient-based method. Start values of the design variables have no influence on the optimum solution of the non-linear problem confirming in such way accuracy and validity of the optimum solutions obtained using the algorithm developed based on the presented improved gradient-based method. The efficiency of the propoced improvement of the gradient-based method has been also confirmed taking into account the number of iterations and absolute value of the maximum violation in the constraints.

**Keywords:** bar system, parametric optimisation, non-linear programming task, gradient-based method, finite-element method

*Пелешко И. Д., Юрченко В. В.*

**УЛУЧШЕННЫЙ ГРАДИЕНТНЫЙ МЕТОД ДЛЯ РЕШЕНИЯ ЗАДАЧ ПАРАМЕТРИЧЕСКОЙ ОПТИМИЗАЦИИ СТЕРЖНЕВЫХ КОНСТРУКЦИЙ**

В статье рассматриваются задачи параметрической оптимизации стержневых конструкций, формулируемые в терминах задачи нелинейного программирования. Объектом исследования выступает метод, базирующийся на вычислении градиентов функции цели и ограничений, а задачей исследования служит разработка математического и алгоритмического обеспечения для решения задач параметрической оптимизации конструкций при ориентации на программную реализацию в системе автоматизированного проектирования.

Для решения задач параметрической оптимизации используется метод проекции градиента функции цели на поверхность активных ограничений с одновременной ликвидацией невязок в ограничениях. В статье предложены эквивалентные преобразования Хаусхолдера для разрешающих уравнений рассматриваемого метода оптимизации, повышающие численную эффективность алгоритма, разработанного на основе градиентного метода. Кроме того, предложены эквивалентные преобразования Гивенса для разрешающих уравнений рассматриваемого метода, обеспечивающие в определенных случаях, оговоренных в статье, ускорение итерационного процесса поиска оптимального решения вследствие уменьшения объема вычислений. Длины векторов градиентов функции цели и ограничений математической модели остаются неизменными при предложенных эквивалентных преобразованиях, что обеспечивает надежность оптимизационного алгоритма.

Сравнение результатов оптимизационных расчетов стержневых систем, представленных в статье, подерждает достоверность оптимальных решений, полученных с использование предложенного улучшения градиентного метода. Эффективность предложенного улучшения рассматриваемого метода оптимизации также подтверждается количеством итераций и абсолютнім значением максимальной невязки в ограничениях.

**Ключевые слова:** стержневая конструкция, параметрическая оптимизация, нелинейное программирование, градиентный метод, метод конечных элементов

УДК 519.853, 624.04
*Пелешко І. Д., Юрченко В. В.* **Модифікація градієнтного методу для розв'язку задач параметричної оптимізації стержневих конструкцій** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 104. – С. 265-288.

*У статті розглядаються задачі параметричної оптимізації стержневих конструкцій, які формулюються в термінах задачі нелінійного програмування. Для розв'язку задач параметричної оптимізації використовується метод проекції градієнта функції мети на поверхню активних обмежень з одночасною ліквідацією нев'язок в обмеженнях. У статті запропоновані еквівалентні перетворення Хаусхолдера для розв'язувальних рівнянь розглядуваного методу оптимізації, які підвищують обчислювальну ефективність алгоритму, розробленого на основі градієнтного методу. Окрім того запропоновані еквівалентні перетворення Гівенса для розв'язувальних рівнянь розглядуваного методу, які для визначених випадків, обумовлених у статті, пришвидшують ітераційний процес пошуку оптимального розв'язку внаслідок скорочення обсягу обчислень. Порівняння результатів оптимізаціних розрахунків стержневих систем, представлене у статті, підтверджує достовірність оптимальних розв'язків, отриманих з використанням запропонованої модифікації градієнтного методу.*
Іл. 6. Табл. 9. Бібліог. 18 назв.

UDC 519.853, 624.04
*Peleshko I. D., Yurchenko V. V.* **An improved gradient-based method to solve parametric optimisation problems of the bar structures** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2020. – Issue 104. – P. 265-288.

*The paper considers parametric optimisation problems for the bar structures formulated as non-linear programming tasks. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used*

*to solve the parametric optimisation problem. Equivalent Householder transformations of the resolving equations of the method have been proposed by the paper. They increase numerical efficiency of the algorithm developed based on the method under consideration. Additionally, proposed improvement for the gradient-based method also consists of equivalent Givens transformations of the resolving equations. They ensure acceleration of the iterative searching process in the specified cases described by the paper due to decreasing the amount of calculations. The comparison of the optimisation results of truss structures presented by the paper confirms the validity of the optimum solutions obtained using proposed improvement of the gradient-based method.*
Figs. 6. Tabs. 9. Refs. 18.

УДК 519.853, 624.04
*Пелешко И. Д., Юрченко В. В.* **Улучшенный градиентный метод для решения задач параметрической оптимизации стержневых конструкций** // Сопротивление материалов и теория сооружений: науч.- тех. сборн. – К.: КНУСА, 2020. – Вып. 104. – С. 265-288.

*В статье рассматриваются задачи параметрической оптимизации стержневых конструкций, формулируемые в терминах задачи нелинейного программирования. Для решения таких задач используется метод проекции градиента функции цели на поверхность активных ограничений с одновременной ликвидацией невязок в ограничениях. Предложены эквивалентные преобразования Хаусхолдера для разрешающих уравнений рассматриваемого метода оптимизации, обеспечивающие численную эффективность алгоритма, разработанного на основе градиентного метода. Кроме того, предложены эквивалентные преобразования Гивенса для разрешающих уравнений рассматриваемого метода, обеспечивающие в определенных случаях, оговоренных в статье, ускорение итерационного процесса поиска оптимального решения вследствие уменьшения объема вычислений. Сравнение результатов оптимизационных расчетов стержневых систем, представленных в статье, подерждает достоверность оптимальных решений, полученных с использование предложенного улучшения градиентного метода.*
Ил. 6. Табл. 9. Библиог. 18 назв.

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UDC 624.012.45:624.012.36

# CALCULATED RELIABILITY OF ECCENTRICALLY COMPRESSED CONCRETE COLUMNS UNDER THE ACTION OF LOW CYCLE LOADING WITH ALTERNATING ECCENTRICITES

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DOI: 10.32347/2410-2547.2020.104.289-298

The paper gives definition of design reliability of eccentric compressed reinforced concrete columns under the action of low-cyclic loads with alternating eccentricities. Based on theoretical researches and using the experimental data the numerical example of defining the design reliability of aforesaid elements was made. The value of eccentricity, level and loading conditions influence essentially on reversal of physical and mechanical properties of materials, since these parameters are considered as random variables from which reliability, constructive reliability and long-term durability of eccentric compressed elements are depended. During estimating the column reliability, it was used the existed method of reliability calculation of building constructions under the action of single-stage steady loads in accordance with existing norms. It was justified statistically the definition of the reversal of physical and mechanical properties of concrete during determining the coefficient of operating conditions. These reversals were taking into account during operating the eccentric compressed elements under action of low-cyclic alternating loads.
**Keywords**: eccentric compression, low-cyclic loads, alternating eccentricity, reliability.

**Introduction.** Construction is one of the materials-intensive manufacturing industries, the development of which requires continuous improvement of calculation methods and design of reinforced concrete structures, aimed at ensuring their reliability in operation while reducing material consumption and other costs. The limit state method, which is contained in the current regulatory documents, allows providing bearing capacity of the reinforced concrete eccentrically compressed elements due to the use of various coefficients of reliability and responsibility of the structure. However, the sufficient bearing capacity of the element does not guarantee its sufficient reliability, the quantitative assessment of which is unknown at the stage of the design of a building. Reliability and economy are the prerequisites for the design, construction and operation of buildings and structures. The need to ensure a high level of reliability of buildings, structures and their structural elements is quite obvious, since their failure, including possible accidents and destruction, leads to great economic losses, dangerous environmental consequences, and sometimes catastrophic casualties. The experience in the construction and operation of construction sites shows that the same type of buildings and structures that are constructed and operated under similar conditions, or their separate structural elements, fail with different random occurrence. It is almost impossible to accurately determine the service life of a building structure or building as a whole, and it is only possible to estimate the likelihood that this building or structure will be used for a given period. Therefore, the methods of assessing the reliability of structural elements requires credible information about the variability of the strength parameters of building materials, the magnitude of loads and their nature as random deviations from the calculation models, etc. Of all the factors that affect the reliability of structures and buildings in general, loading and its actions are the most uncertain values with large statistical errors. Therefore, the study of the variability of loading regimes plays a major role in the issues of reliability and structural safety of structural units, buildings and structures.

**Analysis of recent publications on this topic.** Formation and development of reliability fundamentals in construction originated in the late twenties of the last century in the works of M. Mayer [1] and M. F. Khotsialov [2]. Later, in the late forties of the last century, the modern interpretation of the concept of reliability in the field of construction industry is associated, first and foremost, with the works of M. S. Streletsky [3] and A. R. Rzhanitsyn [4]. It was in these works that the fundamental definitions of the modern theory of the reliability of buildings and structures were laid, in which not only the statistical nature of variability of the strength characteristics of materials, but also the loading parameters were considered, and the necessity of the probabilistic assessment of the reliability of a building or structure was proved. It was A.R. Rajcinin who formulated the basic principles of the concept of security of a building or structure, which are the basic principles of the whole theory of reliability. Somewhat later, basic research on the problem of reliability using probabilistic models was conducted by such scientists as V. V. Bolotin [5], V. D. Reiser [6,7], A. P. Kudzis [8]. Variability studies of the main factors affecting the reliability of structural elements, including statistical analysis of the physical and mechanical characteristics of concrete and reinforcing steel, were carried out by A.Ya. Barashikov [9,10], M. M. Zastava [11], O. S Lychev [12] and others. The significant contribution to the improvement of the methods of calculating the reliability of structures was made by V. A. Perlmuter [13,14], A. I. Lantuh-Lyashchenko [15], S. F. Pichugin [16,17], O. V. Semko [18], V. A. Pashinsky [19], S. B. Usakovsky [20], R. I. Kinash [21] and others. Among the foreign scientists working in the field of reliability, a significant role belongs to A. H - S. Ang [22], A. M. Frendenhal [23], O. Ditlevsen and H. O. Madsen [24], R. E. Melchers, M. A. Ahammed [25] and others.

**Research objective.** To determine, on the basis of experimental and theoretical studies, a calculated estimate of the reliability of reinforced concrete eccentrically compressed columns under the action of low cycle loading with alternating eccentricities.

**Research findings.** The use of eccentrically compressed elements is quite common in buildings and structures. The loads acting on such elements are quite diverse – from constant static to low cyclic repetitions and alternating ones. The latter loads cause special conditions of such elements operation, causing changes in the physical-mechanical and deformative characteristics of materials, affecting the processes of cracking, crack opening and their deformability, which in turn affects their operation reliability and durability. Eccentrically compressed reinforced concrete elements, which are tested in the course of operation, low-cycle alternating loading, include columns of single-storey and multi-storey industrial buildings and various structures, racks of crane and transport trestles, elements of buttress retaining walls. In addition, such a stressful condition in structures may occur during the reconstruction of buildings or structures, as well as in emergency situations.

In the laboratory of the Department of Industrial and Civil Construction, and Engineering Structures, the authors conducted comprehensive experimental studies of the work of eccentrically compressed columns under the action of low-cycle loading of varying intensity with alternating eccentricities. Experimental specimens of $b\times h\times l$ – 100×160×3000 mm in size were made of C16/20 and C20/25 concrete. Reinforcement of columns was carried out by spatial frames. Four rods with a diameter of 12 mm of class A400C were adopted as working reinforcement, and a cross bar of steel of class B500 with a diameter of 4 mm in 150 mm was used as transverse reinforcement. The columns were tested for eccentric compression after 30 days of exploitation and more in special settings. Longitudinal force was alternately applied with eccentricities $e_0$=10 cm through the steel head. The modes of low cycle alternating loads for different columns were at different levels and varied from $\eta$=0.3 to $\eta$=0.85. The methodology of experimental research is presented in detail in [26].

A detailed analysis of the experimental studies showed that alternating low cycle loading at low levels of $\eta$=0.3 slightly increase the load-carrying capacity of eccentrically compressed elements compared to those subjected to eccentric compression by a single short-term static loading. The load carrying capacity, depending on the loading level, was in the range of 8% ... 12%. This is due to a certain compaction of early-age concrete, although at higher loading levels this effect of increasing bearing capacity was counterbalanced. And at the level of $\eta$=0.85, the load carrying capacity even decreased to 7% ... 10%, as cracks appeared in the stretched zone of concrete, which violated the integrity of the section. In this case, only the central part of the section was sealed. In [27], a theoretical determination of the load carrying capacity of eccentrically compressed elements under the action of low-cycle loading with alternating eccentricities is provided, taking into account second-order effects that increase the value of the initial eccentricity of applying compressive forces. The theoretical calculations of the load carrying capacity of the above elements were performed in accordance with the current regulatory documents [28] and [29]. Therefore, let us determine the calculated reliability of eccentrically compressed elements undergoing low cycle loading with alternating eccentricities, using experimental data based on a numerical example. To determine the design reliability, we consider the data of the columns with the lowest load carrying capacity using a statistical analysis of the variability of their parameters (accepted mean values).

The estimated reliability of the columns is evaluated according to the methodology outlined in [17], in accordance with the requirements [30]. The values of [$\beta$]=4,75 and [$\beta$]=3,89 are taken as the normative indicator of reliability according to the current norms in the calculations of building structures, respectively, according to the first and second groups of limit state, considering that they belong to buildings of the CC2 consequences class and the structures of A responsibility category.

**Numerical example of calculation.** Output data: column cross section dimensions – $b\times h$=100×160 mm; $a=a'$=15 mm; working longitudinal reinforcement – 2 rods on each side with a diameter of 12 mm of A400C class ($A_s=A_s'$=2,26 cm$^2$), $f_{yd}$=365 MPa, $f'_{yd}$=300 MPa (the value is taken from the experimental data); concrete grade C20/25, $f_{cd}$=17 MPa; height of columns $H$=300 cm. The compressive force acting on the columns is applied with eccentricity $e_0$=10 cm. The destruction of the column began at $N_u$=130 kN. The collapse of the column occurred at the bending moment. $M_u=N_u\cdot e_0$=130·10=1300 kNcm=13 kNm.

Statistical characteristics of materials: concrete C20/25

- mathematical expectation

$$\bar{\sigma}_c=\frac{f_{cd}}{1-1.64V_c}=\frac{17}{1-1.64\cdot 0.135}=21.83\,\text{MPa}=2.18\,\text{kN/cm}^2,$$

where $V_c$ - coefficient of variation;

- standard

$$\hat{\sigma}_c=\bar{\sigma}_c\cdot V_c=21.83\cdot 0.135=2.95\,\text{MPa}=0.295\,\text{kN/cm}^2;$$

reinforcing steel A400C

- mathematical expectation

$$\bar{\sigma}_s=f_{yd}=365\,\text{MPa}=36.5\,\text{kN/cm}^2;$$

$$\bar{\sigma}_{sc}=f'_{yd}=300\,\text{MPa}=30\,\text{kN/cm}^2;$$

- standard

$$\hat{\sigma}_s=\bar{\sigma}_s\cdot V_s=365\cdot 0.0436=15.91\,\text{MPa}=1.59\,\text{kN/cm}^2;$$

$$\hat{\sigma}_{sc}=\bar{\sigma}_{sc}\cdot V_s=300\cdot 0.0436=13.08\,\text{MPa}=1.31\,\text{kN/cm}^2,$$

where $V_s=0.0436$ according to table 2.31 [17] – coefficient of variation.

Numerical characteristics of stresses:

- mathematical expectation of resisting moment

$$\bar{M}_u = \bar{\sigma}_s A_s d - \bar{\sigma}_{sc} A_s' a' - 0.5\frac{(\bar{\sigma}_s A_s - \bar{\sigma}_{sc} A_s')^2}{\bar{\sigma}_c b} =$$

$$= 36.5 \cdot 2.26 \cdot 14.5 - 30 \cdot 2.26 \cdot 1.5 - 0.5\frac{(36.5 \cdot 2.26 - 30 \cdot 2.26)^2}{2.18 \cdot 10} = 1089.45\,\text{kN cm};$$

- resisting moment standard

$$\hat{M}_u = \hat{\sigma}_s A_s d - \hat{\sigma}_{sc} A_s' a' - 0.5\frac{(\hat{\sigma}_s A_s - \hat{\sigma}_{sc} A_s')^2}{\hat{\sigma}_c b} =$$

$$= 1.59 \cdot 2.26 \cdot 14.5 - 1.31 \cdot 2.26 \cdot 1.5 - 0.5\frac{(1.59 \cdot 2.26 - 1.31 \cdot 2.26)^2}{0.295 \cdot 10} = 46.95\,\text{kN cm};$$

- mathematical expectation of compression force

$$\bar{N}_u = \bar{\sigma}_s A_s + \bar{\sigma}_{sc} A_s' + \bar{\sigma}_c bh = 36.5 \cdot 2.26 + 30 \cdot 2.26 + 2.18 \cdot 10 \cdot 16 = 499.07\,\text{kN};$$

- compression force standard

$$\hat{N}_u = \hat{\sigma}_s A_s + \hat{\sigma}_{sc} A_s' + \hat{\sigma}_c bh = 1.59 \cdot 2.26 + 1.31 \cdot 2.26 + 0.295 \cdot 10 \cdot 16 = 56.75\,\text{kN};$$

Geometric characteristics of the cross section of columns: $A$=160cm$^2$, $W$=426.7cm$^3$, $i_x$=2.67cm.

Using column cross-section parameters, a number of calculations can be performed:

- relative eccentricity

$$\bar{m} = \frac{\bar{M}_u A}{\bar{N}_u W} = \frac{1089.45 \cdot 160}{499.07 \cdot 426.7} = 0.819;$$

- given relative eccentricity

$$\bar{m}_{ef} = \eta \bar{m} = 1.58 \cdot 0.819 = 1.29,$$

where

$\eta = (1.75 - 0.1\bar{m}) - 0.02(5 - \bar{m}) = (1.75 - 0.1 \cdot 0.819) - 0.02(5 - 0.819) = 1.58$ - the coefficient taking into account the shape of the cross section.

Column flexibility – $\lambda = H/b = 300/10 = 30$.

Relative flexibility – $\bar{\lambda} = \lambda\sqrt{f_{yd}/E_s} = 30\sqrt{365/(2 \cdot 10^5)} = 1.28$.

We determine the numerical characteristics of the column stability reserve:

- mathematical expectation

$$\bar{Y} = \bar{f}'_{yd} - \frac{\bar{N}_u \cdot 10}{A \cdot b(1 - C \cdot \lg \bar{m}_{ef})} = 300 - \frac{499.07 \cdot 10}{160 \cdot 0.594(1 - 0.817 \cdot 0.119)} = 241.56\text{MPa},$$

where

$b = K_1 - K_2 \lg\bar{\lambda} = 0.7 - 0.62 \cdot 0.117 = 0.627$, $C = K_3 - K_4/b = 0.943 - 0.075/10 = 0.934$, coefficients $K_1, K_2, K_3, K_4$ are taken according to [17].

To determine the standard of the column stability reserve, we define the coefficients $A_1, A_2, A_3$ under the conditions provided in [17].

$$A_1 = 1,\ A_2 = \frac{C\cdot \lg\overline{m}_{ef} + C\cdot d - 1}{A\cdot b(1 - C\cdot \lg\overline{m}_{ef})^2} = \frac{0.934\cdot 0.119 + 0.934\cdot 0.4343 - 1}{160\cdot 0.627(1 - 0.934\cdot 0.119)^2} = -0.61\cdot 10^{-2}\ \text{cm}^{-1},$$

$$A_3 = \frac{C\cdot d\cdot \eta}{b\overline{m}_{ef}\cdot W(1 - C\cdot \lg\overline{m}_{ef})^2} = \frac{0.934\cdot 0.4343\cdot 1.58}{0.627\cdot 0.119\cdot 426.7(1 - 0.934\cdot 0.119)^2} = -2.0255\cdot 10^{-2}\ \text{cm}^{-1},$$

where $d = 0.4343$ is the module of transition from natural to decimal logarithms.

The standard of the column stability reserve is determined by the formula:

$$\widehat{Y} = \sqrt{\sum_{i=1}^{n} A_2^2 \cdot \widehat{X}_i^2} = \sqrt{A_1^2 \cdot \sigma_{sc}^2 + A_2^2 \cdot \widehat{N}_u^2 + A_3^2 \cdot \widehat{M}_u^2} =$$

$$= 10\sqrt{1^2 \cdot 3^2 + (-0.61 \cdot 10^{-2})^2 \cdot 53.75^2 + (-2.0255 \cdot 10^{-2})^2 \cdot 46.95^2} = 33.924.$$

The safety characteristics proved to be equal: $\beta = \frac{\overline{Y}}{\widehat{Y}} = \frac{241.56}{33.924} = 7.12$, that under the normal stability distribution gives the probability of failure of the column according to the table D3 [17] $Q = 1.30 \cdot 10^{-12}$.

As one can see, the discovered safety characteristics of the investigated columns significantly exceed the normative index: $\beta = 7.12 > [\beta] = 4.75$.

**Conclusions.** Based on the analysis of experimental and theoretical studies of the operation of reinforced concrete eccentrically compressed columns operating under the action of low cycle loading with alternating eccentricities, the calculated reliability of such columns is numerically determined. It should be noted that the reliability of these structures under the above loading during operation will be ensured.

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*Стаття надійшла до редакції 05.03.2020*

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**РОЗРАХУНКОВА НАДІЙНІСТЬ ПОЗАЦЕНТРОВО СТИСНУТИХ ЗАЛІЗОБЕТОННИХ КОЛОН ЗА ДІЇ МАЛО ЦИКЛОВИХ НАВАНТАЖЕНЬ ІЗ ЗНАКОЗМІННИМИ ЕКСЦЕНТРИСИТЕТАМИ**

Стаття присвячена визначенню розрахункової надійності позацентрово стиснутих залізобетонних елементів за дії малоциклових навантажень із знакозмінними ексцентриситетами. Ці елементи є по суті самими малодослідженими з точки зору надійності будівельних конструкцій.

Таке положення обумовлене тим, що окрім труднощів з імовірнісним описом реальних навантажень, особливо тимчасових (кранових, снігових або вітрових), оцінка надійності позацентрово стиснутих елементів, в тому числі і виконаних з такого добре вивченого матеріалу, яким є залізобетон, пов'язана з урахуванням геометричної і фізичної нелінійності.

Важливо підкреслити, що імовірнісний аналіз таких елементів, особливо в частині обґрунтованого розрахунку сполучення зусиль, може дати помітний ефект, оскільки на практиці саме позацентрово стиснуті елементи (стійки, колони та інші) завантажені найбільш широким набором випадкових навантажень.

На основі теоретичних досліджень, використовуючи експериментальні дані випробуваних елементів, виконаний числовий приклад визначення розрахункової надійності вище зазначених елементів. Оскільки величина ексцентриситету, рівень і характер навантажень суттєво впливають на зміну фізико-механічних властивостей матеріалів, так як ці параметри вважаються випадковими величинами від яких і залежить надійність, конструктивна безпека і довговічність позацентрово стиснутих елементів в процесі експлуатації.

При визначенні розрахункової надійності позацентрово стиснутих колон за дії малоциклових навантажень із знакозмінними ексцентриситетами використано існуючу методику розрахунку оцінки надійності будівельних конструкцій за дії однократних статичних навантажень з дотриманням чинних нормативних документів. Визначення зміни фізико-механічних властивостей бетону і арматури, при визначенні коефіцієнта умов роботи за дії вище вказаних навантажень, обґрунтовано статистично. Ці зміни враховано у роботі позацентрово стиснутих колон за дії малоциклових знакозмінних навантажень при визначенні їх розрахункової оцінки надійності.

**Ключові слова:** позацентровий стиск, мало циклове навантаження, знакозмінний ексцентриситет, надійність.

*Masiuk H.Kh., Aleksiievets V.I., Aleksiievets I.I., Masiuk V.H.*

**CALCULATED RELIABILITY OF ECCENTRICALLY COMPRESSED CONCRETE COLUMNS UNDER THE ACTION OF LOW CYCLE LOADING WITH ALTERNATING ECCENTRICITES**

The paper gives definition of design reliability of eccentric compressed reinforced concrete columns under the action of low-cyclic loads with alternating eccentricities. In particular, these elements are underexplored in terms of building constructions reliability.

Such condition is due to the fact that, besides difficulties with the probabilistic description of actual loads, especially temporary ones (crane, snow or wind), the reliability estimation of eccentric

compressed elements, including elements which were made of well-researched material such as reinforced concrete, is connected with taking into account geometric and physical nonlinearity.

It is necessary to emphasize, the probabilistic analysis of such elements, especially in part of justified calculation of coupling forces, can give significant impact since in practice the eccentric compressed elements (studs, columns, etc.) are loaded by the widest set of random loads.

Based on theoretical researches and using the experimental data the numerical example of defining the design reliability of aforesaid elements was made. Since the value of eccentricity, level and loading conditions influence essentially on change of physical and mechanical properties of materials, therefore, these parameters are considered as random variables from which reliability, constructive reliability and long-term durability of eccentric compressed elements are depended during operating process.

During defining the design reliability of eccentric compressed columns under action of low-cyclic alternating loads, it was used the existed method of calculating the estimation of building constructions reliability under the action of single-stage steady loads in accordance with normative documents. It was justified statistically the definition of the change of physical and mechanical properties of concrete and reinforcing steel during determining the coefficient of operating conditions. These changes were taking into account during operating the eccentric compressed columns under action of low-cyclic alternating loads during defining the designing estimate of reliability.

**Keywords**: eccentric compression, low-cyclic loads, alternating eccentricity, reliability.

УДК 624.012.45:624.012.36
*Масюк Г.Х., Алексієвець В.І., Алексієвець І.І., Масюк В.Г.* **Розрахункова надійність позацентрово стиснутих залізобетонних колон за дії мало циклових навантажень із знакозмінними ексцентриситетами** //Опір матеріалів і теорія споруд: наук.-тех. збірник. – К.:КНУБА, 2020. – Вип. 104. – С. 289-298.

*Стаття присвячена визначенню розрахункової надійності позацентрово стиснутих залізобетонних елементів за дії мало циклових навантажень із знакозмінними ексцентриситетами. На основі теоретичних досліджень, використовуючи експериментальні дані, виконаний числовий приклад визначення розрахункової надійності вище зазначених елементів. Величина ексцентриситету, рівень і характер навантажень суттєво впливають на зміну фізико-механічних властивостей матеріалів, так як ці параметри вважаються випадковими величинами від яких і залежить надійність, конструктивна безпека і довговічність позацентрово стиснутих елементів. При визначенні оцінки надійності колон використано існуючу методику розрахунку надійності будівельних конструкцій за дії однократних статичних навантажень з дотриманням чинних норм. Визначення зміни фізико-механічних властивостей бетону при визначенні коефіцієнта умов роботи, обґрунтовано статистично. Враховано ці зміни у роботі позацентрово стиснутих елементів за дії мало циклових знакозмінних навантажень.*
Табл. 0. Іл. 0. Бібліогр. 30 назв.

UDC 624.012.45:624.012.36
*Masiuk H.Kh., Aleksiievets V.I., Aleksiievets I.I., Masiuk V.H.* **Calculated reliability of eccentrically compressed concrete columns under the action of low cycle loading with alternating eccentricities** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2020. – Issue 104. – P. 289-298.

*The paper gives definition of design reliability of eccentric compressed reinforced concrete columns under the action of low-cyclic loads with alternating eccentricities. Based on theoretical researches and using the experimental data the numerical example of defining the design reliability of aforesaid elements was made. The value of eccentricity, level and loading conditions influence essentially on reversal of physical and mechanical properties of materials, since these parameters are considered as random variables from which reliability, constructive reliability and long-term durability of eccentric compressed elements are depended. During estimating the column reliability, it was used the existed method of reliability calculation of building constructions under the action of single-stage steady loads in accordance with existing norms. It was justified statistically the*

*definition of the reversal of physical and mechanical properties of concrete during determining the coefficient of operating conditions. These reversals were taking into account during operating the eccentric compressed elements under action of low-cyclic alternating loads*.
Tabl. 0. Il. 0. Ref. 30.

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UDC 539.3

# THERMOELASTICITY OF ELASTOMERIC CONSTRUCTIONS WITH INITIAL STRESSES

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DOI: 10.32347/2410-2547.2020.104.299-308

The article presents an algorithm for solving linked problems of thermoelasticity of elastomeric structural elements on the basis of a moment scheme of finite elements. To model the processes of thermoelastic deformation of structures with initial stresses the incremental theory of a deformed solid is used. At each step of deformation, the stiffness matrix is adjusted using an incremental geometric stiffness matrix. The use of triple approximation of displacements, deformations and volume change function allows to consider the weak compressibility of elastomers. The components of the stress tensor are calculated according to the Duhamel-Neumann law. To solve the problem of thermal conductivity, a thermal conductivity matrix considering the boundary conditions on the surface of a finite element is constructed. A sequential approximation algorithm is used to solve the thermoelasticity problem. At each stage of the solution, the characteristics of the thermal stress state are calculated. Based on the obtained components of stress and strain tensors, the intensity of internal heat sources is calculated as the dissipative energy averaged over the load cycle. To calculate the dissipative characteristics of the viscoelastic elastomer the parameters of the Rabotnov's relaxation nucleus are used. Solving the problem of thermal conductivity considering the function of internal heat sources allows you to specify the heating temperature of the body. At each cycle of the algorithm, the values of the physical and mechanical characteristics of the thermosensitive material are refined. This approach to solving thermoelastic problems is implemented in the computing complex "MIRELA+". Based on the considered approach, the solutions of a number of problems are obtained. The results obtained satisfactorily coincide with the solutions of other authors. Considering the effect of preload and the dependence of physical and mechanical properties of the material on temperature leads to significant adjustments to the calculated values.

**Key words:** finite element method, elastomer, thermoelasticity, dissipative warming, initial stresses.

**Introduction.** In solving related problems of thermoelasticity of elastomeric structural elements, various theories and approaches are used, which are based on the relations of the thermoelasticity obtained by many researchers [1-6] and others. One of the most important criteria for the study of viscoelastic bodies with a non-uniform temperature field is to take into account the temperature dependence of physical and thermophysical characteristics of the material:

modulus of elasticity or shear modulus, coefficient of thermal expansion and coefficient of thermal conductivity

In the case of a slight increase in temperature, its effect on the mechanical behaviour of the viscoelastic body can be neglected and an unrelated problem of thermovisoelasticity can be considered. This approach is reflected in [7] and others.

When using a coupled linear model, the temperature and thermoelastic states are determined by the solution of the system, which consists of the equations of thermal conductivity, classical equations of motion, Hooke's law equations and classical compatibility equations [8-11], etc.

It is obvious that all thermomechanical processes depend on time and for their research there are problems called non-stationary (dynamic). However, we can identify a number of processes, thermomechanical state, during which, although it changes over time, but from a certain point in time, the system comes to a stationary (static) state, which does not depend on time. In addition, it should be noted that elastomers are a nonlinear viscoelastic material and their stress-strain state depends on the load history.

**1. Problem statement and its solution.** The formulation of the related problem for a quasi-stationary formulation can be represented as the equation Biot and the equation of thermal conductivity [11].

The formulation of the initial provisions for the description of the deformation process considering the initial stresses will begin with the representation of this process as a sequence of equilibrium states:

$$\Omega^{(0)},\Omega^{(1)},\Omega^{(2)},....\ \Omega^{(n)},....\ \Omega^{(L)},$$

where $\Omega^{(0)}$, $\Omega^{(L)}$ - initial and final state of deformation respectively; $\Omega^{(n)}$ - arbitrary intermediate state.

It is considered that for all intermediate states of stress, strain, displacement known throughout the history of deformation to the state $\Omega^{(n)}$. Let the position of an arbitrary point of a body in states $\Omega^{(0)}$, $\Omega^{(n)}$, $\Omega^{(n+1)}$ are determined by the radius vectors of these points $\mathbf{r}^{(0)}$, $\mathbf{r}^{(n)}$, $\mathbf{r}^{(n+1)}$. The body is referred to the basic Cartesian coordinate system and the coordinates of the points are respectively equal $X_i, z_i, Z_i$ $(i = 1, 2, 3)$.

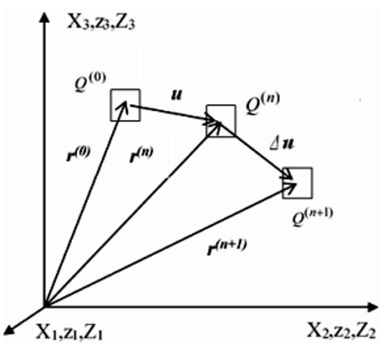

Fig. 1. States $\Omega^{(0)}$, $\Omega^{(n)}$, $\Omega^{(n+1)}$

Let us denote the tensors of Green's deformations in states $\Omega^{(n)}$ and $\Omega^{(n+1)}$ through $\varepsilon_{ij}^{0}$ i $\varepsilon_{ij}^{0}+\varepsilon_{ij}$ respectively:

$$\varepsilon_{ij}^{0}=\frac{1}{2}\left(\mathbf{r}_{,i}^{(n)}\mathbf{r}_{,j}^{(n)}-\mathbf{r}_{,i}^{(0)}\mathbf{r}_{,j}^{(0)}\right)=\frac{1}{2}\left(u_{i,j}+u_{j,i}+u_{k,i}u_{k,j}\right),$$

$$\varepsilon_{ij}^0+\varepsilon_{ij}=\frac{1}{2}\left(\mathbf{r}_{,i}^{(n+1)}\mathbf{r}_{,j}^{(n+1)}-\mathbf{r}_{,i}^{(0)}\mathbf{r}_{,j}^{(0)}\right)=$$

$$=\frac{1}{2}\left((u_i+\Delta u_i)_{,j}+(u_j+\Delta u_j)_{,i}+(u_k+\Delta u_k)_{,i}(u_k+\Delta u_k)_{,j}\right)\ .$$

On the other hand, taking the initial state $\Omega^{(n)}$ and using the rectangular coordinates of the increase in deformation can be determined [12]:

$$\varepsilon_{ij}^*=\frac{1}{2}\left(\frac{\partial\mathbf{r}^{(n+1)}}{\partial z_i}\frac{\partial\mathbf{r}^{(n+1)}}{\partial z_j}-\frac{\partial\mathbf{r}^{(n)}}{\partial z_i}\frac{\partial\mathbf{r}^{(n)}}{\partial z_j}\right)=\frac{1}{2}\left(\frac{\partial\Delta u_i}{\partial z_j}+\frac{\partial\Delta u_j}{\partial z_i}+\frac{\partial\Delta u_k}{\partial z_i}\frac{\partial\Delta u_k}{\partial z_j}\right).$$

The increments of deformations are connected by the relations:

$$\varepsilon_{ij}^*=\frac{\partial X_m}{\partial z_i}\frac{\partial X_n}{\partial z_j}\varepsilon_{mn},\qquad \varepsilon_{ij}=\frac{\partial z_m}{\partial X_i}\frac{\partial z_n}{\partial X_j}\varepsilon_{mn}^*\ .$$

The components of the deformation gain tensor can be represented as linear and nonlinear components:

$$\varepsilon_{ij}^*=\varepsilon_{ij}^{*L}+\varepsilon_{ij}^{*N}=\frac{1}{2}\left(\frac{\partial\Delta u_i}{\partial z_j}+\frac{\partial\Delta u_j}{\partial z_i}\right)+\frac{1}{2}\frac{\partial\Delta u_k}{\partial z_i}\frac{\partial\Delta u_k}{\partial z_j}.$$

Jacobian transformations: $D^{(n)}=\left|z_{i,j}\right|,\ D^{(n+1)}=\left|Z_{i,j}\right|$ ,

$$z_{i,j}=\delta_{ij}+u_{i,j},\ \ Z_{i,j}=\delta_{ij}+(u_i+\Delta u_i)_{,j}\,.$$

To describe a stress state, we introduce the Euler stress tensor. At points $Q^{(n)}$, the components of the stress tensor have the value of $\sigma_0^{ij}$, acting on the faces $z_i-\text{const}$, $(z_i+dz_i)-\text{const}$ and $\sigma_0^{ij}+\sigma^{ij}$, which acting on the faces $Z_i-\text{const}$, $(Z_i+dZ_i)-\text{const}$ an infinitesimal parallelepiped including a point $Q^{(n+1)}$.

At the point $Q^{(n+1)}$ the stress can be described by a modified Kirchhoff tensor [12].

$$\sigma_0^{ij}+\sigma^{ij*}=\frac{1}{D^{(n)}}\frac{\partial z_i}{\partial X_k}\frac{\partial z_j}{\partial X_l}\left(\sigma_0^{kl}+\sigma^{kl}\right),$$

$$\sigma^{ij*}=\frac{1}{D^{(n)}}\frac{\partial z_i}{\partial X_k}\frac{\partial z_j}{\partial X_l}\sigma^{kl}\,.$$

Variational statement of the problem for an elastic body with initial stresses with given additional mass and surface forces in the form of the principle of virtual work in the state $\Omega^{(n+1)}$ looks like:

$$\iiint_V\left[(\sigma_0^{ij}+\sigma^{ij^*})\delta\varepsilon_{ij}^*+(q_i^0+q_i)\delta u_i\right]dV-\iint_S(p_i^0+p_i)\delta\Delta u_i dS^{(n)}=0\ ,$$

where $p_i$, $q_i$– vectors of surface and volumetric additional forces at $n$+1 step.

If $\Omega^{(n)}$ – equilibrium state, then in the equation the terms of the variation of elastic energy and taking into account the initial stress:

$$\iiint_V \left[ \sigma_0^{ij} \delta\varepsilon_{ij}^* - q_i^0 \delta u_i \right] dV^n - \iint_S p_i^0 \delta u_i dS^n = 0 \ .$$

The equilibrium equation takes the form:

$$\iiint_V \left[ \sigma^{ij*} \delta\varepsilon_{ij}^* - \frac{1}{2} \sigma_0^{ij} \delta \left( \frac{\partial \Delta u_k}{\partial z_i} \frac{\partial \Delta u_k}{\partial z_j} \right) - q_i \, \delta\Delta u_i \right] dV^{(n)} - \iint_S p_i \, \delta\Delta u_i dS^{(n)} = 0.$$

To study the stress-strain state of spatial structures made of elastomers, consider the isoparametric finite element in the form of a hexagonal parallelepiped with an edge length of 2. The origin of the basic coordinate system $Z_i$ and an arbitrary local system $x_i$, the axis of which coincides with the direction of its edges, placed in the centre of the cube. Consider the construction of matrices of stiffness of a finite element with initial stresses based on the moment scheme of finite elements. Under the pre-stresses we mean those stresses that arose in the structure in the initial state, i.e. before the deformation process under consideration, before the application of the working load.

The solvating relations in general case can be represented as:

$$\left[ K^{st} + K_0^{st} \right] \{u_t\} = \left\{ P^s \right\},$$

where $K_0^{st}$ – incremental geometric matrix of stiffness, which taking into account the action of prestresses [13].

When forming a matrix $K^{st}$ for a weakly compressible elastomeric layer, a moment scheme of finite elements with triple approximation of displacement fields, deformations, and volume change functions is used. [14].

The components of the stress tensor are determined by the Duhamel – Neumann thermoelasticity law. To solve the problem of calculating the temperature of dissipative heating, it is necessary to solve the problem of thermal conductivity. To construct a thermal conductivity matrix for a layered finite element, the hypothesis of continuity of temperature fields and heat fluxes at the interface is used.

In matrix form, the system of equations for the layer takes the form:

$$[H]\{T\} + [H^{(st)}]\{T\} + \{P\} + \{S\} = 0 \ ,$$

where $H$ – thermal conductivity matrix, $H^{(st)}$ – matrix conditioned to boundary conditions of the 3rd kind on the surface of the construction, $P$ – equivalent load vector conditioned to the internal heat generation source, $S$ – equivalent load vector conditioned to heat fluxes and body surface temperature.

The heat generation function is calculated as averaged over the deformation cycle.

The solution of the linked problem is performed using the method of successive approximations.

**2. Results of the calculation and analysis of solutions.** Let us consider the process of determining the temperature of dissipative heating of elastomeric structures as a solution of the linked problem of thermoelasticity for a stable mode of cyclic deformation and heat exchange with the environment. In this case, the solution of the quasi-static thermoelasticity problem requires the solution of several problems: determination of the function of internal sources in an elastic - hereditary body (solution of the thermoelasticity problem) at the initial temperature; calculation of the temperature field under given boundary conditions (solving the problem of stationary thermal conductivity); solving the problem of thermoelasticity for the final temperature of self-heating. When constructing a mathematical model of the problem, it is assumed that the stress state significantly depends on the coordinates, as a result of which the field of heat and temperature sources is inhomogeneous. In addition, there is a need to take into account the dependence of physical and mechanical properties on temperature. The method of successive approximations is used to solve the related thermoelasticity problem.

The algorithm for solving the related problem is represented by the following sequence:

1. From the solution of the thermoelasticity problem $\left[K^{ij}\right]\left\{u_j\right\}=\left\{P^i\right\}$ the vector of nodal displacements $\{u_i\}$ at the set amplitude of fluctuations is defined. Vector of the right part $\{P^j\}$ determined by the stiffness matrix taking into account the initial stresses and boundary conditions in the form of displacements on the surface of the finite element. For a separated finite element the vector of internal forces is determined by the formula:

$$\left\{P^s\right\}=[A]^T\left[F_{ij}^s\right]^T\left[E^{ijkl}\right]\left[F_{kl}^t\right][A]\{u_t\}+[A]^T\left[F_{(\theta)}^s\right]^T\left[E^{(\theta)}\right]\left[F_{(\theta)}^t\right][A]\{u_t\},$$

where the first term is a vector due to elastic displacements, the second term is a vector of forces due to thermal displacements.

2. To determine the power of internal heat generation sources, it is necessary to determine the amount of scattered energy per load cycle.

The use of the simplest hypotheses about the homogeneity of the displacement field in the direction of reinforcement and the homogeneity of the field of generalized forces for shear stresses and normal stresses to the fibers, allows you to calculate the power of internal heat sources as an average, equal to dissipative energy. The power of internal sources for cyclic loading can also be determined by the formula:

$$w_0^{(k)}=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\sigma_{(k)}^{ij}\dot{\varepsilon}_{ij}^{(k)}dt.$$

3. The temperature field is determined from the solution of the stationary thermal conductivity problem.

The thermal conductivity problem is nonlinear because the matrix $[H]$ and the equivalent heat load vector $\{R\}$ depends on temperature.

The system of solving equations of stationary thermal conductivity using the method of successive approximations is written in the form:

$$\left[H_{(k-1)}\right]\left\{T_{(k)}\right\}=-\left\{R_{(k-1)}\right\}.$$

The iterative process of solving problem continues until the specified calculation accuracy is reached.

After determining the temperature procedure begins with the 1st point. In case when physical and mechanical properties of the material depend on temperature, at each iteration components of the tensor of elastic characteristics, components of the tensor of thermal conductivity, as well as the component of the matrices of rigidity and thermal conductivity of the construction are recalculated.

**3. Results of the calculation and analysis of solutions.** To study the convergence of the results obtained using the proposed approach, we consider solutions of problems for which the literature provides solutions based on other approaches.

*Problem 1.* Consider the deformation of a layered thermosensitive cylindrical shell, the left end of which is rigidly clamped, and the right free, which is under the influence of a non-uniform stationary temperature field of the form [15]:

$$T^{(k)}=T_0^{(k)}(z)+T_1^{(k)}(z),\ k=1, 2, \dots, n,$$

where $n$ – number of layers; $z$ – the coordinate calculated along the outer normal to the middle surface of the cylinder.

Physical and-mechanical characteristics of the layers depends on temperature:

$$E_i^{(k)}=E_{i0}^{(k)}\left(1+\xi_i^{(k)}T^{(k)}\right),\ G^{(k)}=G_0^{(k)(k)}\left(1+\eta^{(k)}T^{(k)}\right),$$

$$\alpha_i^{(k)}=\alpha_{i0}^{(k)}\left(1+\gamma_{1i}^{(k)}T^{(k)}+\gamma_{2i}^{(k)}T^{(k)2}\right),\ i=1, 2, 3,$$

where $\xi_i^{(k)},\eta^{(k)},\gamma_{1i}^{(k)},\gamma_{2i}^{(k)}$ – experimentally determined constants, which characterize the dependence of elastic modules and coefficients of thermal expansion on temperature.

The Poisson's ratio for the thermosensitive materials, which are under consideration, is almost independent of temperature, so we consider it constant. The components of the stress tensor arising from the temperature are determined by the Duhamel-Neumann law.

Physical and mechanical parameters are taken as follows: $E_{10}=E_{30}=1,704\cdot10^{10}$ Pa; $E_{20}=2,808\cdot10^{10}$ Pa; $\mu_0=\dfrac{E_{10}}{50}$; $\eta=\xi_1$; $\nu_{12}=0,106$; $\nu_{21}=\nu_{13}=0,174$; $\nu_{23}=0,064$; $\xi_1=\xi_2=\xi_3=-0,25\cdot10^{-2}$; $\gamma_{2i}=0\ (i=1,2,3)$; $\gamma_{11}=-0,2413\cdot10^{-7}\ \text{K}^{-1}$; $\gamma_{13}=0$; $\gamma_{12}=-0,2445\cdot10^{-7}\ \text{K}^{-1}$; $\alpha_{10}=0,1134\cdot10^{-4}\ \text{K}^{-1}$; $\alpha_{20}=0,1418\cdot10^{-4}\ \text{K}^{-1}$; $\alpha_{30}=0$; $l=0,2$ m; $2h=0,04$ m; $R$=0,4m.

The constructed system of equations in displacements with variable coefficients is solved using the proposed approach. The obtained results satisfactorily coincide with the results obtained in analytical and approximate solutions using shell theory by the authors [15].

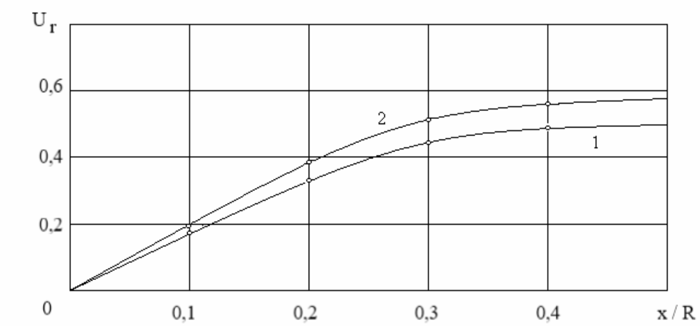


Fig. 2. Distribution of radial displacements

Fig. 2 shows graphs of radial displacements, obtained using the MIRELA+ complex (curve 1) and the authors [15].

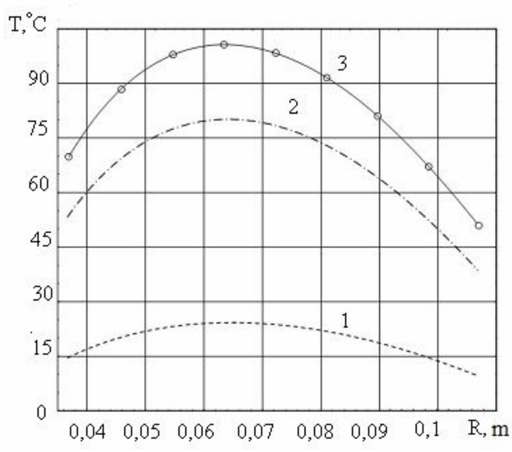


Fig. 3. Temperature distribution in the cross-section $z = h/2$
Preliminary deformation: 1) $\Delta = 0$; 2) $\Delta = 0,01$m; 3) $\Delta = 0,02$m

*Problem 2.* Dissipative heating of a hollow cylindrical shock absorber under preload conditions. The sizes of the shock-absorber: $R_1 = 0,035$ m, $R_2 = 0,1$ m, $h = 0,175$ m. Elastic characteristics of rubber 2959: equilibrium shear modulus $\mu = 0,74$ MPa, instantaneous shift module $\mu_0 = 1,76$ MPa, $\nu = 0,499$; rheological parameters of Rabotnov's relaxation nucleus $\alpha = -0,6$; $\beta = 1,062$; $\chi = 0,64$. Amplitude of axial oscillations $\delta = 0,008$ m, frequency $\omega = 40$ s$^{-1}$.

Thermal conductivity coefficient $\lambda = 0{,}293$ W/(m·K); the heat transfer coefficients with the metal fittings and the environment are respectively equals $H_1 = 5240$ m$^{-1}$, $H_2 = 40$ m$^{-1}$.

In fig. 3 shows graphs of the distribution of the steady-state temperature of self-heating with preliminary deformation (2, 3) and without pre-compression. Analysis of the results shows that the increase in the initial deformation significantly affects the temperature of dissipative heating.

**Conclusions.** An algorithm for solving the related problems of thermoelasticity of elastomeric elements is built on the basis of the moment scheme of finite elements. Incremental theory is used to model the processes of deformation of structures with initial stresses.

The analysis of the obtained results shows that the proposed approach allows to obtain satisfactory calculation results.

Considering the action of prestresses, as well as the dependence of the physical and mechanical properties of the material makes significant adjustments to the values of the calculated values.

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*Стаття надійшла до редакції 05.06.2020*

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**ТЕРМОПРУЖНІСТЬ ЕЛАСТОМЕРНИХ КОНСТРУКЦІЙ З ПОЧАТКОВИМИ НАПРУЖЕННМИ**

У статті представлено алгоритм вирішення зв'язаних задач термопружності еластомерних елементів конструкцій на основі моментної схеми скінченних елементів. Для моделювання процесів термопружного деформування конструкцій з початковими напруженнями використовується інкрементальна теорія деформованого твердого тіла. На кожному кроці деформування виконується коригування матриці жорсткості за допомогою інкрементальної геометричної матриці жорсткості. Використання потрійної апроксимації переміщень, деформацій та функції змінення об'єму дозволяє врахувати слабку стисливість еластомерів. Компоненти тензора напружень обраховуються за законом Дюамеля-Неймана. Для розв'язання задачі теплопровідності побудовано матрицю теплопровідності з урахуванням граничних умов на поверхні скінченного елемента. Для розв'язання задачі термопружності використано алгоритм послідовних наближень. На кожному етапі розв'язку обраховуються характеристики термонапруженого стану. На основі отриманих компонентів тензорів напружень та деформацій обраховується інтенсивність джерел внутрішнього теплоутворення як осереднена за цикл навантаження розсіяна енергія. Для обчислення дисипативних характеристик в'язкопружного еластомера використовуються параметри ядра релаксації Работнова. Розв'язання задачі теплопровідності з урахуванням функції внутрішніх джерел тепла дозволяє уточнити температуру нагрівання тіла. На кожному циклі алгоритму проводиться уточнення значень фізико-механічних характеристик термочутливого матеріалу. Наведений підхід до розв'язання задач термопружності реалізовано в обчислювальному комплексі «МІРЕЛА+». На основі розглянутого підходу отримані розв'язки низки задач. Отримані результати задовільно збігаються з розв'язками інших авторів. Врахування дії попереднього навантаження та залежності фізико-механічних властивостей матеріалу від температури приводить до суттєвих коректив розрахункових величин.

**Ключові слова:** метод скінченних елементів, еластомер, термопружність, дисипативний розігрів, початкові напруження.

УДК 539.3
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Іл. 3. Бібліогр. 15 назв.

UDC 539.3
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UDC 539.3

# THE PARAMETRIC OSCILLATIONS OF ROTATING RODS UNDER ACTION OF THE AXIAL BEAT LOAD

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DOI: 10.32347/2410-2547.2020.104.309-320

The paper presents the results of investigation of the axial beat loads' influence on the transverse rotating rods' oscillations and their stability. The perforator's long drills are considered as objects of investigation.

The analysis of different author's papers that are studded the dynamics of oscillations of shafts and rotating rods is carried out. The relevance of the research topic is substantiated. The model of the considered dynamic system is described and equations of oscillations in space are given.

The technique for investigation is presented. This technique is based on search for new bend forms of rotating rod by solving the equations of oscillations with using the Hubbolt time integration method and the polynomial functions (splines) that are described the current bend form. In it, the spline functions are found by current bend form approximation where each of the found functions is responsible to certain point of rod elastic line and describes the position of nearby points.

Described technique was realized in a computer program with graphic user interface that is developed by author. Program allows to monitor for dynamics of the oscillatory motion of the modeled system in real-time by calculating and drawing the current band forms of the rotating rod during the oscillation.

Diagrams with regions of stable and instable motion of the rods, that were found by different parameters and boundary conditions are shown. The analysis of the results is obtained and the conclusion about possibility of operating the equipment in certain frequency ranges is done. The space oscillating process of rotating rods is considered with account of the gyroscopic loads and geometric nonlinearity.

**Keywords**: numerical differentiation, complex bend forms, spline, geometric nonlinearity, axial loads, hammer drills.

**Introduction**. The tasks of stress-deformed state and oscillations of elastic rotating rods, shafts and rotors have actuality while structural elements of machines and devices are designed. The rotating rods, shafts and rotors are responsible elements in the constructions of engines, turbines, wind and hydropower plants, drill strings and other machines. For these objects the cause of the development of oscillations can be both inertial loads and periodic external loads, such as periodic axial loads.

For example, during the operation of drill string the influence of bottom hole reaction can be periodical as a result of its transverse oscillations, at which the axial moving of its movable end occurs. During the operation of industrial hammer drill the action on the drill is periodic too. During the movement of vessels, the periodic influence on the shaft from propeller can happen, when the vessel passes through turbulent zones. Also in shafts, the periodic influence can be brought from oscillation of the adjacent section, which is transmitted through the coupling due to axial movements.

In recent years, the dynamic tasks of oscillations of shafts and rotating rods were investigated in works of many authors.

The dynamic behavior of drill strings in super-deep wells was considered in papers [4, 5]. The column is modeled by vertical rod, taking into account the longitudinal loads and torque at its lower end. The critical rotational speeds were calculated with various values of external loads that are taken into consideration. The modes of natural oscillations and buckling of the drill string were found. The task is viewed in space taking into account the centrifugal and coriolis inertial loads, also by axial loads with constant values.

The task of rotating shaft with influence of axial loads to the propagation characteristics of the elastic waves is studied in paper [14]. The shaft is viewed with non-uniform cross-sections per length. Axial loads considered with constant values.

The oscillations of shafts and rods under the action of periodic loads were considered in different papers. The paper [10] presents the study of problems with elastic stabilization and long-term strength of the system under cyclically changing external impacts that are appearing because of eccentricities. Task is considered taking into account gyroscopic loads, in linear statement. The paper [9] presents the results of study of space bending oscillations of horizontal rod that is rotating around its axis. Rod is under the action of periodic harmonic force of self-weight per length. The task is considered taking into account gyroscopic loads, too.

Questions about the transverse oscillations of the rods under the action of axial periodic loads, also the tasks of longitudinal-transverse oscillations under the action of beat loads are considered in papers [7, 8]. But in them the investigated rods don't rotate.

In nonlinear statement the dynamics of the drill operation is considered in paper [13], taking into account the axial periodic impact force, but with the aim of studying the vibrations that occur in the coupling.

As result of review we can see, that investigating the dynamics of considered objects, the parametric oscillations of rotating rods under the action of periodic axial beat loads have an opened interest. This is due to actuality of the tasks of vibratory drilling of deep holes.

This paper presents the results of investigating the dynamic behavior of perforators' long drills under action of an axial beat load. It study has interest for long flexible drills.

**Problem statement**. In the process of oscillation of such rotating rods with various lengths, under the action of external periodic forces, the various bending forms that change in time are possible. Beside this, the various character of the oscillatory motion itself for various physical, geometric and dynamic parameters are possible too.

As a dynamic model is considered a rod with length $l$ (Fig. 1) that is under the action of periodic axial load $P(t)$. The rod rotates with an angular rotational speed $\omega$ around the rectilinear axis $O_1X_1$ of the stationary coordinate system $O_1X_1Y_1Z_1$. The rotating coordinate system $OXYZ$ is tied to the rod and rotates with it. The direction of $OX$ axis coincides with direction of $O_1X_1$ axis. Axis of rod in deformed state is coinciding with the $OX$ and $O_1X_1$ axis. The oscillatory motion of the rod in the $OXYZ$ coordinate system is characterized by $y(x,t)$ and $z(x,t)$ displacements of the points, that belong to the axis of rod, in the $OY$ and $OZ$ coordinate axes' direction, respectively.

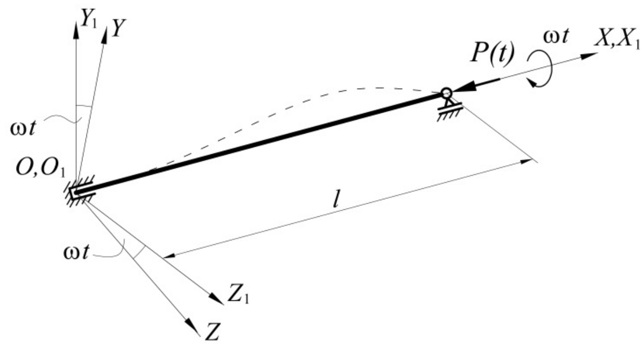


Fig. 1. Dynamic model of system.

In this statement, the oscillations of such rotating objects in space are described by the corresponding system of differential equations [11], which taking into account the geometric nonlinearity and the axial periodic force [3] have a form:

$$\begin{cases} \dfrac{d^2}{dx^2}\left(\dfrac{EI_{1(x)}}{\rho_1}\right) - \overline{m}r^2\left(\dfrac{d^4 y}{dt^2dx^2} + \omega^2\dfrac{d^2 y}{dx^2}\right) - 2\omega\overline{m}\dfrac{dz}{dt} - \overline{m}\,\omega^2 y + \overline{m}\dfrac{d^2 y}{dt^2} + \\ + P(t)\dfrac{d^2 y}{dx^2} = 0 \\ \dfrac{d^2}{dx^2}\left(\dfrac{EI_{2(x)}}{\rho_2}\right) - \overline{m}r^2\left(\dfrac{d^4 z}{dt^2dx^2} + \omega^2\dfrac{d^2 z}{dx^2}\right) + 2\omega\overline{m}\dfrac{dy}{dt} - \overline{m}\,\omega^2 z + \overline{m}\dfrac{d^2 z}{dt^2} + \\ + P(t)\dfrac{d^2 z}{dx^2} = 0, \end{cases} \quad (1)$$

where $E$ – elastic modulus of rod's material; $I_1$, $I_2$ – inertia moments of rod section in mutually perpendicular planes; $r$ – radius of gyration; $\overline{m}$ –mass of unit per length; $\omega$ – rotational speed of rod around the axis that is coincided with the axis of rod in undeformed state; $P(t)$ – periodic axial force; $1/\rho_1$, $1/\rho_2$ – main curvatures of rod's axis in mutually perpendicular planes in form:

$$\frac{1}{\rho_1}=\frac{\dfrac{d^2y}{dx^2}}{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{3/2}},\quad \frac{1}{\rho_2}=\frac{\dfrac{d^2z}{dx^2}}{\left(1+\left(\dfrac{dz}{dx}\right)^2\right)^{3/2}}.$$

If the rod is under the influence of an axial beat load, this action is modeled by function, that looks like as: $P(t)=P_{\max}\cdot(a^{1+\cos\theta t}-a^2)$, for which the changes of amplitude in time is shown in Figure 2.

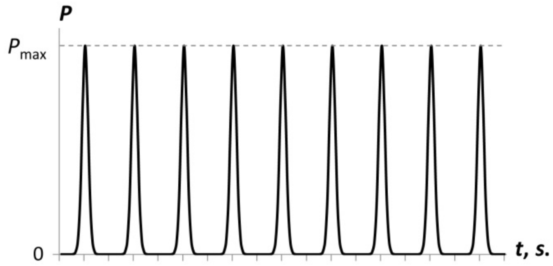

Fig. 2. The diagram of function $P(t)$

**Technique.** For investigation of the dynamics of motion for considered objects in this paper propose to use the technique in which the process of oscillation is modeled based on repeated (cyclic) solving the system of differential equations for every point of system in order to find the new coordinates of positions for these points in each next point of time $t+\Delta t$.

In it, the solving of equations (1) for searching of new bend form for the next point of time is based on the use of polynomial functions (splines) [1] that describe the current bend form and the Houbolt time integration method [12]. The spline functions [6], in turn, are determined by approximating the current bend form, where each of found functions is responsible to certain point of rod elastic line and describes the position of nearby points.

In result of the approximation, the line of the current bend is described by the array of $n$ polynomial functions $f_n(x)=a_{n0}+a_{n1}x+a_{n2}x^2+a_{n3}x^3+a_{n4}x^4$, each of which corresponds to certain point of rod elastic line and tied with nearby points (Fig. 3).

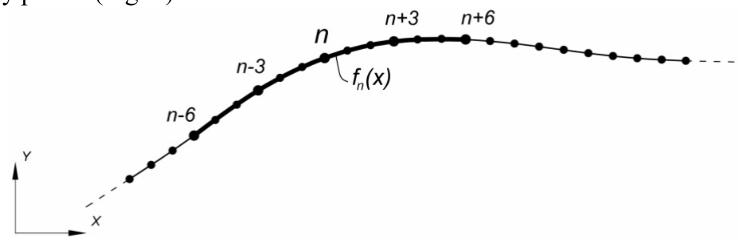

Fig. 3. Bend form approximation for point n to function $f_n(x)$

The search process of each functions $f_n(x)$, namely coefficients $a_{n0}$, $a_{n1}$, $a_{n2}$, $a_{n3}$, $a_{n4}$, can be executed by the coordinates of points $n$–2, $n$–1, $n$+1, $n$+2, that lie near point $n$. However, it is necessary that the functions of all derivatives of spline have been continuous and smooth lines. Therefore, these points are considered as intermediate and used to control of continuity and smoothness. The approximation executes by coordinates of points $n$–6, $n$–3, $n$+3, $n$+6 and current $n$, which are considered like as characteristic.

The calculation of the coefficients $a_{n0}$, $a_{n1}$, $a_{n2}$, $a_{n3}$, $a_{n4}$, for each function, executes by solving the system of five equations (2) using the coordinate values of each of five points that belong to considered part of rod elastic line

$$\begin{cases} a_{n0}+a_{n1}x_{n-6}^{1}+a_{n2}x_{n-6}^{2}+a_{n3}x_{n-6}^{3}+a_{n4}x_{n-6}^{4}=y_{n-6} \\ a_{n0}+a_{n1}x_{n-3}^{1}+a_{n2}x_{n-3}^{2}+a_{n3}x_{n-3}^{3}+a_{n4}x_{n-3}^{4}=y_{n-3} \\ a_{n0}+a_{n1}x_{n}^{1}+a_{n2}x_{n}^{2}+a_{n3}x_{n}^{3}+a_{n4}x_{n}^{4}=y_{n} \\ a_{n0}+a_{n1}x_{n+3}^{1}+a_{n2}x_{n+3}^{2}+a_{n3}x_{n+3}^{3}+a_{n4}x_{n+3}^{4}=y_{n+3} \\ a_{n0}+a_{n1}x_{n+6}^{1}+a_{n2}x_{n+6}^{2}+a_{n3}x_{n+6}^{3}+a_{n4}x_{n+6}^{4}=y_{n+6} \end{cases} \quad (2)$$

Found functions $f_n(x)$ are differentiated and found derivatives are used to solve the system of differential equations for each point of rod elastic line separately for searching of next bend form for next point of time $t+\Delta t$.

**Boundary conditions.** Using the considered technique the boundary conditions at the ends of the rod are modeled by schemes based on imaginary prolongation of the elastic line of the rod. Let's consider few basic boundary conditions.

The hinged boundary at the end of the rod is modeled by analogic scheme, but for the end point. namely, for searching the function for point $m$, use the value $y_m$=0, and assumed that for $x_{m+1}$= $(2x_m–x_{m-1})$, $x_{m+2}$= $(2x_m–x_{m-2})$, the values $y_{m+1}$= $-y_{m-1}$, $y_{m+2}$= $-y_{m-2}$, respectively. the points $m$+1, $m$+2 belong to the line of the imaginary continuation of the rod elastic line and their coordinates are determined by corresponding relations using the coordinate's values in points $m$–1 and $m$–2. such relations give that every time after approximation we get a function whose value in point $m$ will be $y_m$=0, and the value of its second derivative will be $y_m''$=0.

The pinched boundary at the beginning of the rod is modeled by scheme, where, for point 0, use the values $x_0$=0, $y_0$=0, and assumed that for $x_{-1}$= $-x_1$, $x_{-2}$= $-x_2$, the values $y_{-1}$= $y_1$, $y_{-2}$= $y_2$, respectively. such relation gives that every time after approximation we get a function whose value in point 0 will be $y_0$=0, and the value of its second derivative will be $y_0''\neq 0$.

The pinched boundaries at the end of the rod are modeled by scheme, where, for point $m$, use the value $y_m$=0, and assumed that for $x_{m+1}$= $(2x_m–x_{m-1})$, $x_{m+2}$= $(2x_m–x_{m-2})$, the values $y_{m+1}$= $y_{m-1}$, $y_{m+2}$= $y_{m-2}$, respectively. such relations give that every time after approximation we get a function whose value in point $m$ will be $y_m$=0, and the value of its second derivative will be $y_m''\neq0$.

For beginning of the oscillatory motion, it is necessary that the system be out of equilibrium, in which the rod will take the initial bend form. Such action can be caused by the action of random instant load.

During the operation of hammer drills, one of such instant loads can be the bending moment that occurs at the end of the drill due to the uneven strength of particles of concrete that are crashed by beating.

In this representation, the action of the bending moment is modeled by one-time instant load at the end of the rod and the required initial bend form is determined by the method of initial parameters.

The initial bend form is found for the time $t$=0, like a point of start of oscillation, believing, that the action of inertial loads is absent, since before the start of oscillations the axis of the rod in undeformed state passes through the axis of rotation.

So, in this way, the analysis of the geometric position of rotating rod in space, the approximation of bend form (elastic line) and its differentiation is a first component of solving the equations of oscillations and performs in conditionally fixed moment in time.

The oscillatory motion, especially during rotation, is a dynamic process in which at each next moment in time not only the geometric positions of all points of the system have changes, but such parameters as rotation angles, speeds and accelerations have changes too. Therefore, the second component of solving the equations is the solve of task in time, that performs using the numerical Houbolt integration method, in which makes the search of new bend form for each next point of time $t+\Delta t$, based on current bend form and its derivatives.

**Realization note**. The considered technique is the basis component of the algorithm for the numerical solving of the differential equations of oscillations by rotation of rod systems in space and time. The algorithm realized by computer program with graphic user interface that helps to monitor for the dynamics of oscillating process of modeled system in real time. Besides this, the program gives the capacity to make the analysis of behavior of modeled system, find the dynamic instability fields and draw the diagrams of found fields. Moreover, the program draws the graphics of oscillations and changes of angular speeds and accelerations.

**Results**. In this paper shown the results of investigating the dynamic behavior of perforators' long drills under action of an axial beat load that were gotten by described technique and program. Among them: long drills with length 800…1000 mm and diameter 10…22 mm, boers with length 900…1000 mm and diameter of rod 18…22 mm, extensions for drills with length 750…1100 mm and diameter of rod 18 mm.

The priority amounts of industrial hammer drills with an SDS Max type chuck operate on rotational speeds in range of $\omega$ = 25…75 $s^{-1}$ and beat frequencies in range of $\theta$ = 170…380 $s^{-1}$. Household hammer drills with an SDS

Plus type chuck operate on rotational speeds in range of $\omega$ = 75…250 s$^{-1}$ and beat frequencies in range of $\theta$ = 380…580 s$^{-1}$.

During rotation with different speeds under action of an external periodic force with different frequencies the oscillations' amplitude can be constant (with critical parameters), fading out or growing up. This characterizes the stable or instable behavior of an object in such dynamic process.

For example, for steel rod with diameter $d$ = 12 mm and length $l$ = 1 m during rotation with critical speed of the 1-st harmonic that is equal 145.5 s$^{-1}$, but without action of external forces, the transverse oscillations in the rotating coordinate system do not occur (because rod is in equilibrium of inertial and elastic forces). In stationary coordinate system, the oscillations occur with constant amplitude as the projection on the coordinate planes. Herewith, under the action of periodic axial beat load with frequency that is equal to the value of critical rotational speed, the oscillations occur with growing amplitude, which in the projection on the coordinate axes have the form shown in Fig. 4.

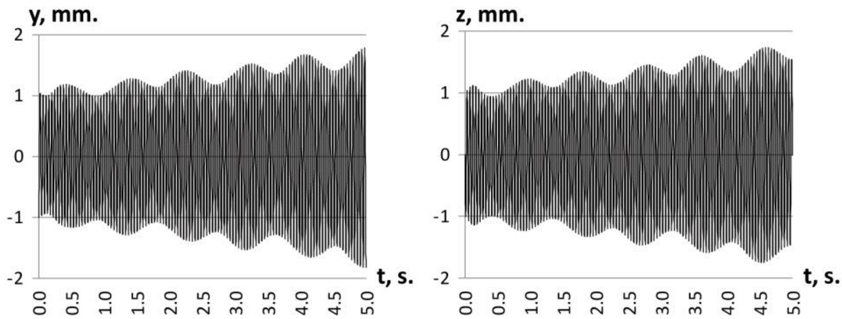


Fig. 4. Oscillations of steel rod that is rotated with speed equal 145.5 s$^{-1}$, under action of the axial beat load with frequency equal 145.5 s$^{-1}$.

Therefore, such oscillatory motion of rotating rod is instable and since over the time it will have been destroyed because of constant amplitude growth.

As results of study of mentioned objects in Figures 5-9 the fields of stable and instable oscillations by rotation are presented. These fields show instable regions in depends of rotational speeds $\omega$ and beat frequencies $\theta$, which were found for the reviewed objects by various geometric, physical parameters and boundaries. Figures 5-7 show the results for rods with hinged boundaries at both ends. Figures 8 and 9 show the results in case when one end of rod is pinched other is hinged.

The fields of instable oscillations are displayed filled gray. White colored regions are the fields of stable oscillations. The rectangles with dashed borders show the operating frequency ranges for priority amounts of industrial hammer drills with SDS Max type chuck and household hammer drills with SDS Plus type chuck. Numerically these ranges are presented upper in this part.

$P(t) = 0 \div 0.6$ kN $P(t) = 0 \div 0.4$ kN

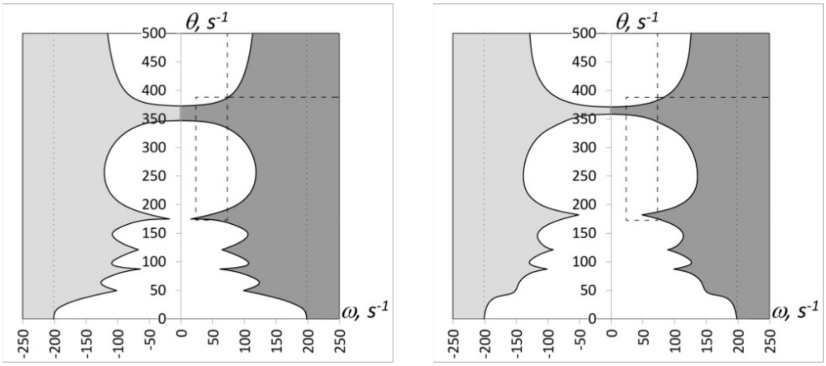


Fig. 5. Dynamic instability fields of rods with diameter $d$ = 10 mm,
length $l$ = 0.8 m, under action of axial beat load $P(t)$, with hinged boundaries

$P(t) = 0 \div 0.6$ kN $P(t) = 0 \div 0.4$ kN

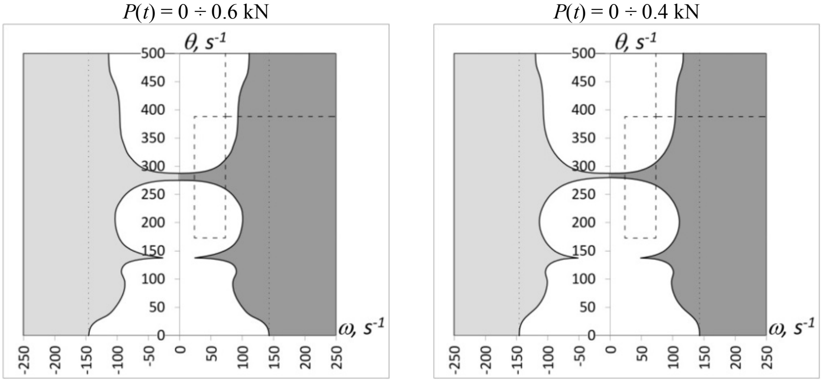


Fig. 6. Dynamic instability fields of rods with diameter $d$ = 12 mm,
length $l$ = 1 m, under action of axial beat load $P(t)$, with hinged boundaries

$P(t) = 0 \div 0.6$ kN $P(t) = 0 \div 0.4$ kN

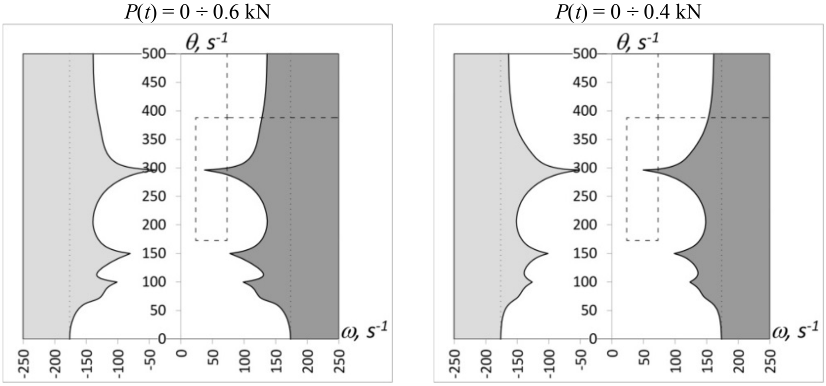


Fig. 7. Dynamic instability fields of rods with diameter $d$ = 18 mm,
length $l$ = 1.2 m, under action of axial beat load $P(t)$, with hinged boundaries

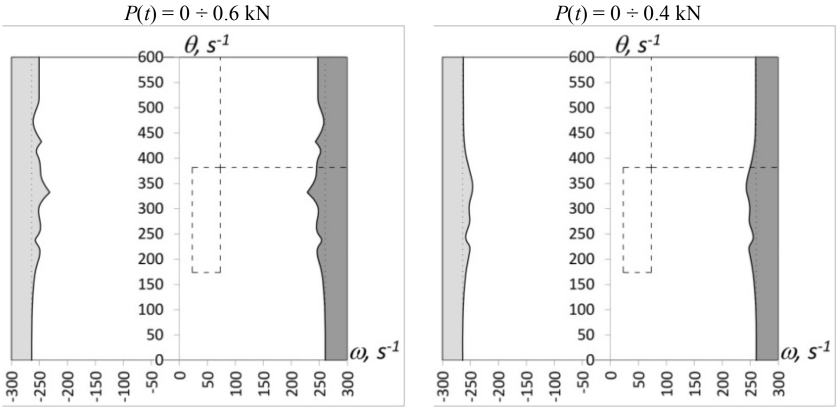


Fig. 8. Dynamic instability fields of rods with diameter $d$ = 10 mm,
length $l$ = 0.8 m, under action of axial beat load $P(t)$, with pinched and hinged boundaries

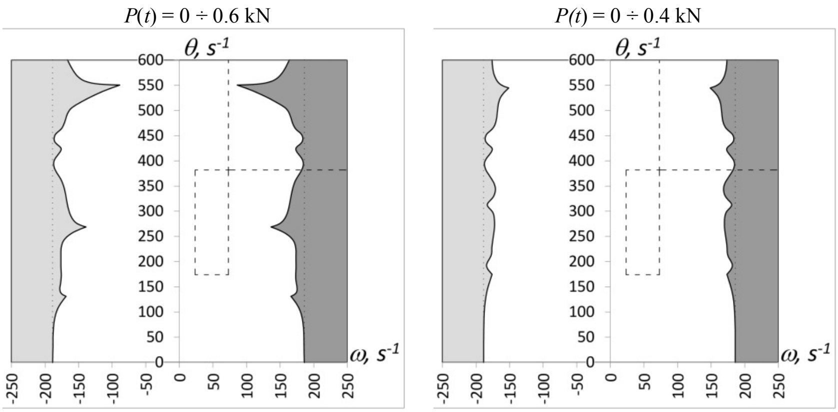


Fig. 9. Dynamic instability fields of rods with diameter $d$ = 12 mm,
length $l$ = 1 m, under action of axial beat load $P(t)$, with pinched and hinged boundaries

As we can see from diagrams, for various steel rods with their parameters and different boundary conditions there are frequency ranges at which the drills of perforator in use will start instable oscillations and over the time they can be destroyed.

So, for example, for industrial hummer drills, such regions are observed for rods with diameter $d$=10 mm and length $l$=0.8 m (Fig. 5), with diameter $d$=12 mm and length $l$=1 m (Fig. 6), with diameter $d$=18 mm and length $l$=1.2 m (Fig. 7), with hinged boundary conditions. And in fact there are in rather narrow ranges.

For household hammer drills that are using in higher frequency ranges, as we can see from the diagrams, the instable regions are much wider and observed both: under hinged boundary conditions at the ends and with boundary conditions where one end is pinched other is conditionally hinged.

**Conclusion**. The considered results of investigation of the axial beat loads' influence on the stability of rotating rods in space, studding perforator's long drills as example, show, that for certain ratios of rotation and beat frequencies there are regions of instable oscillatory motion in which running the equipment can inevitably lead to its destruction, that can turn to undesirable injuries or tragic consequences either at work or at home.

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*Стаття надійшла до редакції 05.06.2020*

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**ПАРАМЕТРИЧНІ КОЛИВАННЯ СТЕРЖНІВ, ЩО ОБЕРТАЮТЬСЯ ПІД ДІЄЮ ПОЗДОВЖНЬОГО УДАРНОГО НАВАНТАЖЕННЯ**

В роботі наведені результати дослідження впливу поздовжніх ударних навантажень на характер поперечних коливань стержнів, що обертаються та їх стійкість. В якості об’єктів дослідження обрані довгомірні робочі органи перфораторів, що мають значну гнучкість.

Здійснено аналіз публікацій різних авторів, які займаються дослідженням динаміки коливань валів та стержнів, що обертаються та обґрунтована актуальність обраної тематики дослідження. Описана модель динамічної системи, що розглядається, наведені рівняння коливального руху у просторі.

Представлена методика дослідження, яка будується на пошуку нових форм вигину стержнів при обертанні, через розв’язання рівнянь коливального руху з використанням поліноміальних функцій (сплайнів), що описують форму вигину, та методі інтегрування за часом Хубболта. В цій методиці сплайн-функції отримуються апроксимацією поточної форми вигину, де кожна з найдених функцій відповідає за певну точку пружної лінії стержня та описує положення сусідніх точок.

Наведені діаграми, що відображають області стійкого та нестійкого руху стержнів при різних параметрах та граничних умовах. Здійснено аналіз отриманих результатів та висновок про можливість експлуатації обладнання у певних діапазонах частот. Процес коливального руху розглянуто у просторі з урахуванням геометричної нелінійності стержня та гіроскопічних навантажень.

**Ключові слова**: чисельне диференціювання, складні форми вигину, сплайн, геометрична нелінійність, поздовжні навантаження, ударні навантаження, перфоратори.

*Недин В.О.*

**ПАРАМЕТРИЧЕСКИЕ КОЛЕБАНИЯ ВРАЩАЮЩИХСЯ СТЕРЖНЕЙ ПОД ДЕЙСТВИЕМ ПРОДОЛЬНОЙ УДАРНОЙ НАГРУЗКИ**

Рассматриваются некоторые результаты исследования влияния продольных ударных нагрузок на характер поперечных колебаний вращающихся стержней и их устойчивость. В качестве объектов исследования были выбраны длинномерные рабочие органы перфораторов со значительной гибкостью.

В работе осуществлён анализ публикаций разных авторов, занимающихся исследованием динамики колебаний валов и стержней, вращающиеся и обоснована актуальность тематики исследования. Описана модель рассматриваемой динамической системы, приведены уравнения колебательного движения в пространстве.

Представлена методика исследования, которая строится на поиске новых форм изгиба вращающихся стержней решением уравнений колебательного движения с использованием полиномиальных функций (сплайнов), описывающих текущую форму изгиба, и методе интегрирования по времени Хубболта. В этой методике сплайн-функции находятся аппроксимацией текущей формы изгиба, где каждая из найденных функций отвечает за определенную точку упругой линии стержня и описывает положение соседних точек.

Приведены диаграммы, отражающие области устойчивого и неустойчивого движения стержней при различных параметрах и граничных условиях. Осуществлен анализ полученных результатов и сделан вывод о возможности эксплуатации оборудования в определенных диапазонах частот. Процесс колебательного движения рассмотрен в пространстве с учетом гироскопических инерционных нагрузок на вращающийся стержень, а также с учётом геометрической нелинейности.

**Ключевые слова:** численное дифференцирование, сложные формы изгиба, сплайн, геометрическая нелинейность, продольные нагрузки, ударные нагрузки, перфораторы.

UDC 539.3
*Nedin V.O.* **The parametric oscillations of rotating rods under action of the axial beat load** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2020. – Issue 104. – P. 309-320.
*The paper presents the results of investigation of the axial beat loads' influence on the transverse rotating rods' oscillations and their stability.*
Tabl. 0. Fig. 8. Ref. 7.

УДК 539.3
*Недін В.О.* **Параметричні коливання стержнів, що обертаються під дією поздовжнього ударного навантаження** // Опір матеріалів і теорія споруд: наук.-тех. збірник. – К.: КНУБА, 2020. – Вип. 104. – С. 309-320.
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Табл. 0. Іл. 8. Бібліогр. 7 назв.

*Недин В.О.* **Параметрические колебания вращающихся стержней под действием продольной ударной нагрузки** // Сопротивление материалов и теория сооружений. – 2020. – Вып. 104. С. 309-320.
*В работе представлены результаты исследования влияния продольных ударных нагрузок на характер поперечных колебаний вращающихся стержней и их устойчивость.*
Табл. 0. Ил. 8. Библиогр. 7 назв.

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ISSN 2410-2547
Опір матеріалів і теорія споруд/Strength of Materials and Theory of Structures. 2020. № 104


UDC 539.3

# BOUNDARY ELEMENT APPROACHES TO THE PROBLEM OF 2-D NON-STATIONARY ELASTIC VIBRATIONS

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DOI: 10.32347/2410-2547.2020.104.321-327

Two boundary element approaches are used to solve the problem on non-stationary vibrations of elastic solids. The first approach is based on the transition to the frequency domain by means of a Fourier series expansion. The second approach is associated with the direct solution of a system of time-dependent boundary integral equations, with a piecewise constant approximation of the dependence of the unknowns on time. In both cases, a collocation scheme is used to algebraize the integral equations, and the difficulties associated with the calculation of singular integrals are overcome by replacing the kernels with the initial segment of the Maclaurin series. After such a replacement, the kernels take the form of a sum, the first term of which is the corresponding fundamental solution of the statics problem while other terms are regular. Since integration of static kernels is not difficult the problem of calculating the diagonal coefficients of the SLAE turns out to be solved. The developed techniques are compared in the process of dynamics analysis solving of elastic media with two cylindrical cavities. The boundary of one of the cavities is subjected to a radial impulse load, which varies according to the parabolic law. Both approaches have shown the similar effectiveness and qualitative consistency.
**Keywords:** time-dependent boundary integral equations, frequency domain, Hankel functions, Maclaurin series, impulse loading.

The Boundary Element Method (BEM) is an effective tool for solving the problem of vibrations of elastic bodies with different cavities and inclusions. In this case, the solution of the problem can be obtained in two ways. In the first case, the Fourier transform is used to convert the problem into a frequency domain, while in the second case, a time-step procedure is used. In both cases, the algorithmic basis of the problem is the boundary analogue of the Somigliana identity for displacements. The identity links known displacements and stresses at the boundary points with those not unspecified by boundary conditions. The resolving equation describing two-dimensional harmonic vibrations of an elastic mass body, in the absence of mass loads, is as follows

$$\frac{1}{2}u_j^*(\vec{x},\omega)=\int_\Gamma \tau_k^*(\vec{y},\omega)U_{jk}(\vec{x},\vec{y},\omega)d\Gamma_y-\int_\Gamma u_k^*(\vec{y},\omega)T_{jk}(\vec{x},\vec{y},\omega)d\Gamma_y,\ \ j,k=1,2\ , \quad (1)$$

where $\omega$ is the circular frequency of vibrations; $\vec{x}\{x_1,x_2\}$, $\vec{y}\{y_1,y_2\}\in\Gamma$; $\Gamma$ – boundary of the area occupied by the body; $u_j(\vec{x},\omega)$, $q_j(\vec{x},\omega)$ are respectively complex amplitudes of displacements and stresses at the boundary; $U_{jk}(\vec{x},\vec{y},\omega)$ - the fundamental solution of the problem [1, 2], which is given by the expression

$$U_{jk}(\vec{x},\vec{y},\omega)=\frac{i}{4\mu}\left\{\delta_{jk}\left[H_0^{(1)}(\varphi_1)-\frac{H_1^{(1)}(\varphi_2)}{\varphi_2}+\alpha\frac{H_1^{(1)}(\varphi_1)}{\varphi_1}\right]+\right.$$
$$\left.+\frac{i}{4\mu}r,_j r,_k\left[H_2^{(1)}(\varphi_2)-\alpha H_2^{(1)}(\varphi_1)\right]\right., \quad (2)$$

$\varphi_j=\frac{\omega r}{C_j}$; $C_2=\sqrt{\frac{\mu}{\rho}}$; $C_1=\sqrt{\frac{\lambda+2\mu}{\rho}}$; $\alpha=\frac{C_2^2}{C_1^2}$; $\rho$ - material density; $\lambda$ and $\mu$ - Lame constants; $r,_j=\frac{\partial r}{\partial y_j}=\frac{y_j-x_j}{r}$; $r^2=(y_1-x_1)^2+(y_2-x_2)^2$; $T_{jk}(\vec{x},\vec{y},\omega)$ - generalized derivative of the fundamental solution (stresses at the sites with normal $n_j(\vec{y})$ that arise in elastic two-dimensional media due to the action in the point $\vec{x}$ of concentrated unit force in the direction $x_k$; $H_m^{(1)}$ is the Hankel function of the first kind.

In accordance with the collocation procedure of BEM the boundary is represented as a set of elements and a hypothesis defining how the unknown quantities change within each BE (i.e. basic function) is introduced. Then a set of poles $\vec{x}_l$ is assigned, agreed with the introduced hypothesis, and then equations (1) are written in each of the poles in turn. The result of this procedure is a system of linear algebraic equations with respect to unknown values of amplitudes of displacements and stresses at collocation points $\vec{x}_l$. The system coefficients are integrals over boundary elements of product of kernel $U_{jk}(\vec{x},\vec{y},\omega)$ or $T_{jk}(\vec{x},\vec{y},\omega)$ and the corresponding basic function.

When the distance *r* between the source $\vec{x}$ point and the integration point $\vec{y}$ tends to zero, i.e., if the pole is located at the same BE on which the integration is carried out, the Hankel functions, and with them the sub-integral expressions, take infinitely large values, which makes it impossible to calculate the diagonal system matrix coefficients using numerical integration. To overcome this obstacle, the Hankel functions are approximately replaced by the initial segments of the Maclaurin series. After such a replacement the kernels can be represented as follows [3]

$$U_{jk}(\vec{x},\vec{y},\omega)\approx U_{jk}^s(\vec{x},\vec{y})+U_{jk}^*(\vec{x},\vec{y},\omega);\ T_{jk}(\vec{x},\vec{y},\omega)\approx T_{jk}^s(\vec{x},\vec{y})+T_{jk}^*(\vec{x},\vec{y},\omega), \quad (3)$$

where $U_{jk}^s(\vec{x},\vec{y})$ and $T_{jk}^s(\vec{x},\vec{y})$ are respectively the fundamental solution of the static 2D problem of and its generalized derivative.

Since integration of the kernels of the static problem does not cause difficulties, and regular additives $U_{jk}^*(\vec{x},\vec{y},\omega)$, $T_{jk}^*(\vec{x},\vec{y},\omega)$ can be determined with any degree of accuracy, the problem of algebraization of the system of boundary integral equations (1) can be considered solved in the first approach.

The second approach requires solving a system of time-dependent boundary integral equations, which under zero initial conditions and the absence of mass forces can be written as follows

$$\frac{1}{2}u_j(\vec{x},t) = \int_0^{t^+} \int_\Gamma U_{jk}(\vec{x},\vec{y},t-\tau)q_k(\vec{y},\tau)d\Gamma_y d\tau - \\ -\int_0^{t^+} \int_\Gamma T_{jk}(\vec{x},\vec{y},t-\tau)u_k(\vec{y},\tau)d\Gamma_y d\tau, \quad (4)$$

where $U_{kj}(r,t) = \frac{1}{2\pi\rho}\left[\frac{H(C_1 t - r)}{C_1 r^2} f_{kj}^{(1)}(\vec{x},\vec{y},t) + \frac{H(C_2 t - r)}{C_2 r^2} f_{kj}^{(2)}(\vec{x},\vec{y},t)\right]$, $H(\tau)$ is Heaviside step function; $f_{kj}^{(1)}(r,t) = \left(2d_1 + \frac{r^2}{d_1}\right) r_{,k}\, r_{,j} - \delta_{kj} d_1$, $f_{kj}^{(2)}(r,t) = \left(2d_2 + \frac{r^2}{d_2}\right) r_{,k}\, r_{,j} - \delta_{kj}\left(d_2 + \frac{r^2}{d_2}\right)$, $d_1 = \sqrt{(C_1 t)^2 - r^2}$, $d_2 = \sqrt{(C_2 t)^2 - r^2}$, $T_{jk}(r,\tau)$ is a generalized derivative of the fundamental solution $U_{jk}(r,\tau)$.

The peculiarity of this approach is that time is one of variables, on which the solution depends. Accordingly, it is necessary to divide the time interval into separate intervals and approximate the unknown within each interval. Assuming that the boundary stresses at each step remain constant, time integration of the both terms in the right side of equation (4) can be done analytically [4]. Moreover, the resulting expressions can also be presented in a form similar to that of representation (3), i.e., as a sum of the corresponding static kernel and some regular additive. Thus, even in this case, it is possible to overcome all the fundamental obstacles to the numerical solution of the problem [4].

Comparison of the developed algorithms was carried out using the problem about non-stationary vibrations of elastic space with two circular cylindrical cavities of radius R=3m. Boundary of the left cavity is exposed to radial influence of parabolic impulse, set by the formula $q_R(t) = 4\frac{qt(T-t)}{T^2}$ (Fig. 1). Time of the impulse action is $T = 20R / C_1 = 0.0108\,\text{s}$.

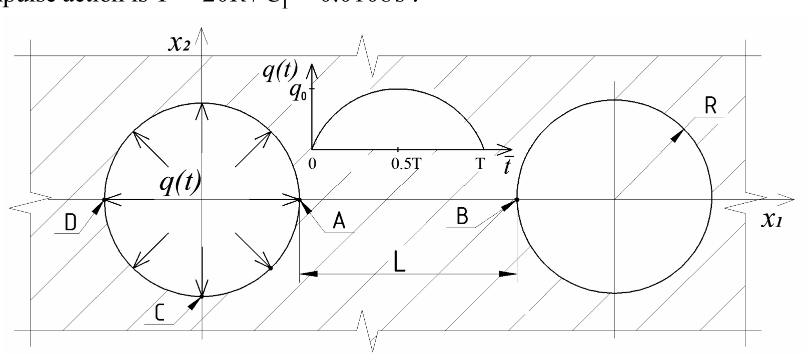


Fig. 1. Calculated area

The results of calculations are shown in Figures 2, 3. Graphs marked with figures 1 and 4 are the curves obtained as a result of the system (4) solution. Figures 2 and 5 denote the graphs built within the first approach using 12 terms of the Fourier series, and figures 3 and 6 denote the graphs, for the construction of which 4 terms of the series were kept. At the same time, the diagrams 1, 2, 3 characterize the stress-strain state parameters at the point *A*, and the diagrams 4, 5, 6 correspond to the same parameters at the point *B* of the boundary (Fig. 1).

Extreme values of normalized parameters of stress-strain state, obtained with the help of two approaches, are contained in Tables 1-4.

The values of radial displacements $u_R$ and tangential stresses $\sigma_S$ were normalized according to the formulas $U_R^n = u_R \dfrac{\mu}{qR}$ and $\sigma_s^n = \dfrac{\sigma_s}{q}$ respectively. The values $U_R^{n,t}$, $\sigma_s^{n,t}$ are obtained by solving the system of time-dependent boundary integral equations (2nd approach) and $U_R^{n,\omega}(m)$, $\sigma_s^{n,\omega}(m)$ are obtained using transition to the frequency domain (1st approach) while value *m* corresponds to the number sine waves.

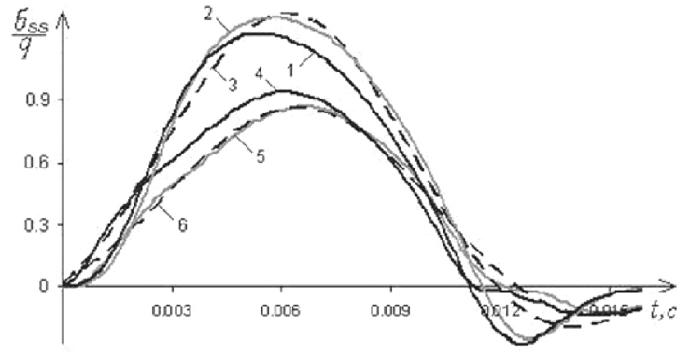


Fig. 2. Radial displacements in points *A* and *B* of the boundary when *L*=3m

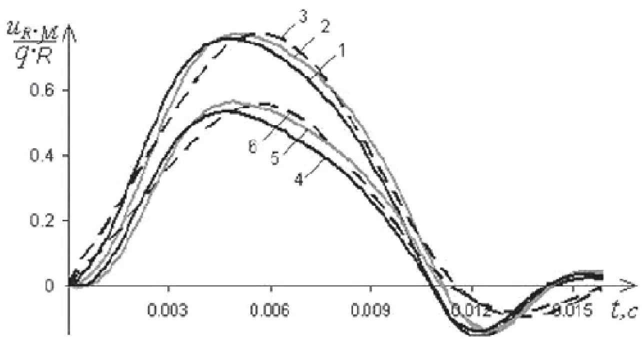


Fig. 3. Tangential stresses in points *A* and *B* of the boundary when *L*=3m

Table 1

Normalized radial displacements at the points of the boundary

| Point | $U^{n,t}_{R,\max}$ | $U^{n,w}_{R,\max}$ (4) | $U^{n,w}_{R,\max}$ (12) | $U^{n,t}_{R,\min}$ | $U^{n,w}_{R,\min}$ (4) | $U^{n,w}_{R,\min}$ (12) |
|---|---|---|---|---|---|---|
| *A* | 0.764 | 0.777 | 0.775 | -0.155 | -0.096 | -0.144 |
| *B* | 0.502 | 0.521 | 0.502 | -0.043 | -0.055 | -0.036 |
| *C* | 0.526 | 0.528 | 0.524 | -0.080 | -0.057 | -0.071 |
| *D* | 0.536 | 0.562 | 0.567 | -0.134 | -0.078 | -0.135 |

Table 2

Normalized tangential stresses at the border points. *L*=3m

| Point | $\sigma^{n,t}_{s,\max}$ | $\sigma^{n,w}_{s,\max}$ (4) | $\sigma^{n,w}_{s,\max}$ (12) | $\sigma^{n,t}_{s,\min}$ | $\sigma^{n,w}_{s,\min}$ (4) | $\sigma^{n,w}_{s,\min}$ (12) |
|---|---|---|---|---|---|---|
| *A* | 0.929 | 0.855 | 0.806 | -0.144 | -0.138 | -0.131 |
| *B* | 1.288 | 1.248 | 1.250 | -0.262 | -0.168 | -0.249 |
| *C* | 1.046 | 1.065 | 0.993 | -0.130 | -0.118 | -0.114 |
| *D* | 1.202 | 1.302 | 1.283 | -0.273 | -0.199 | -0.255 |

Table 3

Normalized radial displacements at border points. *L*=6m

| Point | $U^{n,t}_{R,\max}$ | $U^{n,w}_{R,\max}$ (4) | $U^{n,w}_{R,\max}$ (12) | $U^{n,t}_{R,\min}$ | $U^{n,w}_{R,\min}$ (4) | $U^{n,w}_{R,\min}$ (12) |
|---|---|---|---|---|---|---|
| *A* | 0.619 | 0.625 | 0.625 | -0.105 | -0.085 | -0.102 |
| *B* | 0.511 | 0.527 | 0.509 | -0.053 | -0.048 | -0.045 |
| *C* | 0.531 | 0.527 | 0.530 | -0.077 | -0.059 | -0.070 |
| *D* | 0.312 | 0.323 | 0.325 | -0.104 | -0.063 | -0.098 |

Table 4

Normalized tangential stresses at the border points. *L*=6m

| Point | $\sigma^{n,t}_{s,\max}$ | $\sigma^{n,w}_{s,\max}$ (4) | $\sigma^{n,w}_{s,\max}$ (12) | $\sigma^{n,t}_{s,\min}$ | $\sigma^{n,w}_{s,\min}$ (4) | $\sigma^{n,w}_{s,\min}$ (12) |
|---|---|---|---|---|---|---|
| *A* | 1.054 | 1.030 | 0.992 | -0.120 | -0.116 | -0.970 |
| *B* | 1.234 | 1.183 | 1.182 | -0.225 | -0.169 | -0.211 |
| *C* | 1.122 | 1.076 | 1.060 | -0.147 | -0.106 | -0.129 |
| *D* | 0.550 | 0.576 | 0.583 | -0.151 | -0.119 | -0.144 |

The data presented in the figures and tables show satisfactory qualitative consistency of the data obtained using different approaches. It should be noted that as the number of members of the Fourier series increases, the diagrams obtained in the first approach become more and more similar to the curves using the solution of the time-dependent boundary integral equations (4), i.e., using relations of the second approach.

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*Стаття надійшла до редакції 05.06.2020*

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**ГРАНИЧНОЕЛЕМЕНТНІ ПІДХОДИ ДО ЗАДАЧІ ПРО НЕСТАЦІОНАРНІ ПРУЖНІ КОЛИВАННЯ У ДВОВИМІРНІЙ ПОСТАНОВЦІ**

Для аналізу нестаціонарних коливань пружних масивів використовуються два граничноелементні підходи. Перший пов'язаний з переходом в частотну область, а другий реалізує процедуру інтегрування за часом. Проведено порівняння методів при вирішенні задачі про імпульсне навантаження пружного середовища з двома циліндричними порожнинами.

**Ключові слова**: гранично-часові інтегральні рівняння, частотна область, функції Ганкеля, ряд Маклорена, імпульсне навантаження.

*Ворона Ю.В., Козак А.А.*

**ГРАНИЧНОЭЛЕМЕНТНЫЕ ПОДХОДЫ К ЗАДАЧЕ ПРО НЕСТАЦИОНАРНЫЕ УПРУГИЕ КОЛЕБАНИЯ В ДВУМЕРНОЙ ПОСТАНОВКЕ**

Для анализа нестационарных колебаний упругих массивов используются два граничноэлементных подхода. Первый из них связан с переходом в частотную область, а второй реализует процедуру интегрирования по времени. Проведено сравнение методов при решении задачи об импульсном нагружении упругой среды с двумя цилиндрическими полостями.

**Ключевые слова**: гранично-временные интегральные уравнения, частотная область, функции Ганкеля, ряд Маклорена, импульсное нагружение.

УДК 539.
*Ворона Ю.В., Козак А.А.* **Граничноелементні підходи до задачі про нестаціонарні пружні коливання у двовимірній постановці** // Опір матеріалів і теорія споруд: наук.-техн. збірник. – К.: КНУБА, 2020. – Вип. 104. – С. 321-327.

*В процесі дослідження нестаціонарних коливань пружного середовища з двома циліндричними порожнинами під дією імпульсного навантаження порівнюються між собою два граничноелементні підходи.*

Таб. 4. Рис. 1. Бібліогр. 4 назв.

UDC 539.3
*Vorona Yu.V., Kozak A.A.* **Boundary element approaches to the problem of 2-D non-stationary elastic vibrations** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 104. – P. 321-327.
*Two boundary element approaches are compared in the process of analysis of elastic media with two cylindrical cavities response to impulse loading.*
Tab. 4. Fig. 1. Ref. 4 items.

УДК 539.
*Ворона Ю.В., Козак А.А.* **Граничноэлементные подходи к задаче про нестационарные упругие колебания в двумерной постановке** // Опір матеріалів і теорія споруд: наук.-техн. збірник. – К.: КНУБА, 2020. – Вип. 104. – С. 321-327.
*При исследовании нестационарных колебаний упругой среды с двумя цилиндрическими полостями под действием импульсной нагрузки сравниваются между собой два граничноэлементных подхода.*
Таб. 4. Рис. 1. Библиогр. 4 назв.

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## ЗМІСТ

Наукове видання

**ОПІР МАТЕРІАЛІВ І ТЕОРІЯ СПОРУД**
**STRENGTH OF MATERIALS AND THEORY OF STRUCTURES**

Науково-технічний збірник
Scientific-and-technical collected articles

Випуск 104

Головний редактор В.А.Баженов

Підп. до друку 10.09.20. Формат 60x90 $^{1}/_{16}$. Папір друк №1.
Друк офсетний. Умовн. друк. арк.
Тираж 100.

КНУБА
м. Київ, Повітрофлотський пр., 31. 03680